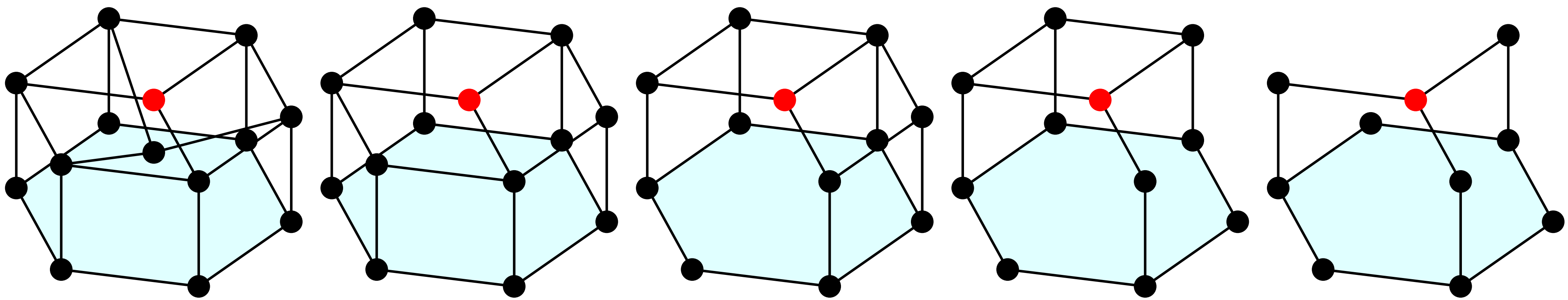
Oriented matroids and beyond

Kolja Knauer
LIS, Aix-Marseille Université

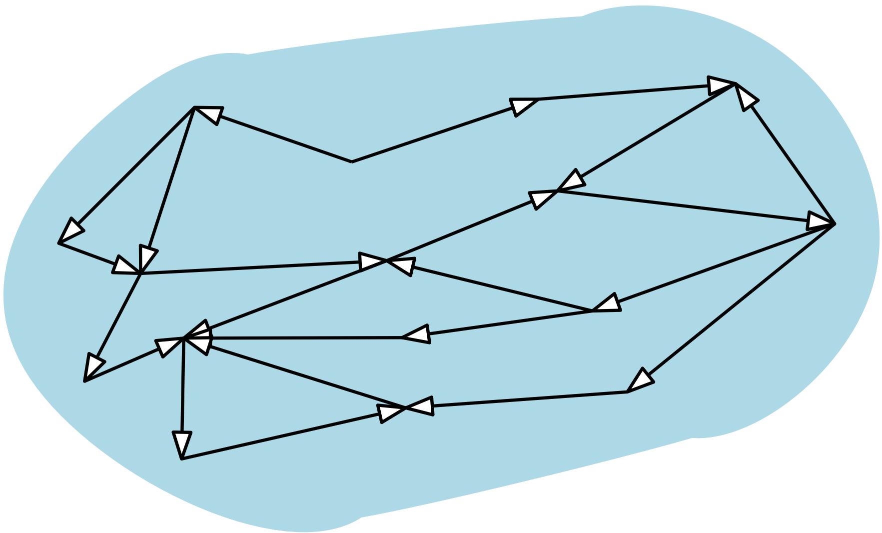
Hans-Jürgen Bandelt
Universität Hamburg
Victor Chepoi
LIS, Aix-Marseille Université
Tilen Marc
FMF, Univerza v Ljubljani



Séminaire Francilien de Géométrie Algorithmique et Combinatoire
Février 8, 2018

Graphic oriented matroids

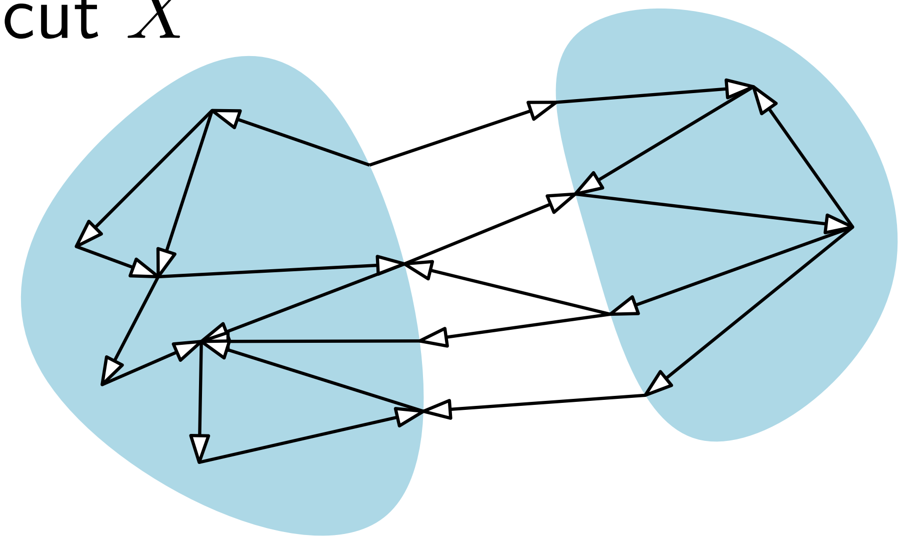
digraph $D = (V, E)$



Graphic oriented matroids

digraph $D = (V, E)$

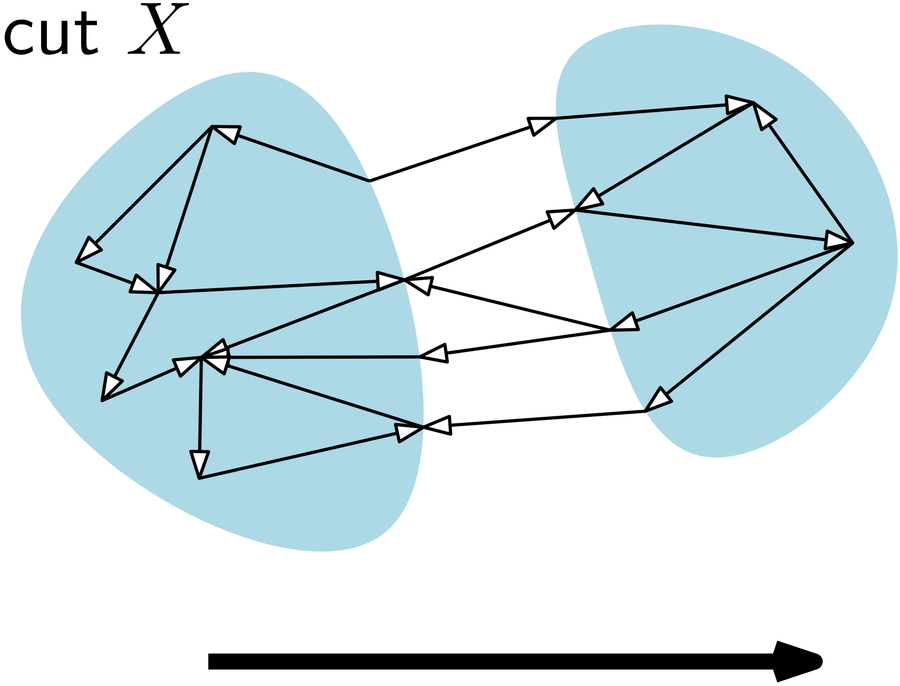
minimal edge cut X



Graphic oriented matroids

digraph $D = (V, E)$

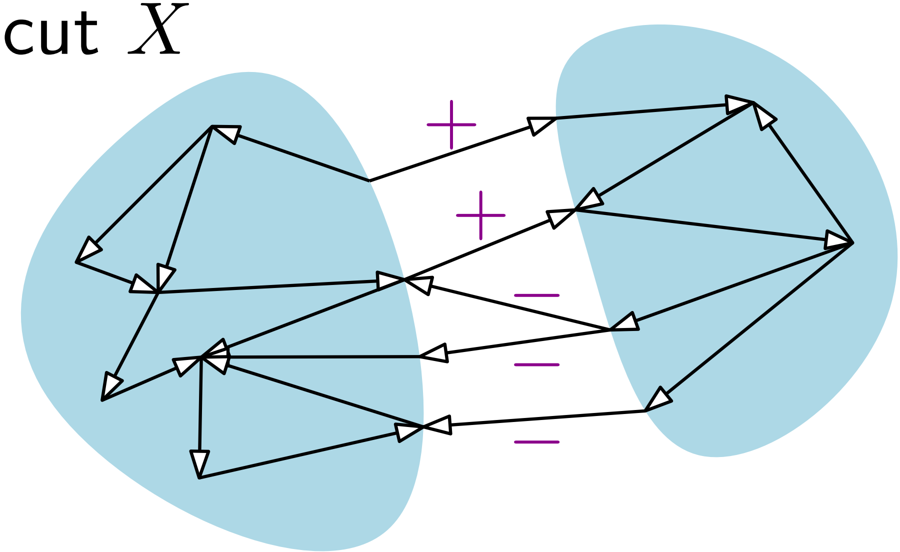
minimal edge cut X



Graphic oriented matroids

digraph $D = (V, E)$

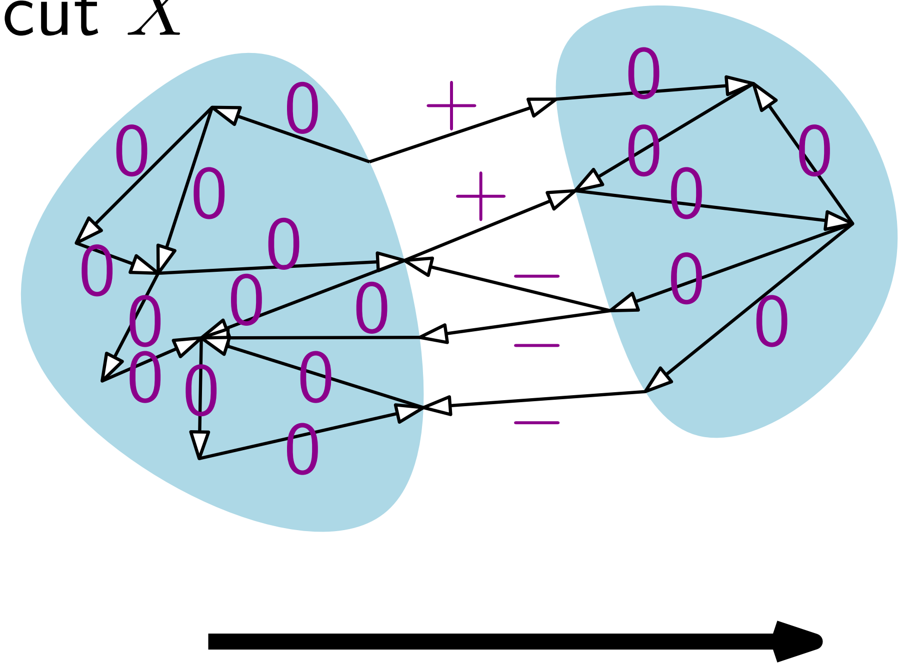
minimal edge cut X



Graphic oriented matroids

digraph $D = (V, E)$

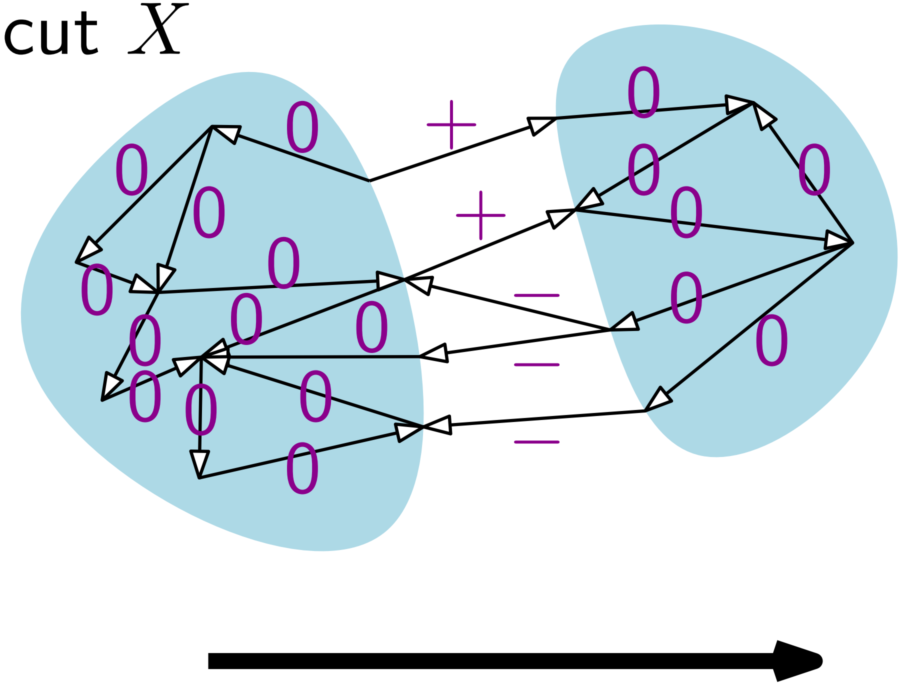
minimal edge cut X



Graphic oriented matroids

digraph $D = (V, E)$

minimal edge cut X



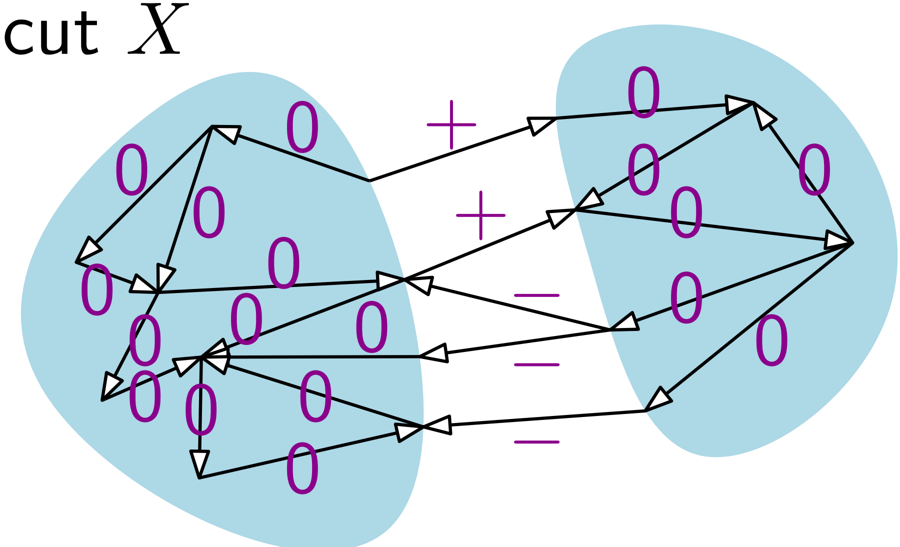
$$\begin{pmatrix} 0 \\ + \\ 0 \\ \vdots \\ 0 \\ - \\ - \\ - \\ + \end{pmatrix} \in \{\pm, 0\}^E$$

Graphic oriented matroids

digraph $D = (V, E)$

minimal edge cut X

cocircuit $X \in \mathcal{C}^*$



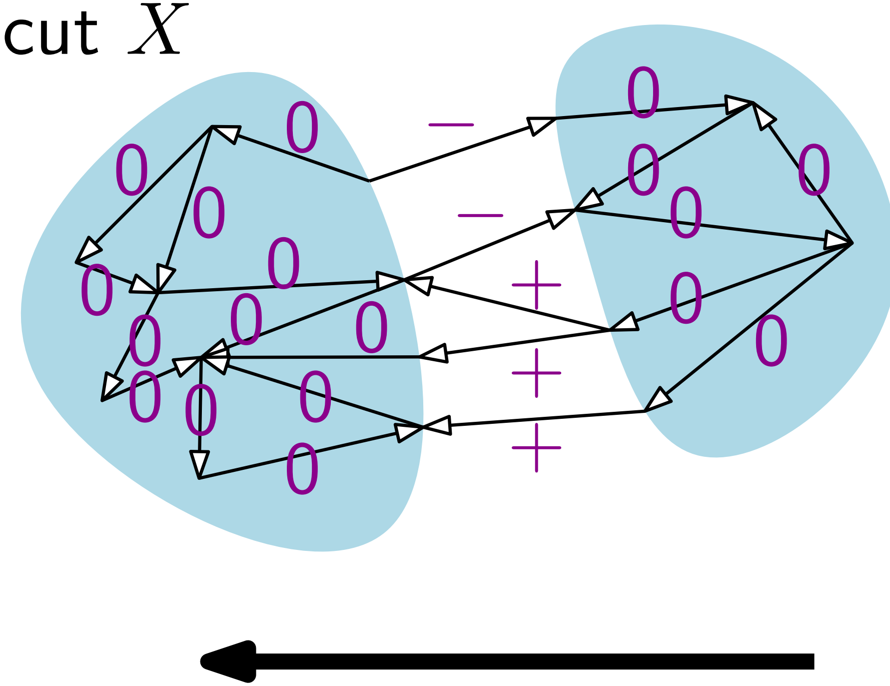
$$\begin{pmatrix} 0 \\ + \\ 0 \\ \vdots \\ 0 \\ - \\ - \\ - \\ + \end{pmatrix} \in \{\pm, 0\}^E$$

Graphic oriented matroids

digraph $D = (V, E)$

minimal edge cut X

cocircuit $X \in \mathcal{C}^*$



$$\begin{pmatrix} 0 \\ + \\ 0 \\ \vdots \\ 0 \\ - \\ - \\ - \\ + \end{pmatrix} \in \{\pm, 0\}^E$$

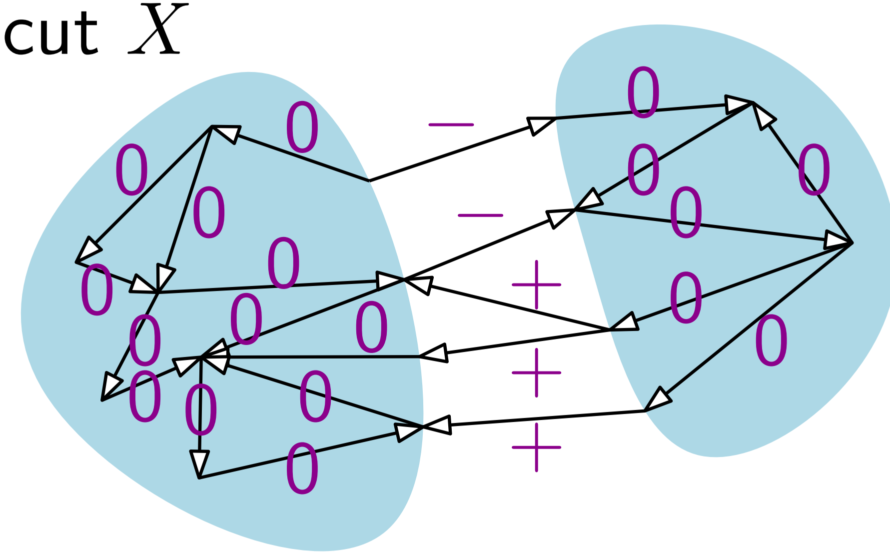
Graphic oriented matroids

digraph $D = (V, E)$

minimal edge cut X

cocircuit $X \in \mathcal{C}^*$

cocircuit $-X \in \mathcal{C}^*$



$$\begin{pmatrix} 0 \\ - \\ 0 \\ \vdots \\ 0 \\ + \\ + \\ + \\ - \end{pmatrix}, \begin{pmatrix} 0 \\ + \\ 0 \\ \vdots \\ 0 \\ - \\ - \\ - \\ + \end{pmatrix} \in \{\pm, 0\}^E$$

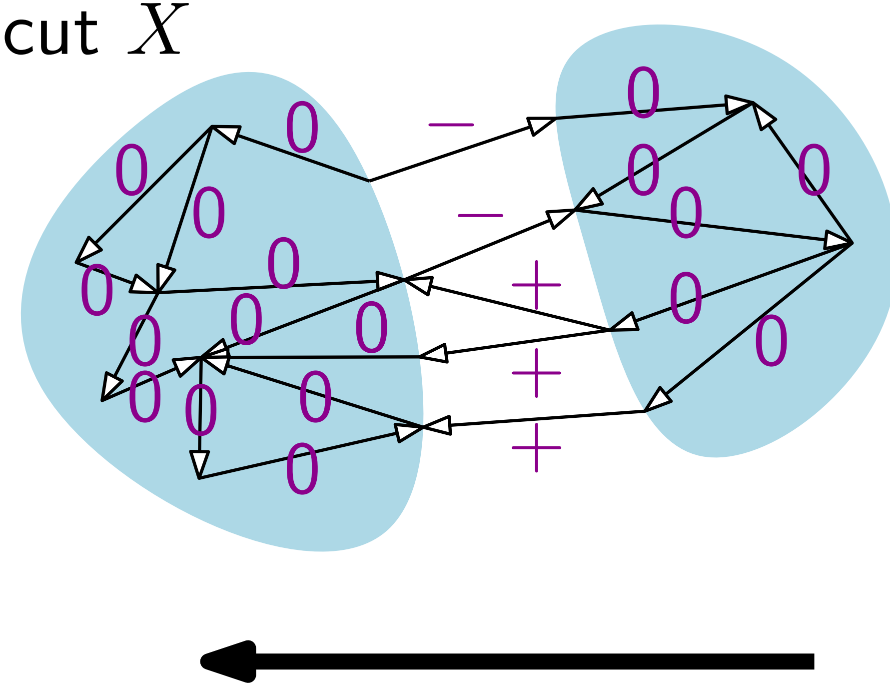
Graphic oriented matroids

digraph $D = (V, E)$

minimal edge cut X

cocircuit $X \in \mathcal{C}^*$

cocircuit $-X \in \mathcal{C}^*$



$$\begin{pmatrix} 0 \\ - \\ 0 \\ \vdots \\ 0 \\ + \\ + \\ + \\ - \end{pmatrix}, \begin{pmatrix} 0 \\ + \\ 0 \\ \vdots \\ 0 \\ - \\ - \\ - \\ + \end{pmatrix} \in \{\pm, 0\}^E$$

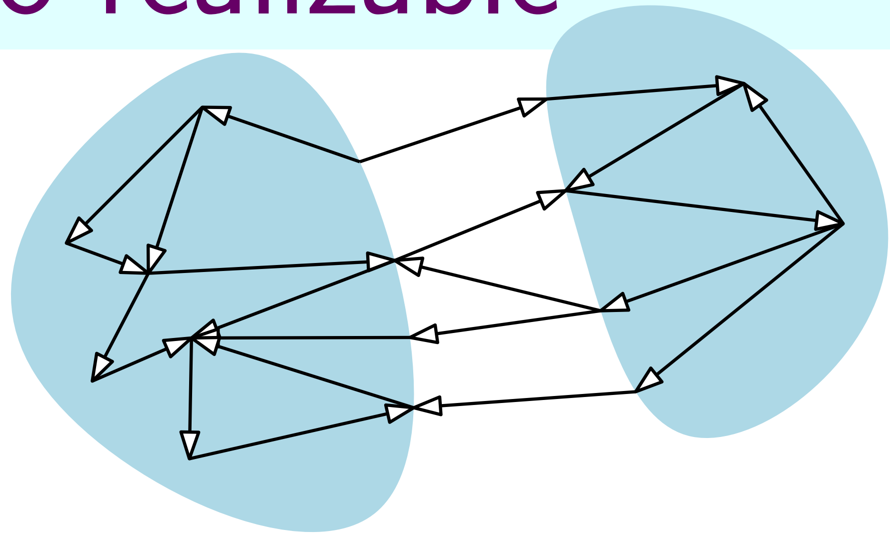
$\rightsquigarrow$ graphic oriented matroid of D: $\mathcal{M} = (E, \mathcal{C}^*)$

ground set

cocircuits

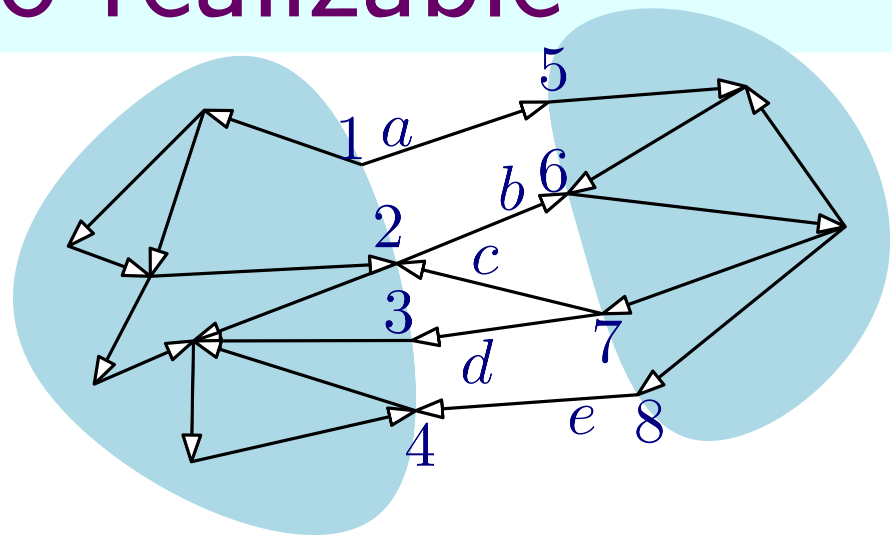
From graphic to realizable

digraph $D = (V, E)$
minimal edge cut X



From graphic to realizable

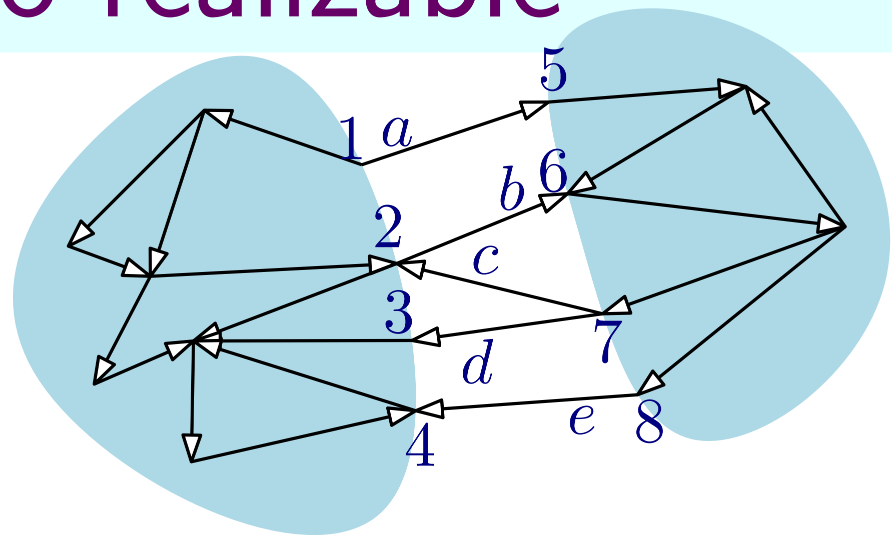
digraph $D = (V, E)$
minimal edge cut X
incidence matrix $I \in \{\pm 1, 0\}^{V \times E}$



From graphic to realizable

digraph $D = (V, E)$
minimal edge cut X
incidence matrix $I \in \{\pm 1, 0\}^{V \times E}$

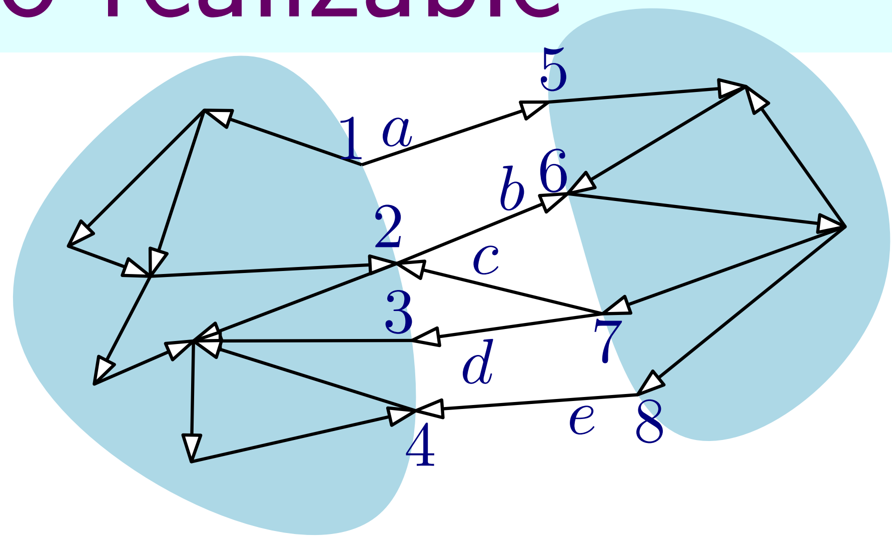
$$
\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ \vdots \end{array}
\begin{array}{c}
\begin{array}{cccccc} a & b & c & d & e & \cdots \end{array} \\
\begin{pmatrix}
- & 0 & 0 & 0 & 0 & \cdots \\
0 & - & + & 0 & 0 & \cdots \\
0 & 0 & 0 & + & 0 & \cdots \\
0 & 0 & 0 & 0 & + & \cdots \\
+ & 0 & 0 & 0 & 0 & \cdots \\
0 & + & 0 & 0 & 0 & \cdots \\
0 & 0 & - & - & 0 & \cdots \\
0 & 0 & 0 & 0 & - & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix}
\end{array}
$$



From graphic to realizable

digraph $D = (V, E)$
minimal edge cut X
incidence matrix $I \in \{\pm 1, 0\}^{V \times E}$

$$
\begin{array}{r}
\\
+1\cdot\, 1 \\
+1\cdot\, 2 \\
+1\cdot\, 3 \\
+1\cdot\, 4 \\
-1\cdot\, 5 \\
-1\cdot\, 6 \\
-1\cdot\, 7 \\
-1\cdot\, 8 \\
\vdots
\end{array}
\begin{array}{c}
\begin{array}{cccccc} a & b & c & d & e & \cdots \end{array} \\
\begin{pmatrix}
- & 0 & 0 & 0 & 0 & \cdots \\
0 & - & + & 0 & 0 & \cdots \\
0 & 0 & 0 & + & 0 & \cdots \\
0 & 0 & 0 & 0 & + & \cdots \\
+ & 0 & 0 & 0 & 0 & \cdots \\
0 & + & 0 & 0 & 0 & \cdots \\
0 & 0 & - & - & 0 & \cdots \\
0 & 0 & 0 & 0 & - & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix}
\end{array}
$$

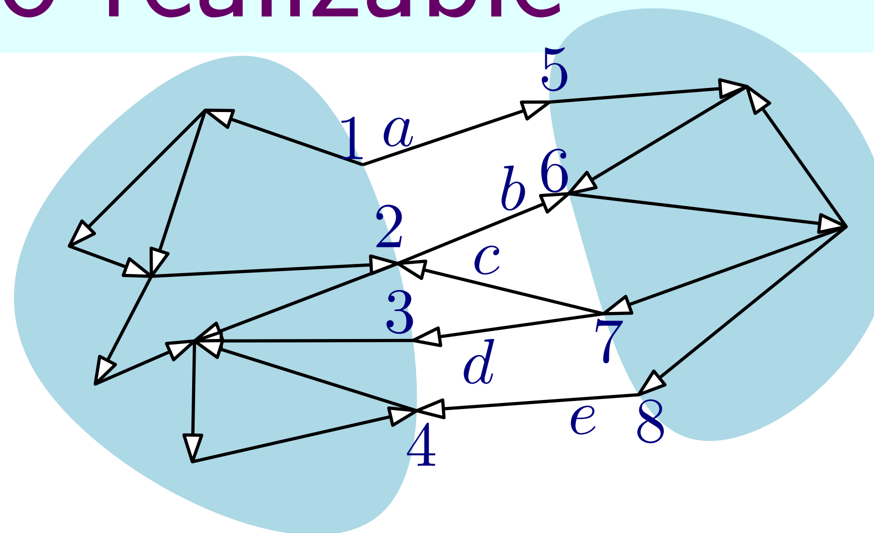


From graphic to realizable

digraph $D = (V, E)$
minimal edge cut X
incidence matrix $I \in \{\pm 1, 0\}^{V \times E}$

$$
\begin{array}{r}
 \\
+1 \cdot 1 \\
+1 \cdot 2 \\
+1 \cdot 3 \\
+1 \cdot 4 \\
-1 \cdot 5 \\
-1 \cdot 6 \\
-1 \cdot 7 \\
-1 \cdot 8 \\
\vdots
\end{array}
\begin{array}{c}
\begin{array}{cccccc} a & b & c & d & e & \cdots \end{array} \\
\begin{pmatrix}
- & 0 & 0 & 0 & 0 & \cdots \\
0 & - & + & 0 & 0 & \cdots \\
0 & 0 & 0 & + & 0 & \cdots \\
0 & 0 & 0 & 0 & + & \cdots \\
+ & 0 & 0 & 0 & 0 & \cdots \\
0 & + & 0 & 0 & 0 & \cdots \\
0 & 0 & - & - & 0 & \cdots \\
0 & 0 & 0 & 0 & - & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix}
\end{array}
$$

$$\Sigma = (-2 \; -2 \; +2 \; +2 \; +2 \quad ? \quad)$$



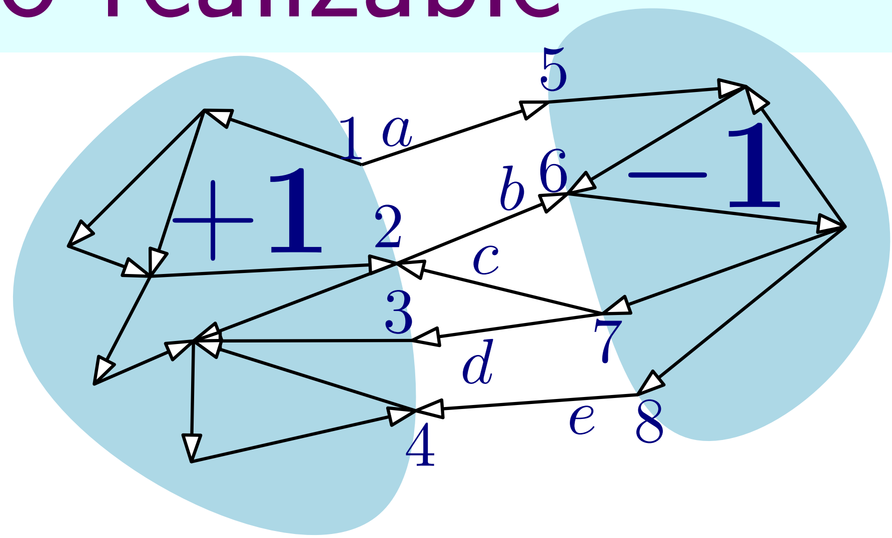
From graphic to realizable

digraph $D = (V, E)$
minimal edge cut X
incidence matrix $I \in \{\pm 1, 0\}^{V \times E}$

$$
\begin{array}{r}
 \\
+1\cdot 1 \\
+1\cdot 2 \\
+1\cdot 3 \\
+1\cdot 4 \\
-1\cdot 5 \\
-1\cdot 6 \\
-1\cdot 7 \\
-1\cdot 8 \\
\vdots
\end{array}
\begin{array}{cccccc}
a & b & c & d & e & \cdots \\
\end{array}
$$

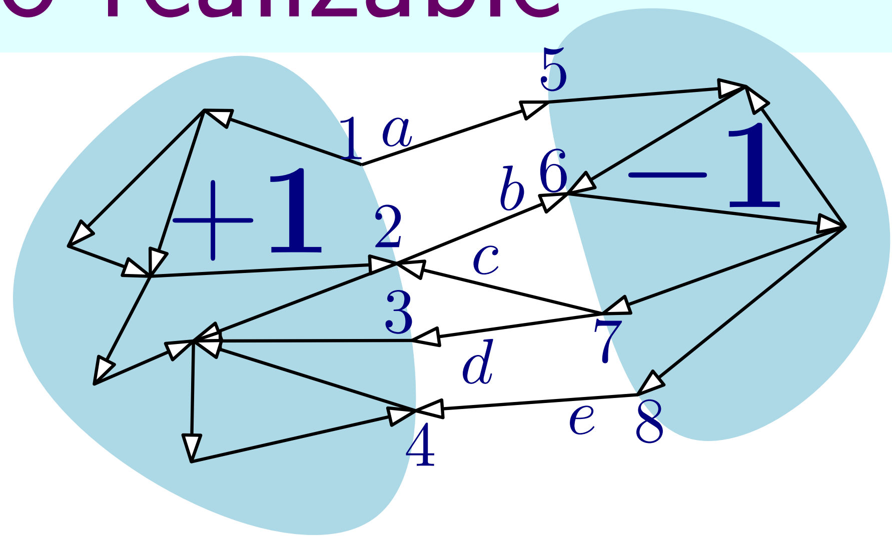
$$
\begin{pmatrix}
- & 0 & 0 & 0 & 0 & \cdots \\
0 & - & + & 0 & 0 & \cdots \\
0 & 0 & 0 & + & 0 & \cdots \\
0 & 0 & 0 & 0 & + & \cdots \\
+ & 0 & 0 & 0 & 0 & \cdots \\
0 & + & 0 & 0 & 0 & \cdots \\
0 & 0 & - & - & 0 & \cdots \\
0 & 0 & 0 & 0 & - & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix}
$$

$$\Sigma = (-2 \; -2 \; +2 \; +2 \; +2 \; 0 \ldots 0)$$



From graphic to realizable

digraph $D = (V, E)$
minimal edge cut X
incidence matrix $I \in \{\pm 1, 0\}^{V \times E}$



$$
\begin{array}{r c}
 & \begin{array}{cccccc} a & b & c & d & e & \cdots \end{array} \\
\begin{array}{r} +1\cdot 1 \\ +1\cdot 2 \\ +1\cdot 3 \\ +1\cdot 4 \\ -1\cdot 5 \\ -1\cdot 6 \\ -1\cdot 7 \\ -1\cdot 8 \\ \vdots \end{array} &
\begin{pmatrix}
- & 0 & 0 & 0 & 0 & \cdots \\
0 & - & + & 0 & 0 & \cdots \\
0 & 0 & 0 & + & 0 & \cdots \\
0 & 0 & 0 & 0 & + & \cdots \\
+ & 0 & 0 & 0 & 0 & \cdots \\
0 & + & 0 & 0 & 0 & \cdots \\
0 & 0 & - & - & 0 & \cdots \\
0 & 0 & 0 & 0 & - & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix}
\end{array}
$$

$$\Sigma = (-2 \ \ -2 \ \ +2 \ \ +2 \ \ +2 \ \ 0 \ldots 0)$$

$$\text{sgn}(\Sigma) = (- \ \ - \ \ + \ \ + \ \ + \ \ 0 \ldots 0)$$

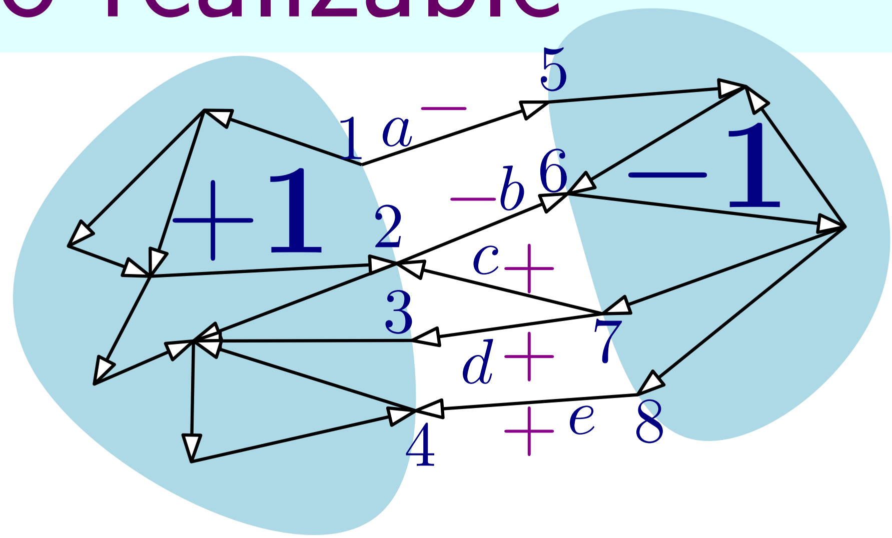
From graphic to realizable

digraph $D = (V, E)$
minimal edge cut X
incidence matrix $I \in \{\pm 1, 0\}^{V \times E}$

$$
\begin{array}{r} \\ +1\cdot 1 \\ +1\cdot 2 \\ +1\cdot 3 \\ +1\cdot 4 \\ -1\cdot 5 \\ -1\cdot 6 \\ -1\cdot 7 \\ -1\cdot 8 \\ \vdots \end{array}
\begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & \dots \end{array} \\
\begin{pmatrix}
- & 0 & 0 & 0 & 0 & \dots \\
0 & - & + & 0 & 0 & \dots \\
0 & 0 & 0 & + & 0 & \dots \\
0 & 0 & 0 & 0 & + & \dots \\
+ & 0 & 0 & 0 & 0 & \dots \\
0 & + & 0 & 0 & 0 & \dots \\
0 & 0 & - & - & 0 & \dots \\
0 & 0 & 0 & 0 & - & \dots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix} \end{array}
$$

$$\Sigma = (-2 \ -2 \ +2 \ +2 \ +2 \ 0 \dots 0)$$

$$\text{sgn}(\Sigma) = (- \ - \ + \ + \ + \ 0 \dots 0) = X \in \mathcal{C}^*$$



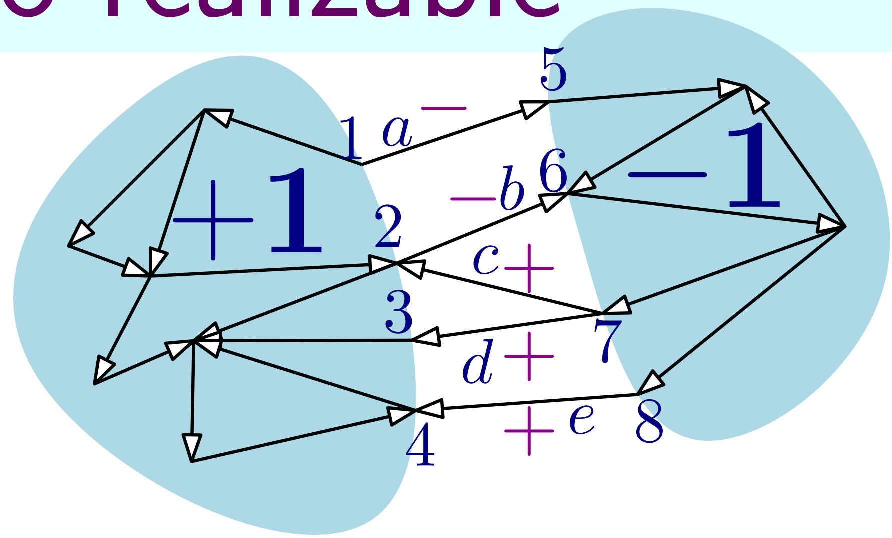
From graphic to realizable

digraph $D = (V, E)$
minimal edge cut X
incidence matrix $I \in \{\pm 1, 0\}^{V \times E}$

$$
\begin{array}{r}
\\
+1\cdot 1\\
+1\cdot 2\\
+1\cdot 3\\
+1\cdot 4\\
-1\cdot 5\\
-1\cdot 6\\
-1\cdot 7\\
-1\cdot 8\\
\vdots
\end{array}
\begin{array}{c}
\begin{array}{cccccc} a & b & c & d & e & \cdots \end{array}\\
\begin{pmatrix}
- & 0 & 0 & 0 & 0 & \cdots\\
0 & - & + & 0 & 0 & \cdots\\
0 & 0 & 0 & + & 0 & \cdots\\
0 & 0 & 0 & 0 & + & \cdots\\
+ & 0 & 0 & 0 & 0 & \cdots\\
0 & + & 0 & 0 & 0 & \cdots\\
0 & 0 & - & - & 0 & \cdots\\
0 & 0 & 0 & 0 & - & \cdots\\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix}
\end{array}
$$

$$\Sigma = (-2 \;\; -2 \;\; +2 \;\; +2 \;\; +2 \;\; 0 \dots 0)$$

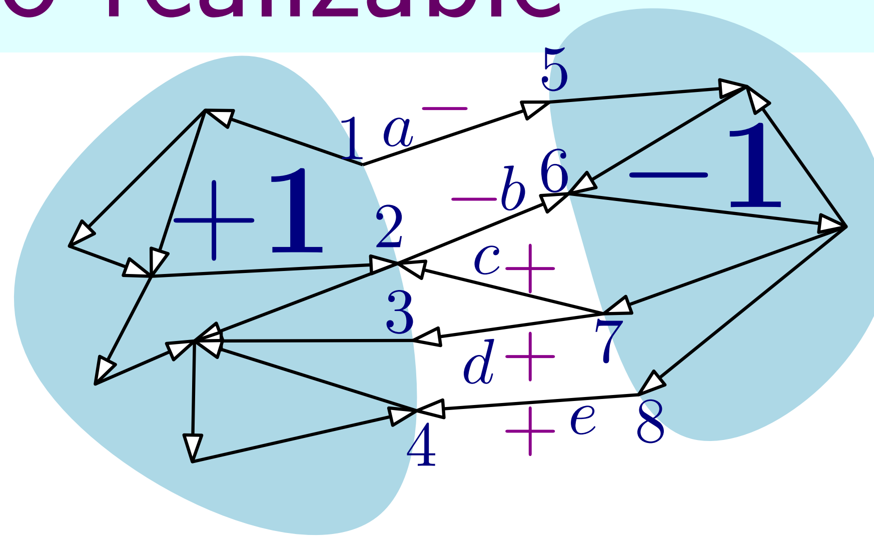
$$\mathrm{sgn}(\Sigma) = (- \;\; - \;\; + \;\; + \;\; + \;\; 0 \dots 0) = X \in \mathcal{C}^*$$



$\mathcal{C}^* :=$ support-minimal sign vectors of elements of row-space of I

From graphic to realizable

digraph $D = (V, E)$
minimal edge cut X
incidence matrix $I \in \{\pm 1, 0\}^{V \times E}$



$$
\begin{array}{r}
 \\ +1\cdot 1 \\ +1\cdot 2 \\ +1\cdot 3 \\ +1\cdot 4 \\ -1\cdot 5 \\ -1\cdot 6 \\ -1\cdot 7 \\ -1\cdot 8 \\ \vdots
\end{array}
\begin{array}{c}
\begin{array}{cccccc} a & b & c & d & e & \cdots \end{array} \\
\left(\begin{array}{cccccc}
- & 0 & 0 & 0 & 0 & \cdots \\
0 & - & + & 0 & 0 & \cdots \\
0 & 0 & 0 & + & 0 & \cdots \\
0 & 0 & 0 & 0 & + & \cdots \\
+ & 0 & 0 & 0 & 0 & \cdots \\
0 & + & 0 & 0 & 0 & \cdots \\
0 & 0 & - & - & 0 & \cdots \\
0 & 0 & 0 & 0 & - & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{array}\right)
\end{array}
$$

$$\Sigma = (-2 \;\; -2 \;\; +2 \;\; +2 \;\; +2 \;\; 0 \ldots 0)$$

$$\text{sgn}(\Sigma) = (\underbrace{- \;\; - \;\; + \;\; + \;\; +}\;\; 0 \ldots 0) = X \in \mathcal{C}^*$$

$\mathcal{C}^* :=$ support-minimal sign vectors
of elements of row-space of I

From graphic to realizable

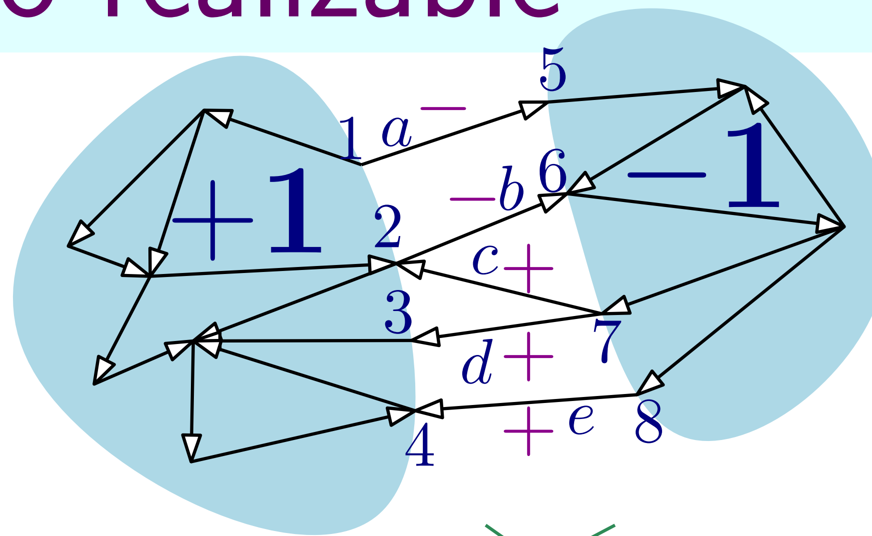
digraph $D = (V, E)$
minimal edge cut X
incidence matrix $I \in \{\pm 1, 0\}^{V \times E}$

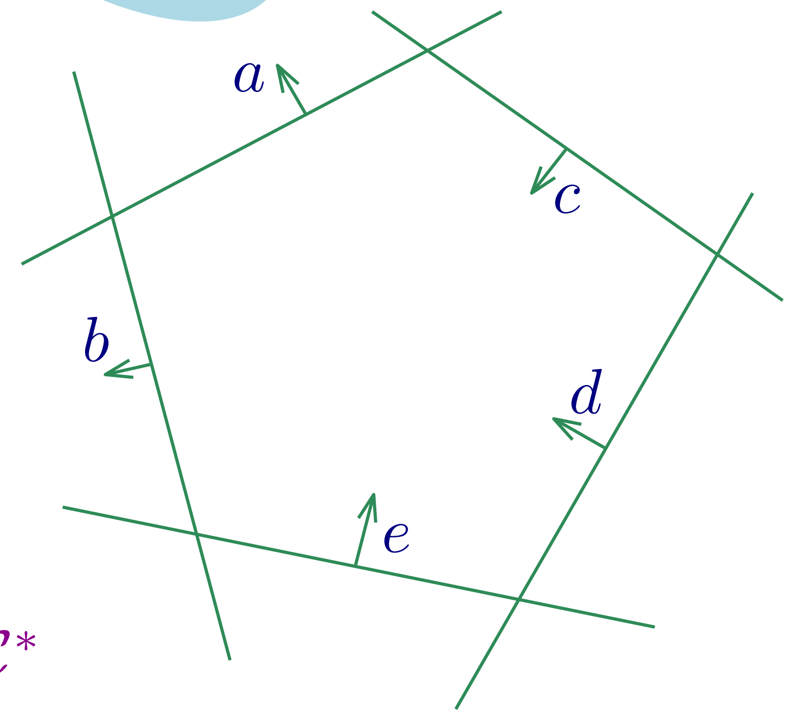
$$
\begin{array}{r}
 \\ +1\cdot 1 \\ +1\cdot 2 \\ +1\cdot 3 \\ +1\cdot 4 \\ -1\cdot 5 \\ -1\cdot 6 \\ -1\cdot 7 \\ -1\cdot 8 \\ \vdots
\end{array}
\begin{array}{c}
\begin{array}{cccccc} a & b & c & d & e & \cdots \end{array} \\
\begin{pmatrix}
- & 0 & 0 & 0 & 0 & \cdots \\
0 & - & + & 0 & 0 & \cdots \\
0 & 0 & 0 & + & 0 & \cdots \\
0 & 0 & 0 & 0 & + & \cdots \\
+ & 0 & 0 & 0 & 0 & \cdots \\
0 & + & 0 & 0 & 0 & \cdots \\
0 & 0 & - & - & 0 & \cdots \\
0 & 0 & 0 & 0 & - & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix}
\end{array}
$$

$$\Sigma = (-2\ -2\ +2\ +2\ +2\ 0 \ldots 0)$$

$$\mathrm{sgn}(\Sigma) = (-\ -\ +\ +\ +\ 0 \ldots 0) = X \in \mathcal{C}^*$$

$\mathcal{C}^* :=$ support-minimal sign vectors of elements of row-space of I





From graphic to realizable

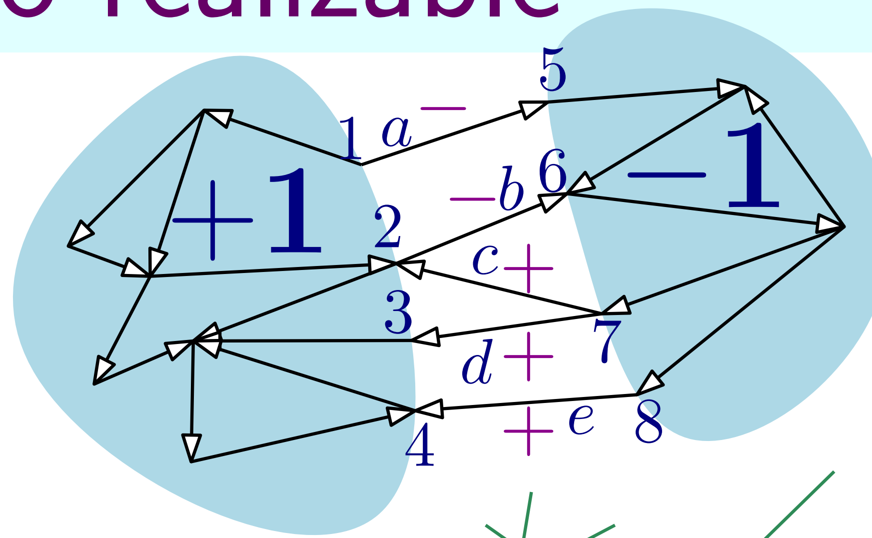
digraph $D = (V, E)$
minimal edge cut X
incidence matrix $I \in \{\pm 1, 0\}^{V \times E}$

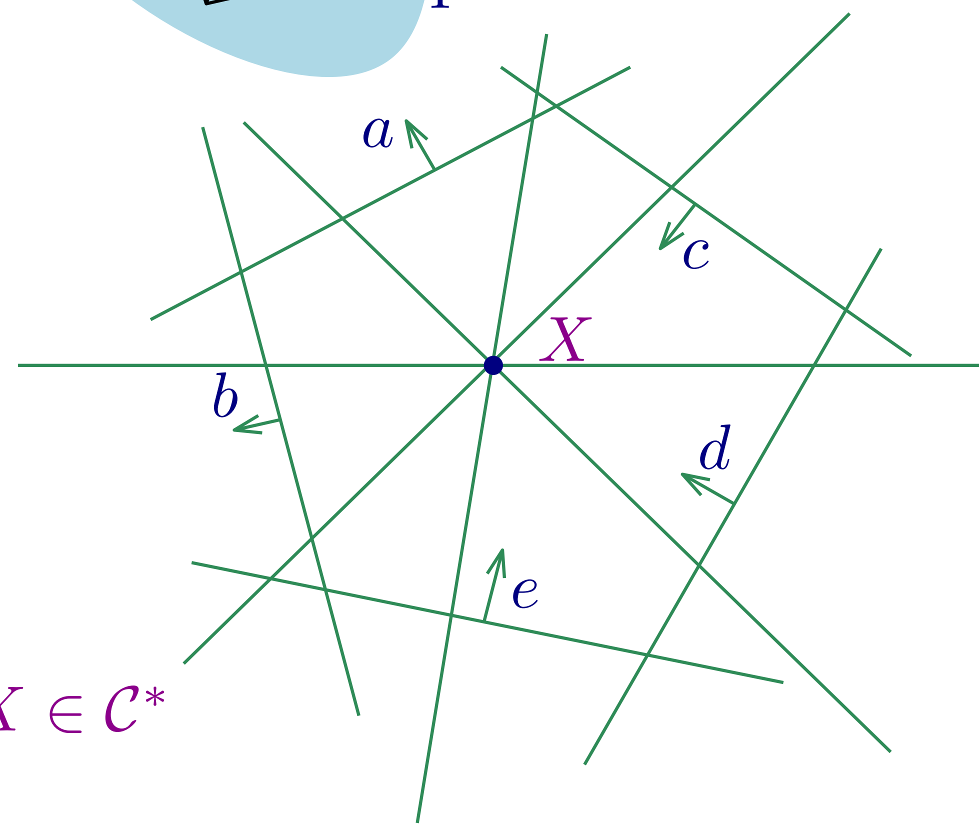
$$
\begin{array}{r}
 \\ +1\cdot 1 \\ +1\cdot 2 \\ +1\cdot 3 \\ +1\cdot 4 \\ -1\cdot 5 \\ -1\cdot 6 \\ -1\cdot 7 \\ -1\cdot 8 \\ \vdots
\end{array}
\begin{array}{c}
\begin{array}{cccccc} a & b & c & d & e & \cdots \end{array} \\
\left(\begin{array}{cccccc}
- & 0 & 0 & 0 & 0 & \cdots \\
0 & - & + & 0 & 0 & \cdots \\
0 & 0 & 0 & + & 0 & \cdots \\
0 & 0 & 0 & 0 & + & \cdots \\
+ & 0 & 0 & 0 & 0 & \cdots \\
0 & + & 0 & 0 & 0 & \cdots \\
0 & 0 & - & - & 0 & \cdots \\
0 & 0 & 0 & 0 & - & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{array}\right)
\end{array}
$$

$$\Sigma = (-2 \; -2 \; +2 \; +2 \; +2 \; 0 \ldots 0)$$

$$\mathrm{sgn}(\Sigma) = (- \; - \; + \; + \; + \; 0 \ldots 0) = X \in \mathcal{C}^*$$

$\mathcal{C}^* :=$ support-minimal sign vectors
of elements of row-space of I





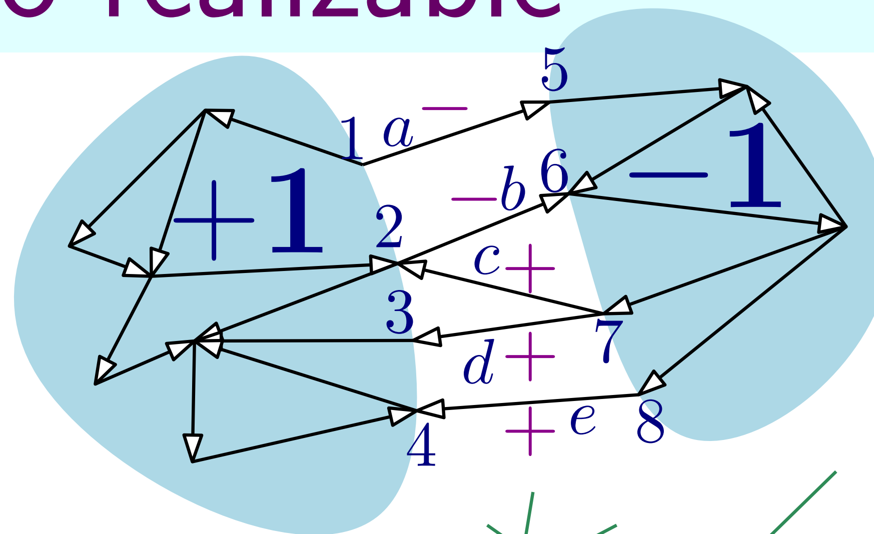
From graphic to realizable

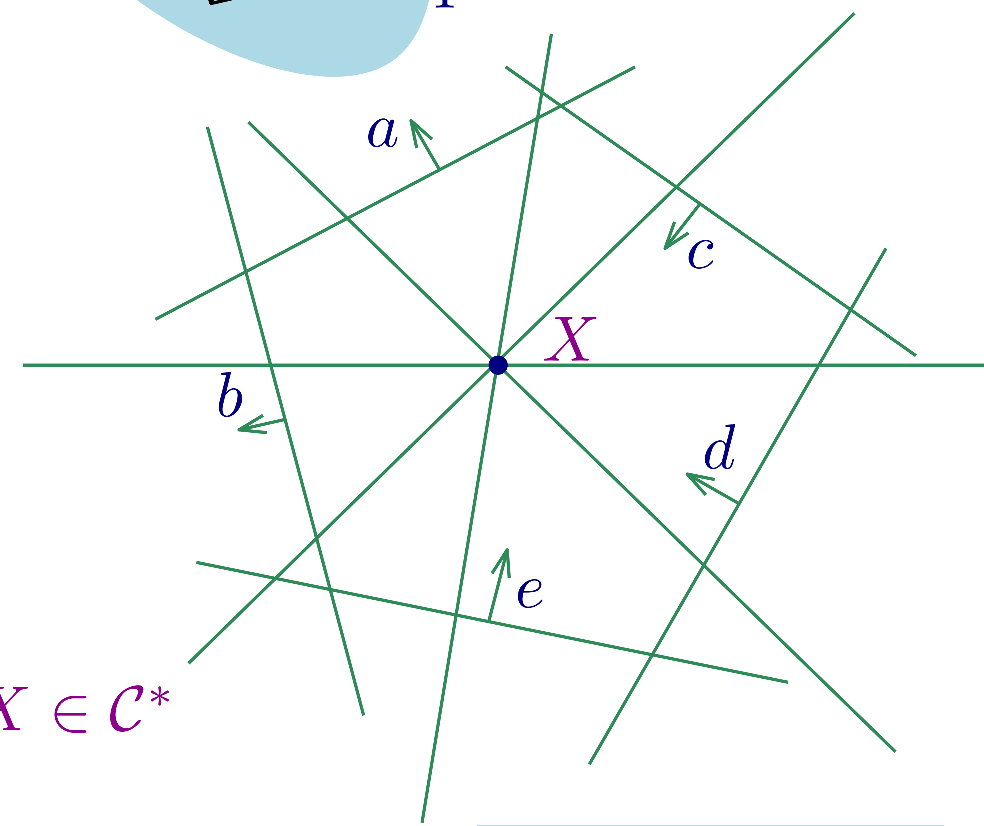
digraph $D = (V, E)$
minimal edge cut X
incidence matrix $I \in \{\pm 1, 0\}^{V \times E}$

$$
\begin{array}{r}
 \\ +1\cdot 1 \\ +1\cdot 2 \\ +1\cdot 3 \\ +1\cdot 4 \\ -1\cdot 5 \\ -1\cdot 6 \\ -1\cdot 7 \\ -1\cdot 8 \\ \vdots
\end{array}
\begin{array}{c}
\begin{array}{cccccc} a & b & c & d & e & \cdots \end{array} \\
\begin{pmatrix}
- & 0 & 0 & 0 & 0 & \cdots \\
0 & - & + & 0 & 0 & \cdots \\
0 & 0 & 0 & + & 0 & \cdots \\
0 & 0 & 0 & 0 & + & \cdots \\
+ & 0 & 0 & 0 & 0 & \cdots \\
0 & + & 0 & 0 & 0 & \cdots \\
0 & 0 & - & - & 0 & \cdots \\
0 & 0 & 0 & 0 & - & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix}
\end{array}
$$

$$\Sigma = (-2 \; -2 \; +2 \; +2 \; +2 \; 0 \ldots 0)$$

$$\mathrm{sgn}(\Sigma) = (- \; - \; + \; + \; + \; 0 \ldots 0) = X \in \mathcal{C}^*$$





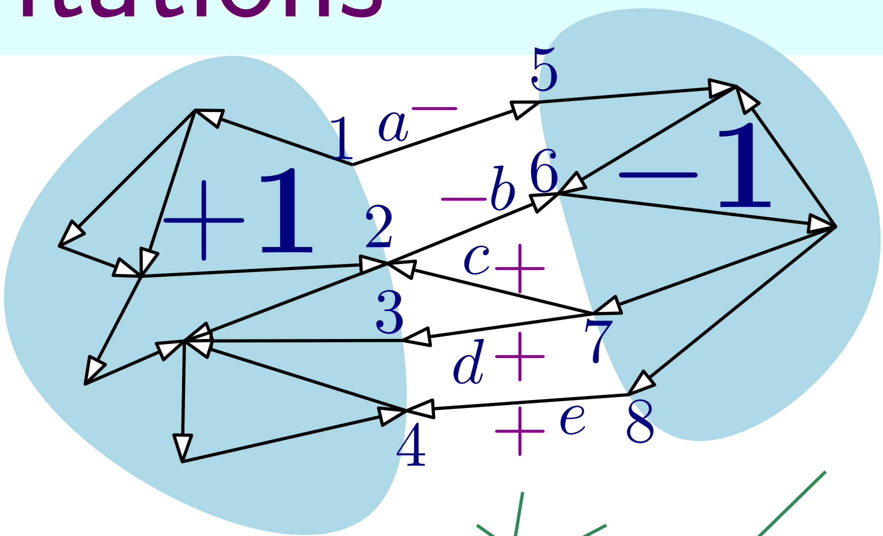
$\mathcal{C}^* :=$ support-minimal sign vectors of elements of row-space of I $=$ sign vectors of min-dimensional cells of hyperplane arrangement

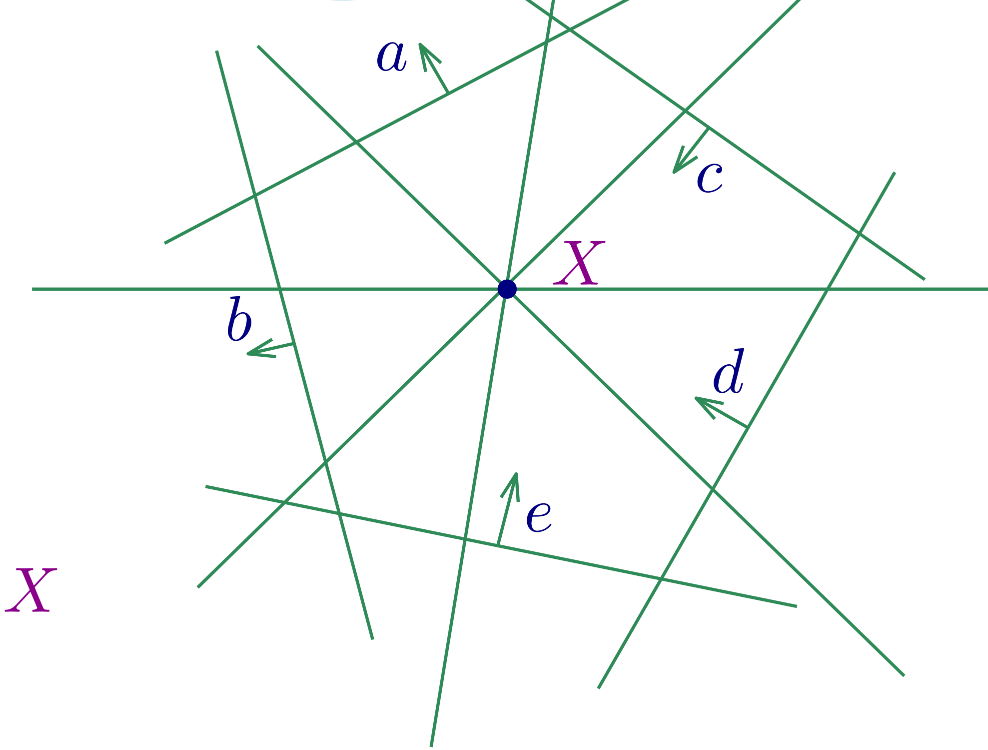
Acyclic orientations

digraph $D = (V, E)$
minimal edge cut X
incidence matrix $I \in \{\pm 1, 0\}^{V \times E}$

$$
\begin{array}{r} \\ +1\cdot 1 \\ +1\cdot 2 \\ +1\cdot 3 \\ +1\cdot 4 \\ -1\cdot 5 \\ -1\cdot 6 \\ -1\cdot 7 \\ -1\cdot 8 \\ \vdots \end{array}
\begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & \cdots \end{array} \\
\left(\begin{array}{cccccc}
- & 0 & 0 & 0 & 0 & \cdots \\
0 & - & + & 0 & 0 & \cdots \\
0 & 0 & 0 & + & 0 & \cdots \\
0 & 0 & 0 & 0 & + & \cdots \\
+ & 0 & 0 & 0 & 0 & \cdots \\
0 & + & 0 & 0 & 0 & \cdots \\
0 & 0 & - & - & 0 & \cdots \\
0 & 0 & 0 & 0 & - & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{array} \right) \end{array}
$$

$$\mathrm{sgn}(\Sigma) = (- \quad - \quad + \quad + \quad + \quad 0 \ldots 0) = X$$



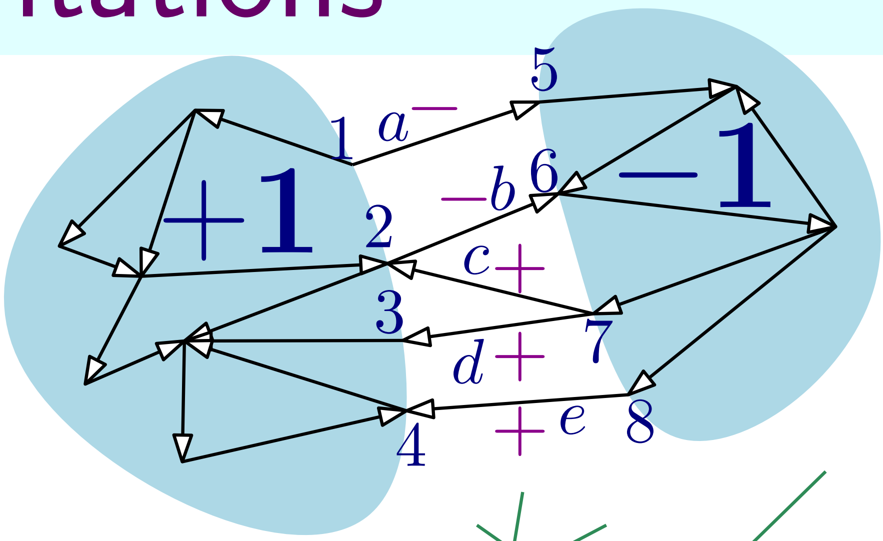


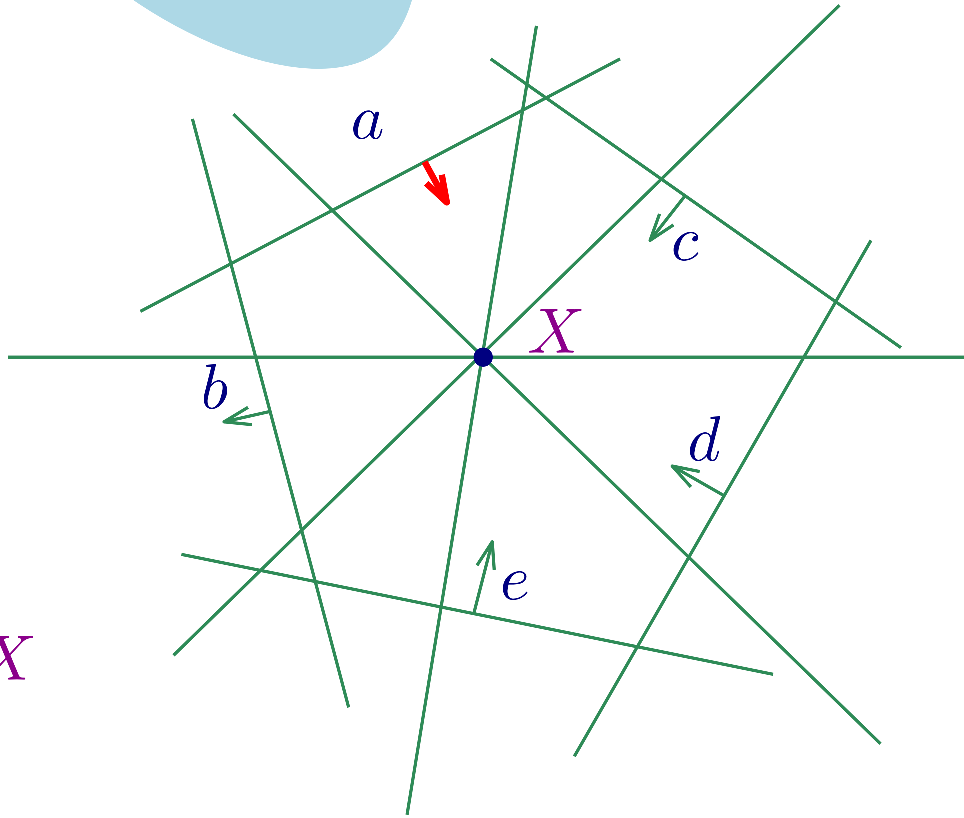
Acyclic orientations

digraph $D = (V, E)$
minimal edge cut X
incidence matrix $I \in \{\pm 1, 0\}^{V \times E}$

$$
\begin{array}{r} \\ +1\cdot 1 \\ +1\cdot 2 \\ +1\cdot 3 \\ +1\cdot 4 \\ -1\cdot 5 \\ -1\cdot 6 \\ -1\cdot 7 \\ -1\cdot 8 \\ \vdots \end{array}
\begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & \cdots \end{array} \\
\begin{pmatrix}
- & 0 & 0 & 0 & 0 & \cdots \\
0 & - & + & 0 & 0 & \cdots \\
0 & 0 & 0 & + & 0 & \cdots \\
0 & 0 & 0 & 0 & + & \cdots \\
+ & 0 & 0 & 0 & 0 & \cdots \\
0 & + & 0 & 0 & 0 & \cdots \\
0 & 0 & - & - & 0 & \cdots \\
0 & 0 & 0 & 0 & - & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix} \end{array}
$$

$$\mathrm{sgn}(\Sigma) = (- \quad - \quad + \quad + \quad + \quad 0 \ldots 0) = X$$



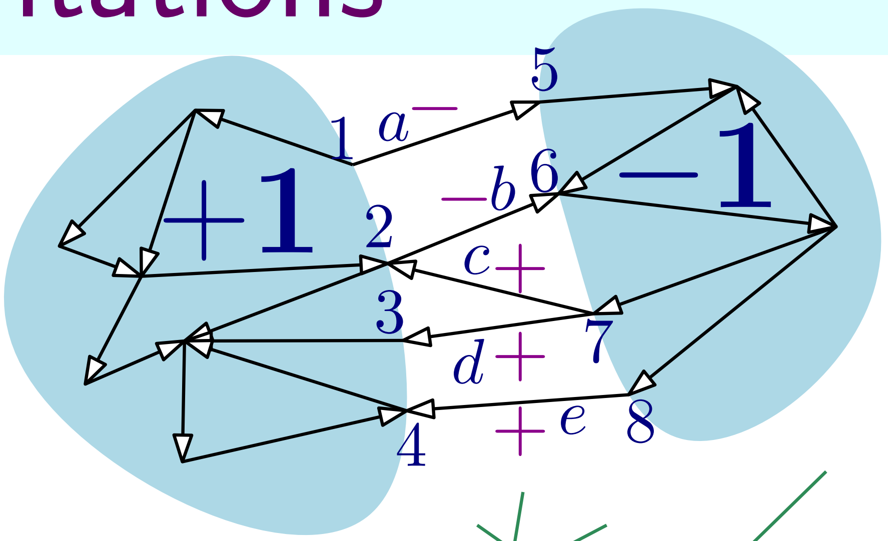


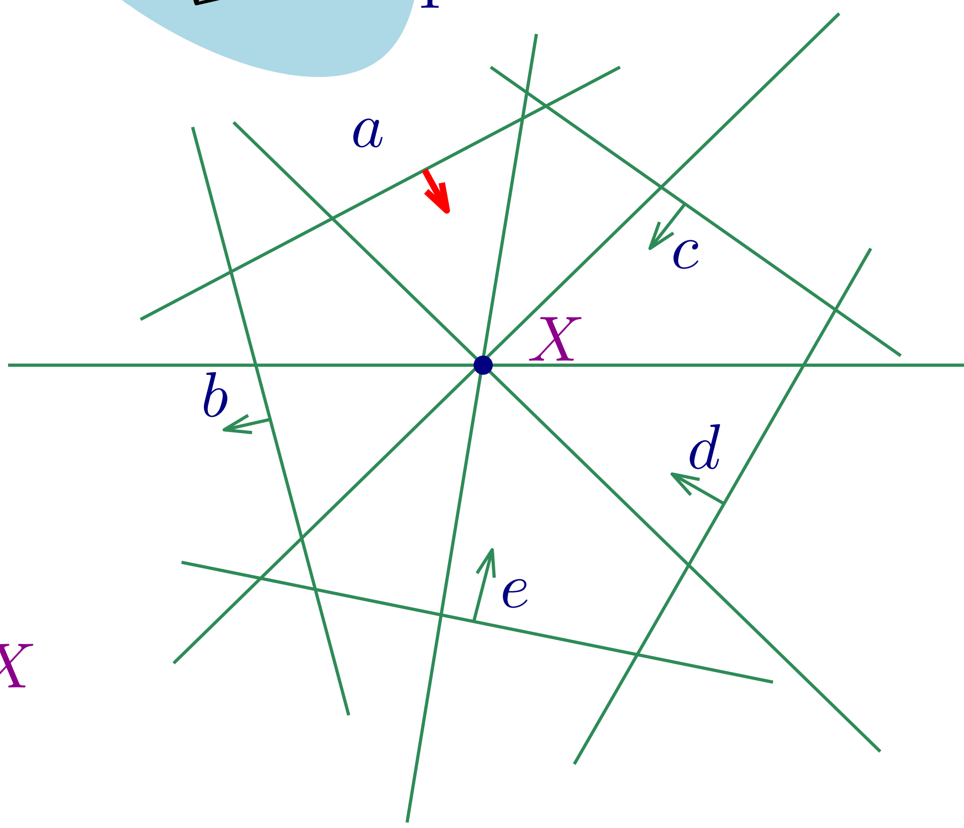
Acyclic orientations

digraph $D = (V, E)$
minimal edge cut X
incidence matrix $I \in \{\pm 1, 0\}^{V \times E}$

$$
\begin{array}{r}
 \\ +1\cdot\, 1 \\ +1\cdot\, 2 \\ +1\cdot\, 3 \\ +1\cdot\, 4 \\ -1\cdot\, 5 \\ -1\cdot\, 6 \\ -1\cdot\, 7 \\ -1\cdot\, 8 \\ \vdots
\end{array}
\begin{array}{c}
\begin{array}{cccccc} a & b & c & d & e & \cdots \end{array} \\
\begin{pmatrix}
+ & 0 & 0 & 0 & 0 & \cdots \\
0 & - & + & 0 & 0 & \cdots \\
0 & 0 & 0 & + & 0 & \cdots \\
0 & 0 & 0 & 0 & + & \cdots \\
- & 0 & 0 & 0 & 0 & \cdots \\
0 & + & 0 & 0 & 0 & \cdots \\
0 & 0 & - & - & 0 & \cdots \\
0 & 0 & 0 & 0 & - & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix}
\end{array}
$$

$$\mathrm{sgn}(\Sigma) = (- \quad - \quad + \quad + \quad + \quad 0 \ldots 0) = X$$



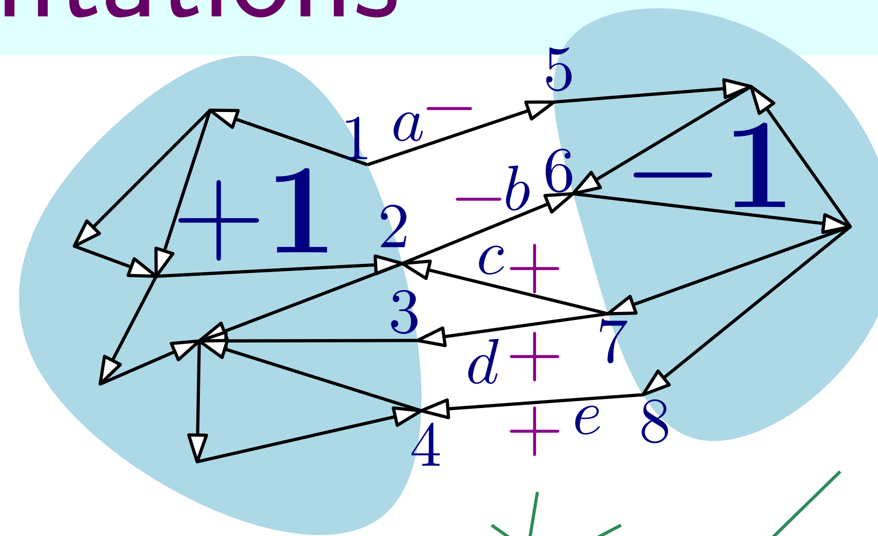


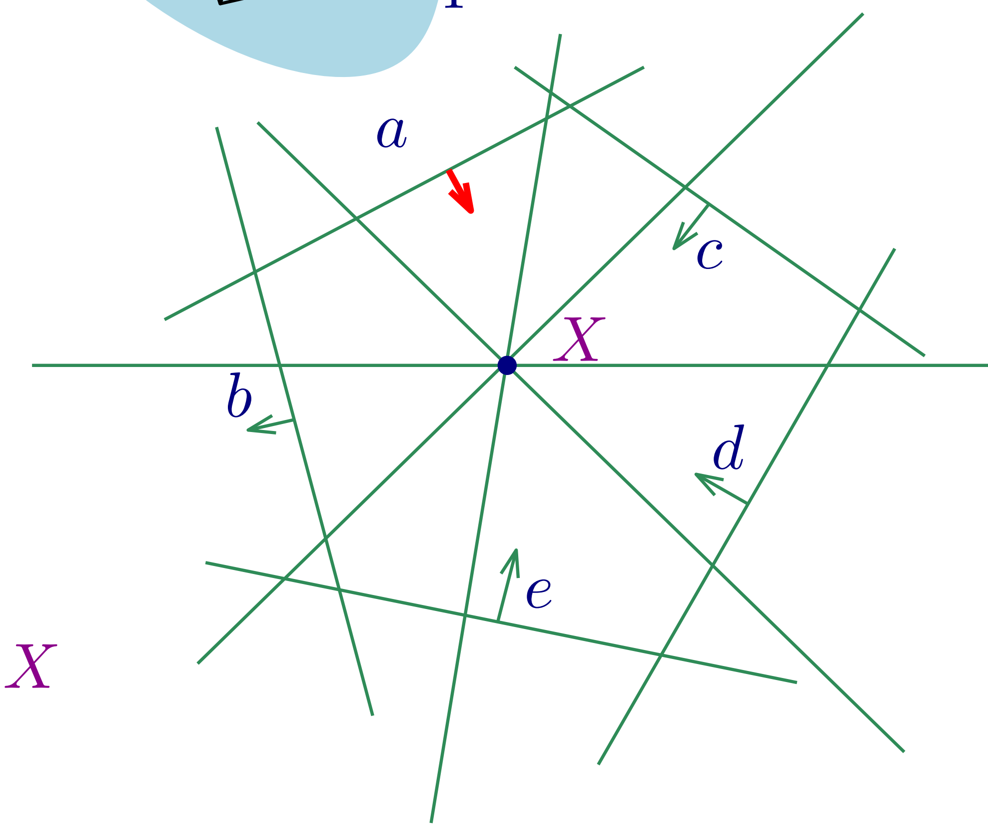
Acyclic orientations

digraph $D = (V, E)$
minimal edge cut X
incidence matrix $I \in \{\pm 1, 0\}^{V \times E}$

$$
\begin{array}{r}
\\
+1\cdot 1\\ +1\cdot 2\\ +1\cdot 3\\ +1\cdot 4\\ -1\cdot 5\\ -1\cdot 6\\ -1\cdot 7\\ -1\cdot 8\\ \vdots
\end{array}
\begin{array}{c}
\begin{array}{cccccc} a & b & c & d & e & \cdots \end{array}\\
\begin{pmatrix}
+ & 0 & 0 & 0 & 0 & \cdots\\
0 & - & + & 0 & 0 & \cdots\\
0 & 0 & 0 & + & 0 & \cdots\\
0 & 0 & 0 & 0 & + & \cdots\\
- & 0 & 0 & 0 & 0 & \cdots\\
0 & + & 0 & 0 & 0 & \cdots\\
0 & 0 & - & - & 0 & \cdots\\
0 & 0 & 0 & 0 & - & \cdots\\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix}
\end{array}
$$

$$\mathrm{sgn}(\Sigma) = (+ \quad - \quad + \quad + \quad + \quad 0 \ldots 0) = X$$



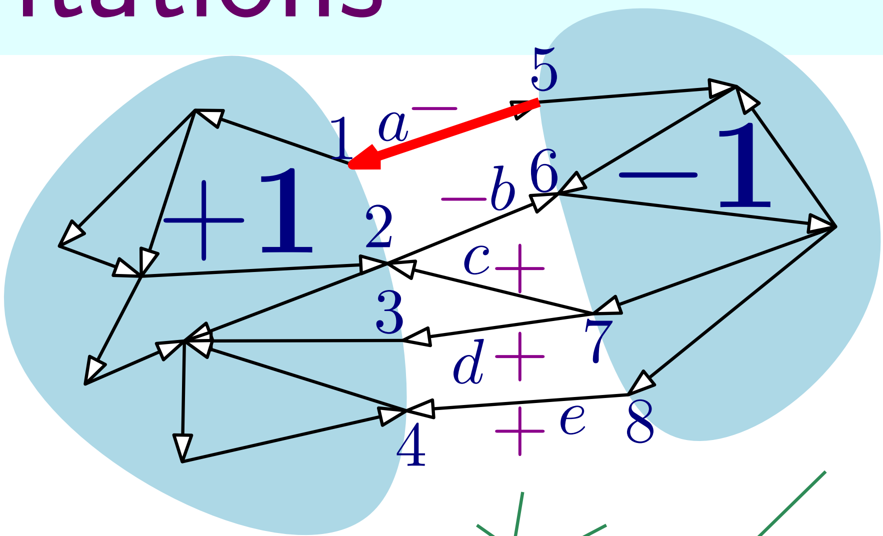


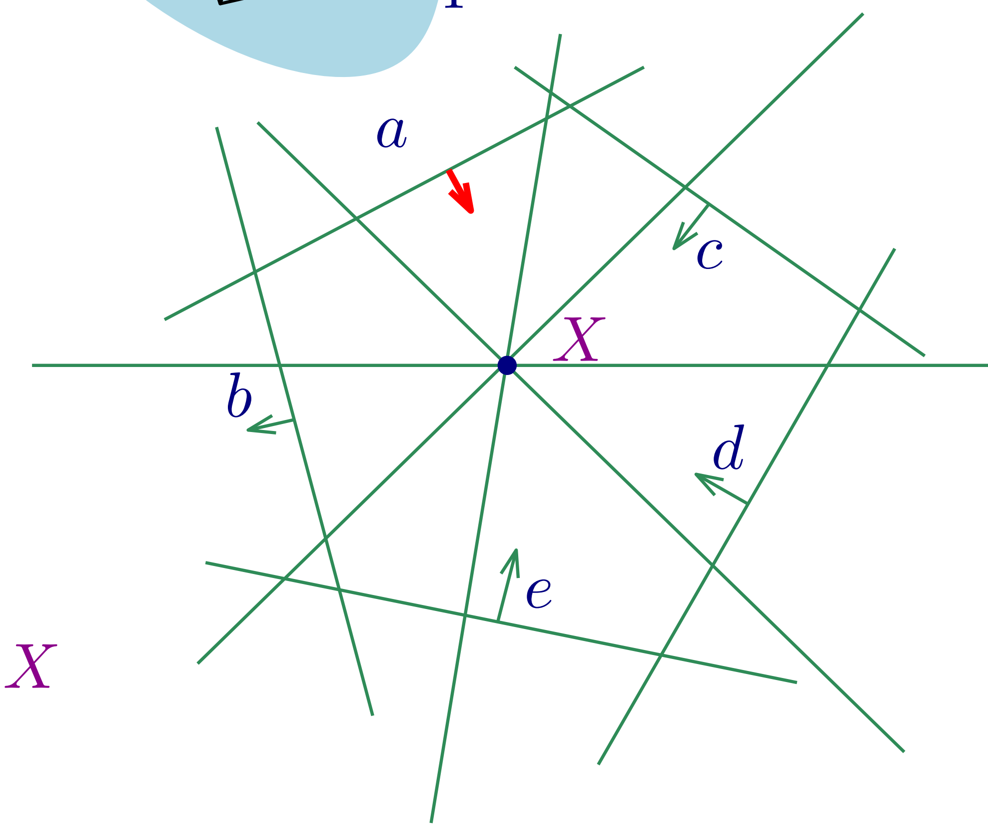
Acyclic orientations

digraph $D = (V, E)$
minimal edge cut X
incidence matrix $I \in \{\pm 1, 0\}^{V \times E}$

$$
\begin{array}{r}
\\
+1\cdot 1 \\ +1\cdot 2 \\ +1\cdot 3 \\ +1\cdot 4 \\ -1\cdot 5 \\ -1\cdot 6 \\ -1\cdot 7 \\ -1\cdot 8 \\ \vdots
\end{array}
\begin{array}{c}
\begin{array}{cccccc} a & b & c & d & e & \cdots \end{array} \\
\left(\begin{array}{cccccc}
+ & 0 & 0 & 0 & 0 & \cdots \\
0 & - & + & 0 & 0 & \cdots \\
0 & 0 & 0 & + & 0 & \cdots \\
0 & 0 & 0 & 0 & + & \cdots \\
- & 0 & 0 & 0 & 0 & \cdots \\
0 & + & 0 & 0 & 0 & \cdots \\
0 & 0 & - & - & 0 & \cdots \\
0 & 0 & 0 & 0 & - & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{array}\right)
\end{array}
$$

$$\mathrm{sgn}(\Sigma) = (+ \quad - \quad + \quad + \quad + \quad 0 \ldots 0) = X$$



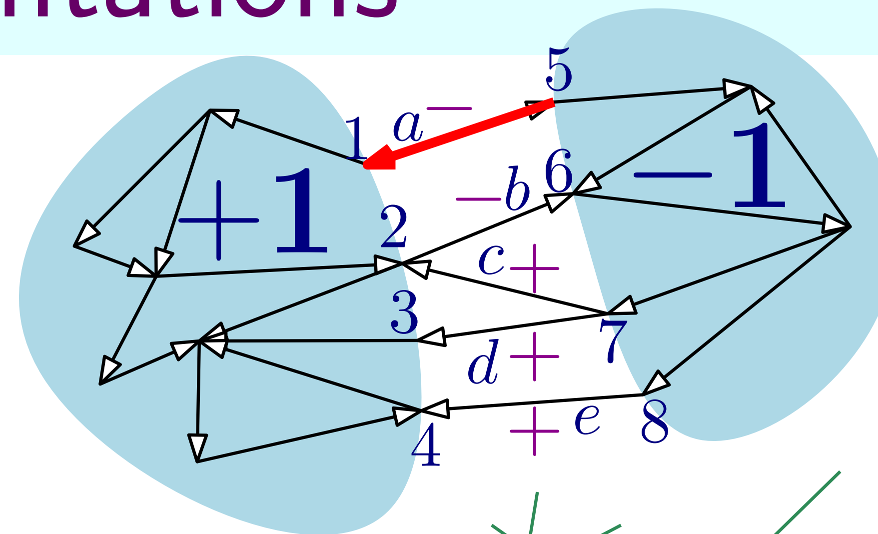


Acyclic orientations

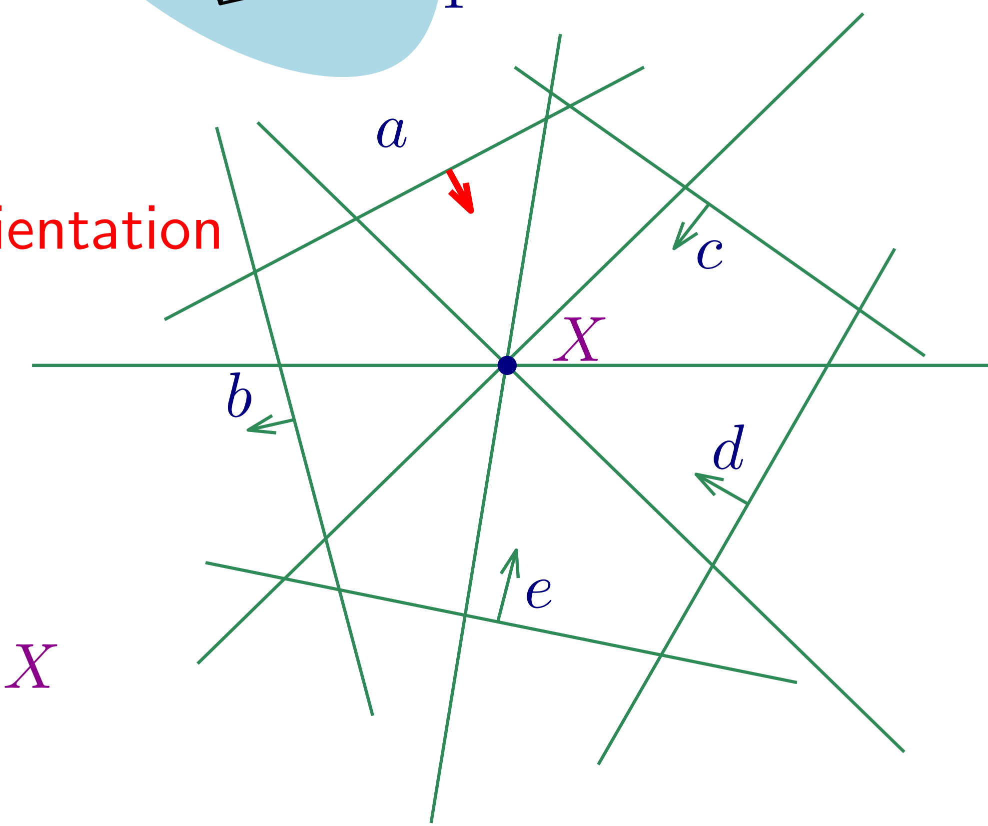
digraph $D = (V, E)$
minimal edge cut X
incidence matrix $I \in \{\pm 1, 0\}^{V \times E}$

$$
\begin{array}{r}
 \\
+1\cdot 1 \\
+1\cdot 2 \\
+1\cdot 3 \\
+1\cdot 4 \\
-1\cdot 5 \\
-1\cdot 6 \\
-1\cdot 7 \\
-1\cdot 8 \\
\vdots
\end{array}
\begin{array}{c}
\begin{array}{cccccc} a & b & c & d & e & \cdots \end{array} \\
\begin{pmatrix}
+ & 0 & 0 & 0 & 0 & \cdots \\
0 & - & + & 0 & 0 & \cdots \\
0 & 0 & 0 & + & 0 & \cdots \\
0 & 0 & 0 & 0 & + & \cdots \\
- & 0 & 0 & 0 & 0 & \cdots \\
0 & + & 0 & 0 & 0 & \cdots \\
0 & 0 & - & - & 0 & \cdots \\
0 & 0 & 0 & 0 & - & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix}
\end{array}
$$

$$\mathrm{sgn}(\Sigma) = (+ \quad - \quad + \quad + \quad + \quad 0 \ldots 0) = X$$



reorientation

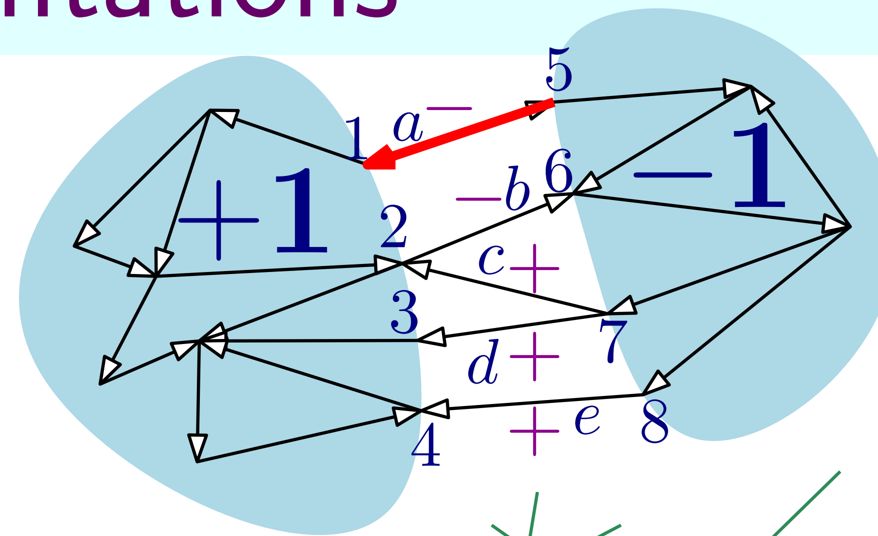


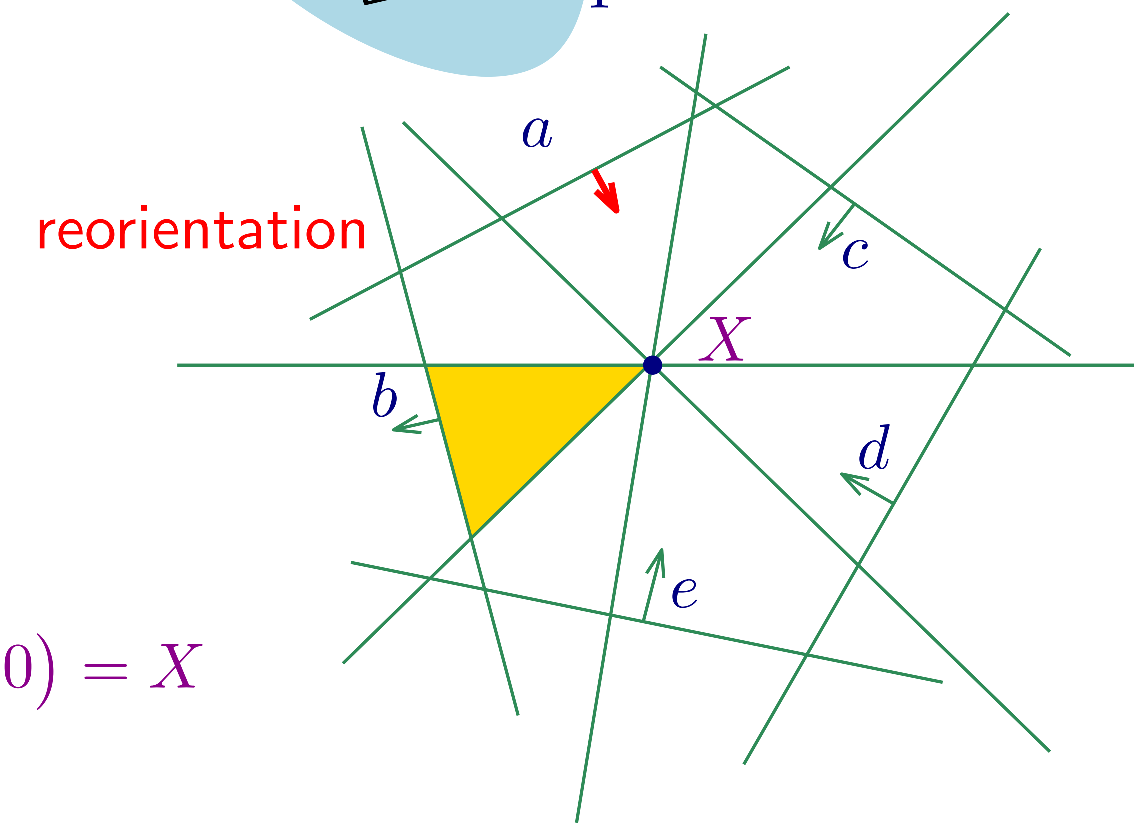
Acyclic orientations

digraph $D = (V, E)$
minimal edge cut X
incidence matrix $I \in \{\pm 1, 0\}^{V \times E}$

$$
\begin{array}{r} \\ +1\cdot 1 \\ +1\cdot 2 \\ +1\cdot 3 \\ +1\cdot 4 \\ -1\cdot 5 \\ -1\cdot 6 \\ -1\cdot 7 \\ -1\cdot 8 \\ \vdots \end{array}
\begin{array}{cccccc} a & b & c & d & e & \cdots \\ + & 0 & 0 & 0 & 0 & \cdots \\ 0 & - & + & 0 & 0 & \cdots \\ 0 & 0 & 0 & + & 0 & \cdots \\ 0 & 0 & 0 & 0 & + & \cdots \\ - & 0 & 0 & 0 & 0 & \cdots \\ 0 & + & 0 & 0 & 0 & \cdots \\ 0 & 0 & - & - & 0 & \cdots \\ 0 & 0 & 0 & 0 & - & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{array}
$$

$$\mathrm{sgn}(\Sigma) = (+ \quad - \quad + \quad + \quad + \quad 0 \ldots 0) = X$$



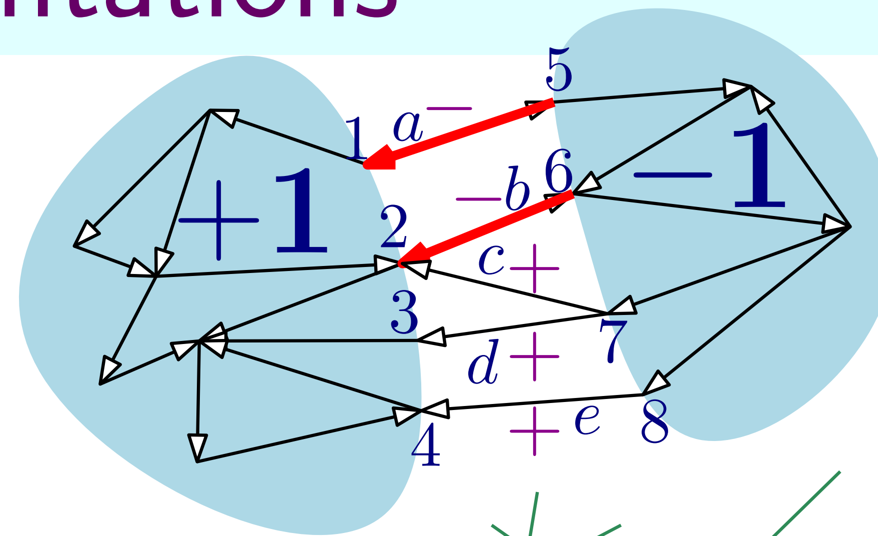


Acyclic orientations

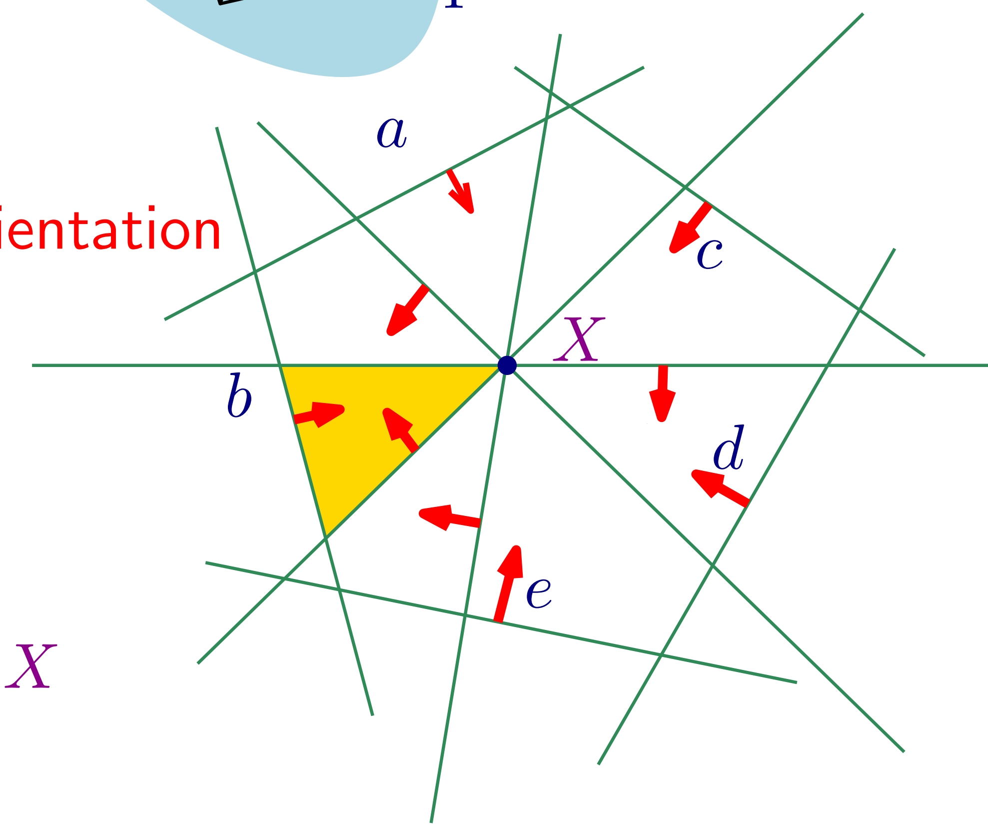
digraph $D = (V, E)$
minimal edge cut X
incidence matrix $I \in \{\pm 1, 0\}^{V \times E}$

$$
\begin{array}{r}
 \\
+1\cdot 1 \\
+1\cdot 2 \\
+1\cdot 3 \\
+1\cdot 4 \\
-1\cdot 5 \\
-1\cdot 6 \\
-1\cdot 7 \\
-1\cdot 8 \\
\vdots
\end{array}
\begin{array}{c}
\begin{array}{cccccc} a & b & c & d & e & \cdots \end{array} \\
\begin{pmatrix}
+ & 0 & 0 & 0 & 0 & \cdots \\
0 & + & + & 0 & 0 & \cdots \\
0 & 0 & 0 & + & 0 & \cdots \\
0 & 0 & 0 & 0 & + & \cdots \\
- & 0 & 0 & 0 & 0 & \cdots \\
0 & - & 0 & 0 & 0 & \cdots \\
0 & 0 & - & - & 0 & \cdots \\
0 & 0 & 0 & 0 & - & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix}
\end{array}
$$

$$\mathrm{sgn}(\Sigma) = (+ \quad + \quad + \quad + \quad + \quad 0 \ldots 0) = X$$



reorientation

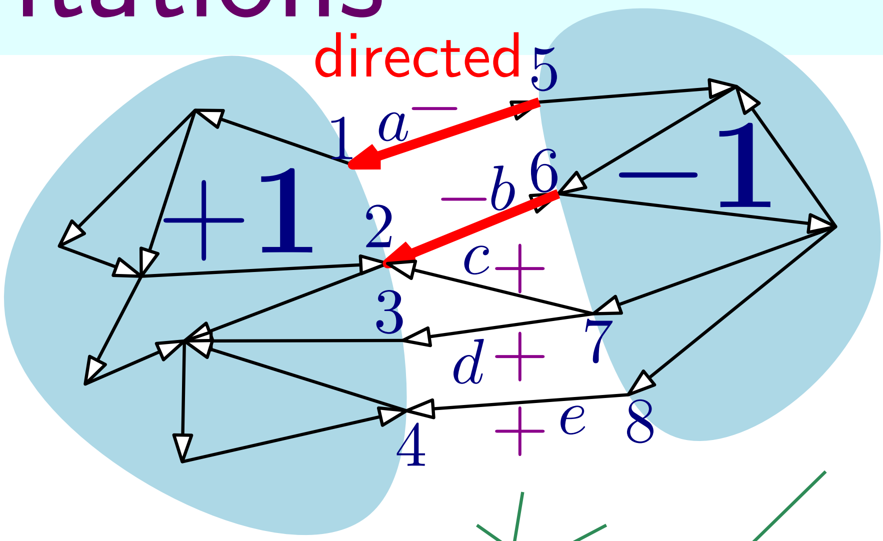


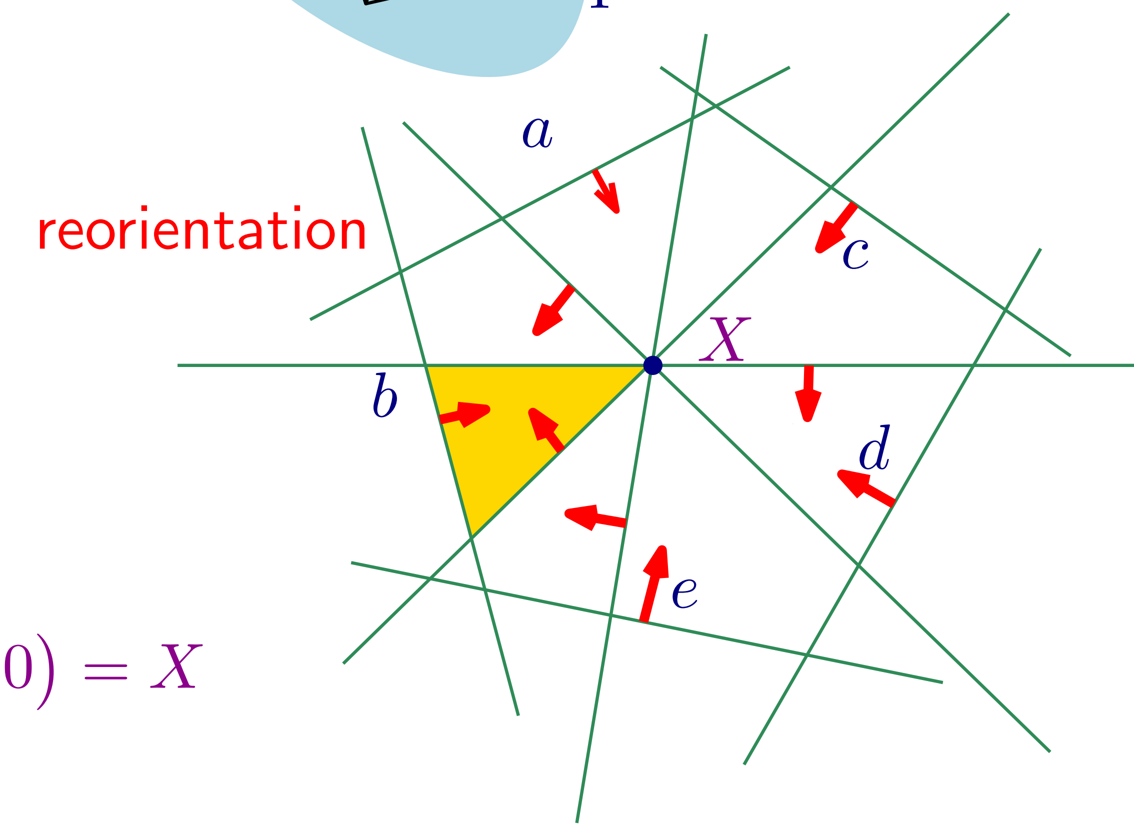
Acyclic orientations

digraph $D = (V, E)$
minimal edge cut X
incidence matrix $I \in \{\pm 1, 0\}^{V \times E}$

$$
\begin{array}{r} \\ +1\cdot 1 \\ +1\cdot 2 \\ +1\cdot 3 \\ +1\cdot 4 \\ -1\cdot 5 \\ -1\cdot 6 \\ -1\cdot 7 \\ -1\cdot 8 \\ \vdots \end{array}
\begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & \cdots \end{array} \\
\begin{pmatrix}
+ & 0 & 0 & 0 & 0 & \cdots \\
0 & + & + & 0 & 0 & \cdots \\
0 & 0 & 0 & + & 0 & \cdots \\
0 & 0 & 0 & 0 & + & \cdots \\
- & 0 & 0 & 0 & 0 & \cdots \\
0 & - & 0 & 0 & 0 & \cdots \\
0 & 0 & - & - & 0 & \cdots \\
0 & 0 & 0 & 0 & - & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix} \end{array}
$$

$$\mathrm{sgn}(\Sigma) = (+ \quad + \quad + \quad + \quad + \quad 0 \ldots 0) = X$$



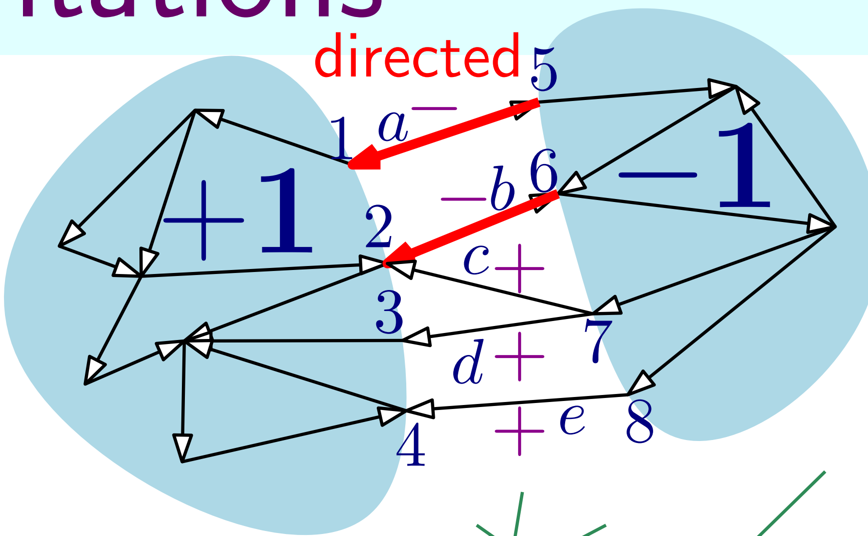


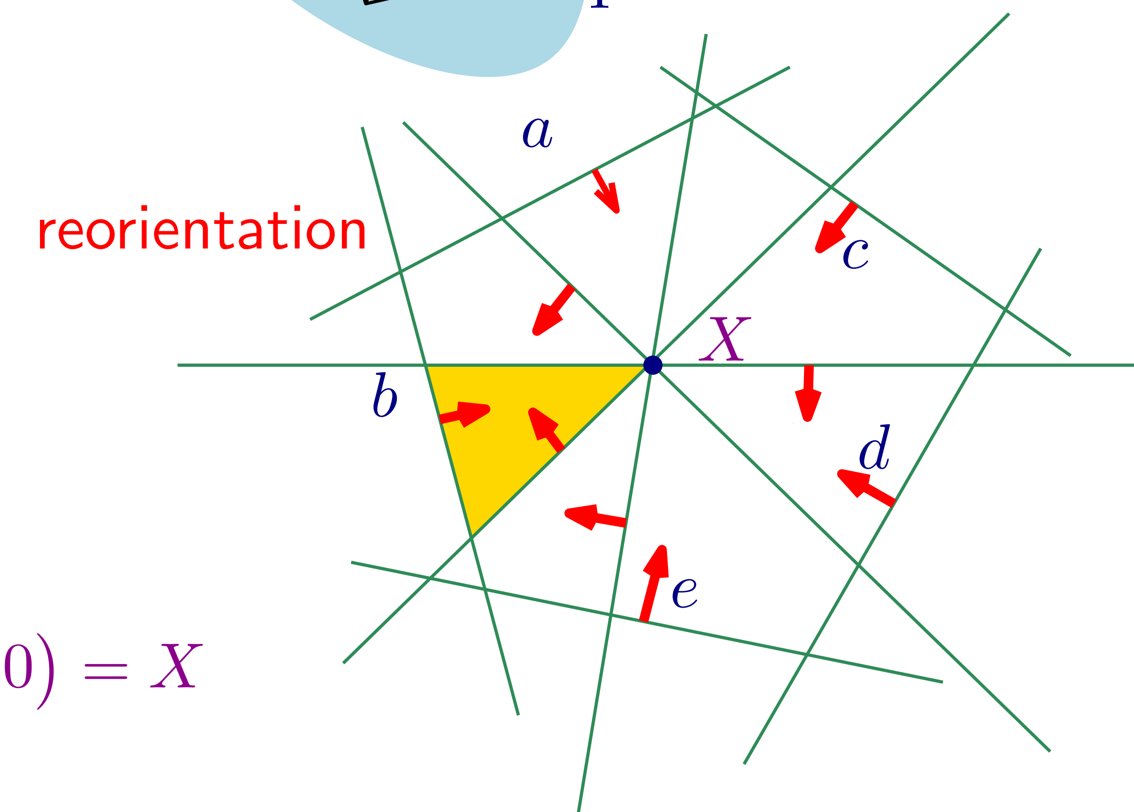
Acyclic orientations

digraph $D=(V,E)$
minimal edge cut X
incidence matrix $I \in \{\pm 1, 0\}^{V\times E}$

$$
\begin{array}{r}
\\
+1\cdot 1\\ +1\cdot 2\\ +1\cdot 3\\ +1\cdot 4\\ -1\cdot 5\\ -1\cdot 6\\ -1\cdot 7\\ -1\cdot 8\\ \vdots
\end{array}
\begin{array}{c}
\begin{array}{cccccc} a & b & c & d & e & \cdots \end{array}\\
\begin{pmatrix}
+ & 0 & 0 & 0 & 0 & \cdots\\
0 & + & + & 0 & 0 & \cdots\\
0 & 0 & 0 & + & 0 & \cdots\\
0 & 0 & 0 & 0 & + & \cdots\\
- & 0 & 0 & 0 & 0 & \cdots\\
0 & - & 0 & 0 & 0 & \cdots\\
0 & 0 & - & - & 0 & \cdots\\
0 & 0 & 0 & 0 & - & \cdots\\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix}
\end{array}
$$

$$\mathrm{sgn}(\Sigma) = (+ \quad + \quad + \quad + \quad + \quad 0\ldots 0) = X$$





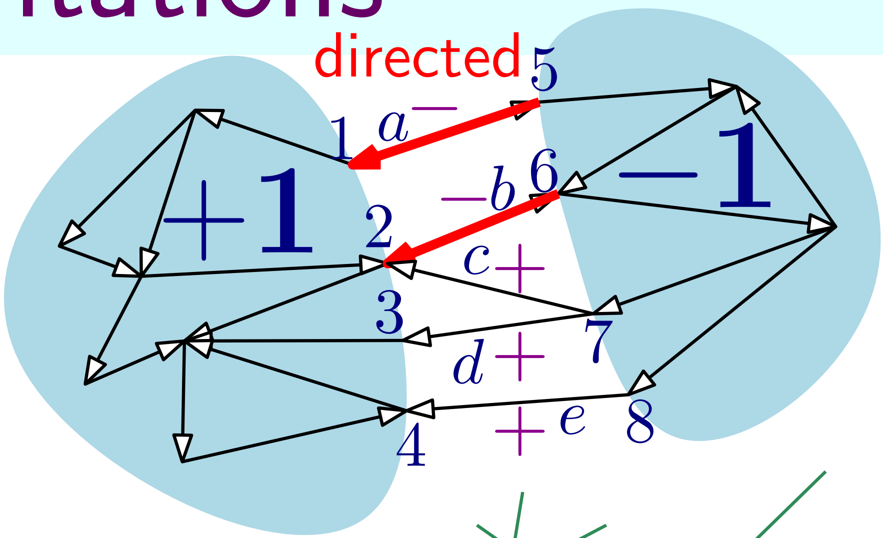
every edge in a directed cut

Acyclic orientations

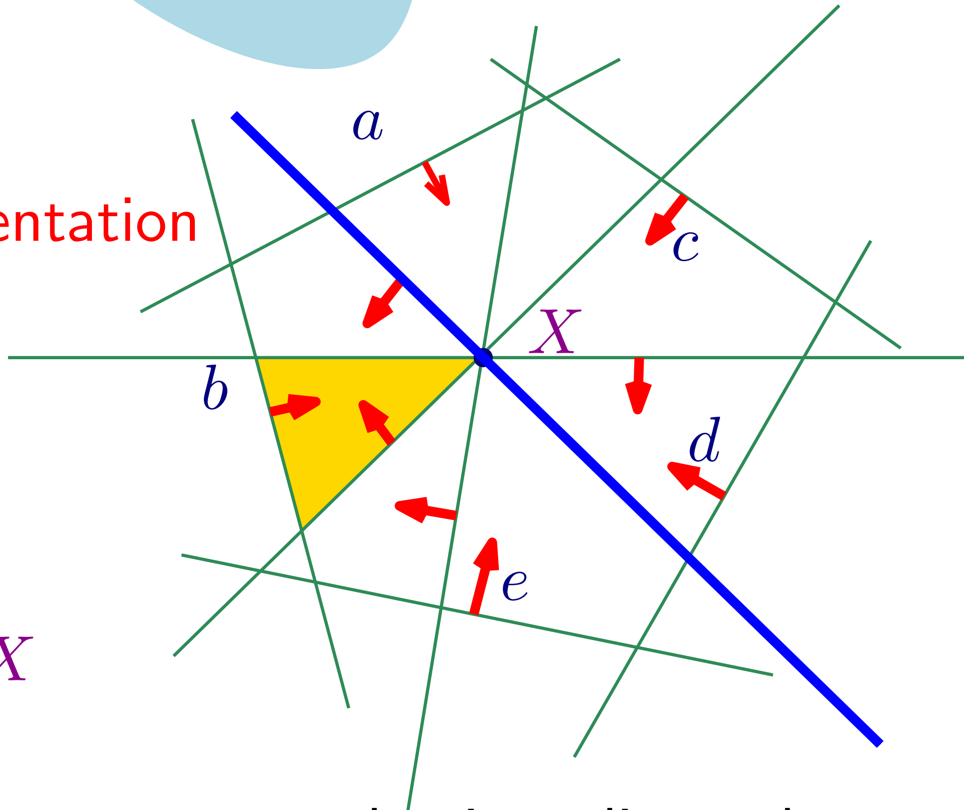
digraph $D = (V, E)$
minimal edge cut X
incidence matrix $I \in \{\pm 1, 0\}^{V \times E}$

$$
\begin{array}{r}
\\
+1 \cdot 1 \\
+1 \cdot 2 \\
+1 \cdot 3 \\
+1 \cdot 4 \\
-1 \cdot 5 \\
-1 \cdot 6 \\
-1 \cdot 7 \\
-1 \cdot 8 \\
\vdots
\end{array}
\begin{array}{c}
\begin{array}{cccccc} a & b & c & d & e & \cdots \end{array} \\
\left(\begin{array}{cccccc}
+ & 0 & 0 & 0 & 0 & \cdots \\
0 & + & + & 0 & 0 & \cdots \\
0 & 0 & 0 & + & 0 & \cdots \\
0 & 0 & 0 & 0 & + & \cdots \\
- & 0 & 0 & 0 & 0 & \cdots \\
0 & - & 0 & 0 & 0 & \cdots \\
0 & 0 & - & - & 0 & \cdots \\
0 & 0 & 0 & 0 & - & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{array}\right)
\end{array}
$$

$$\text{sgn}(\Sigma) = (+ \quad + \quad + \quad + \quad + \quad 0 \ldots 0) = X$$



reorientation



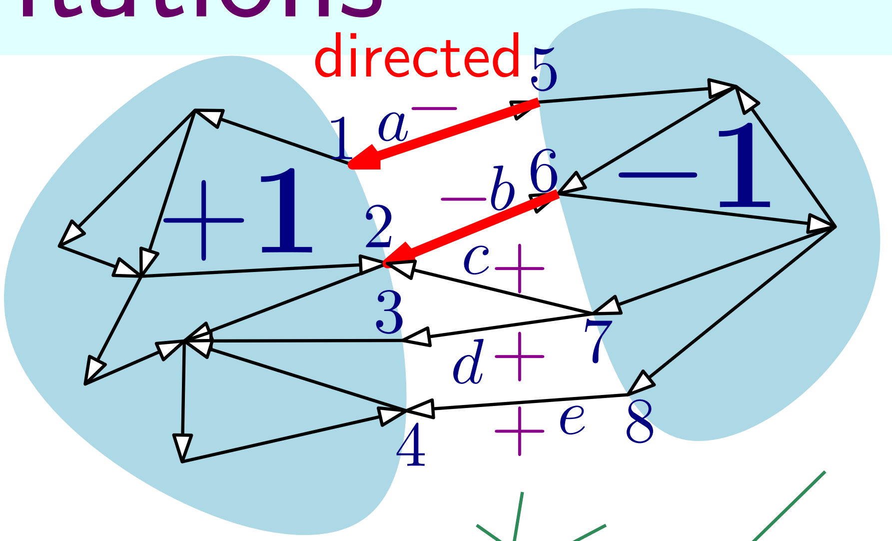
every edge in a directed cut

Acyclic orientations

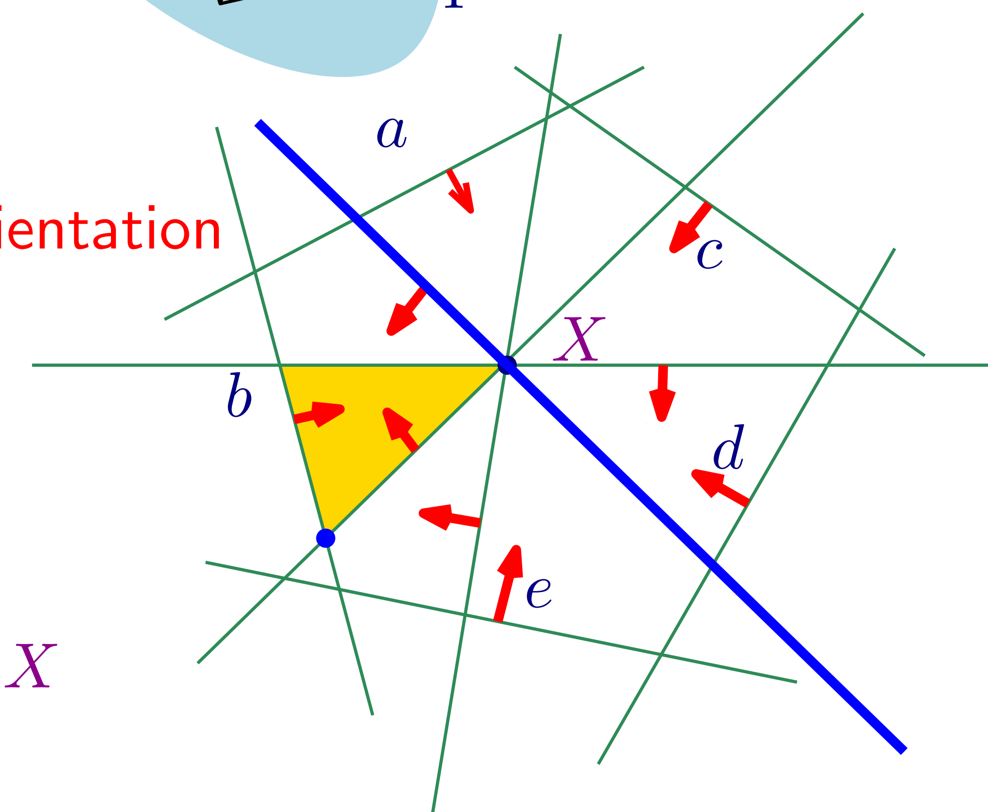
digraph $D = (V, E)$
minimal edge cut X
incidence matrix $I \in \{\pm 1, 0\}^{V \times E}$

$$
\begin{array}{r}
 \\ +1\cdot 1 \\ +1\cdot 2 \\ +1\cdot 3 \\ +1\cdot 4 \\ -1\cdot 5 \\ -1\cdot 6 \\ -1\cdot 7 \\ -1\cdot 8 \\ \vdots
\end{array}
\begin{array}{c}
\begin{array}{cccccc} a & b & c & d & e & \cdots \end{array} \\
\left(\begin{array}{cccccc}
+ & 0 & 0 & 0 & 0 & \cdots \\
0 & + & + & 0 & 0 & \cdots \\
0 & 0 & 0 & + & 0 & \cdots \\
0 & 0 & 0 & 0 & + & \cdots \\
- & 0 & 0 & 0 & 0 & \cdots \\
0 & - & 0 & 0 & 0 & \cdots \\
0 & 0 & - & - & 0 & \cdots \\
0 & 0 & 0 & 0 & - & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{array}\right)
\end{array}
$$

$\mathrm{sgn}(\Sigma) = (+ \quad + \quad + \quad + \quad + \quad 0 \ldots 0) = X$



reorientation



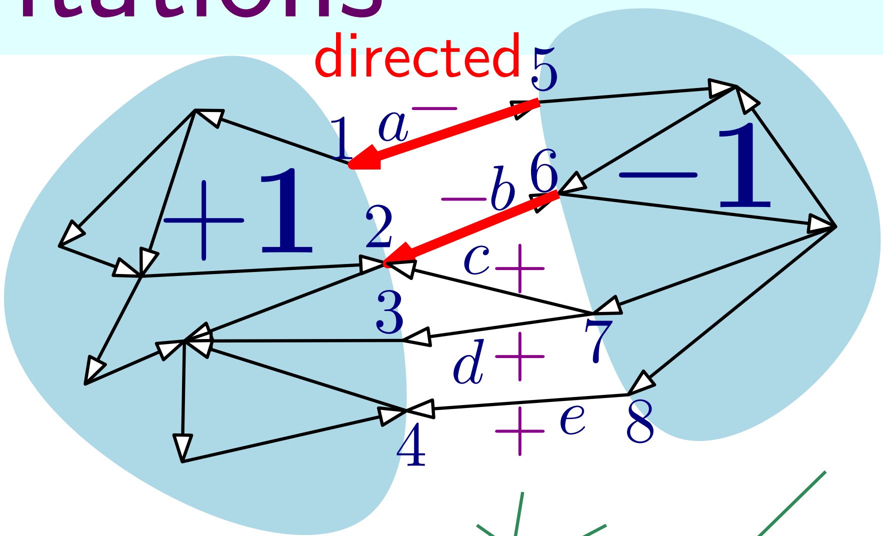
every edge in a directed cut

Acyclic orientations

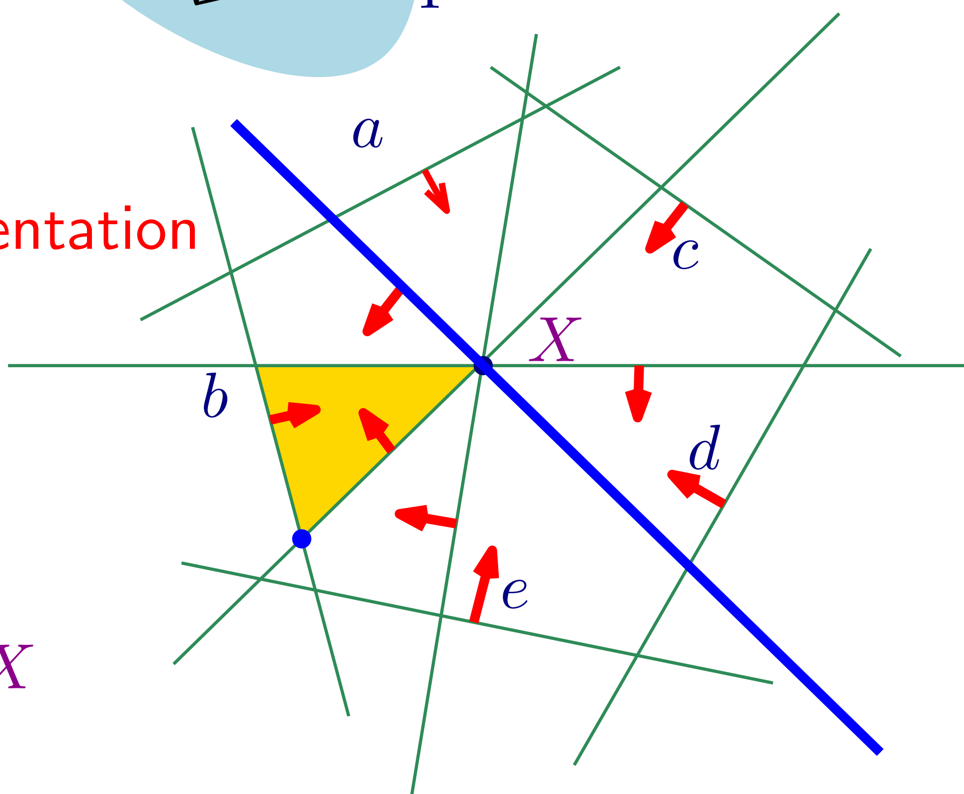
digraph $D=(V,E)$
minimal edge cut X
incidence matrix $I \in \{\pm 1, 0\}^{V\times E}$

$$
\begin{array}{r}
 \\
+1\cdot 1 \\
+1\cdot 2 \\
+1\cdot 3 \\
+1\cdot 4 \\
-1\cdot 5 \\
-1\cdot 6 \\
-1\cdot 7 \\
-1\cdot 8 \\
\vdots
\end{array}
\begin{array}{c}
\begin{array}{cccccc} a & b & c & d & e & \cdots \end{array} \\
\begin{pmatrix}
+ & 0 & 0 & 0 & 0 & \cdots \\
0 & + & + & 0 & 0 & \cdots \\
0 & 0 & 0 & + & 0 & \cdots \\
0 & 0 & 0 & 0 & + & \cdots \\
- & 0 & 0 & 0 & 0 & \cdots \\
0 & - & 0 & 0 & 0 & \cdots \\
0 & 0 & - & - & 0 & \cdots \\
0 & 0 & 0 & 0 & - & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix}
\end{array}
$$

$$\text{sgn}(\Sigma) = (+ \quad + \quad + \quad + \quad + \quad 0\ldots 0) = X$$



reorientation



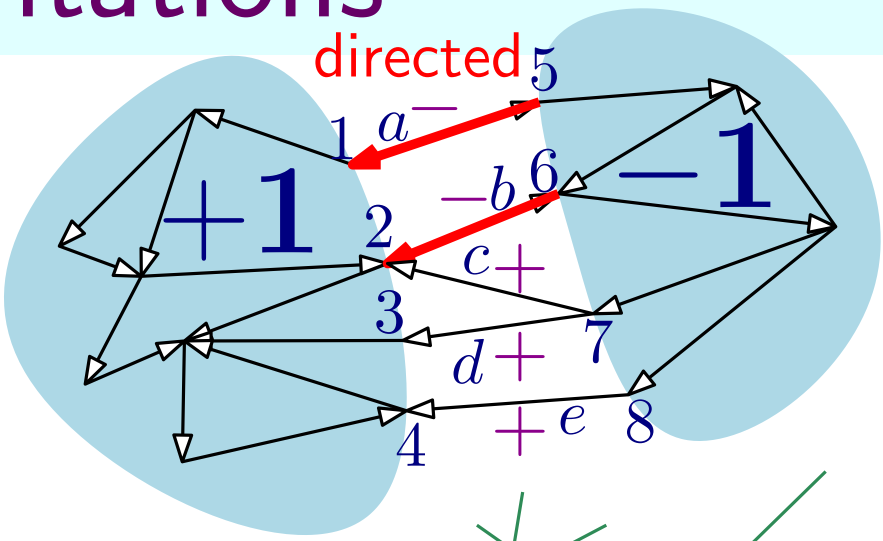
every edge in a directed cut
$\Longleftrightarrow$ acyclic orientation

Acyclic orientations

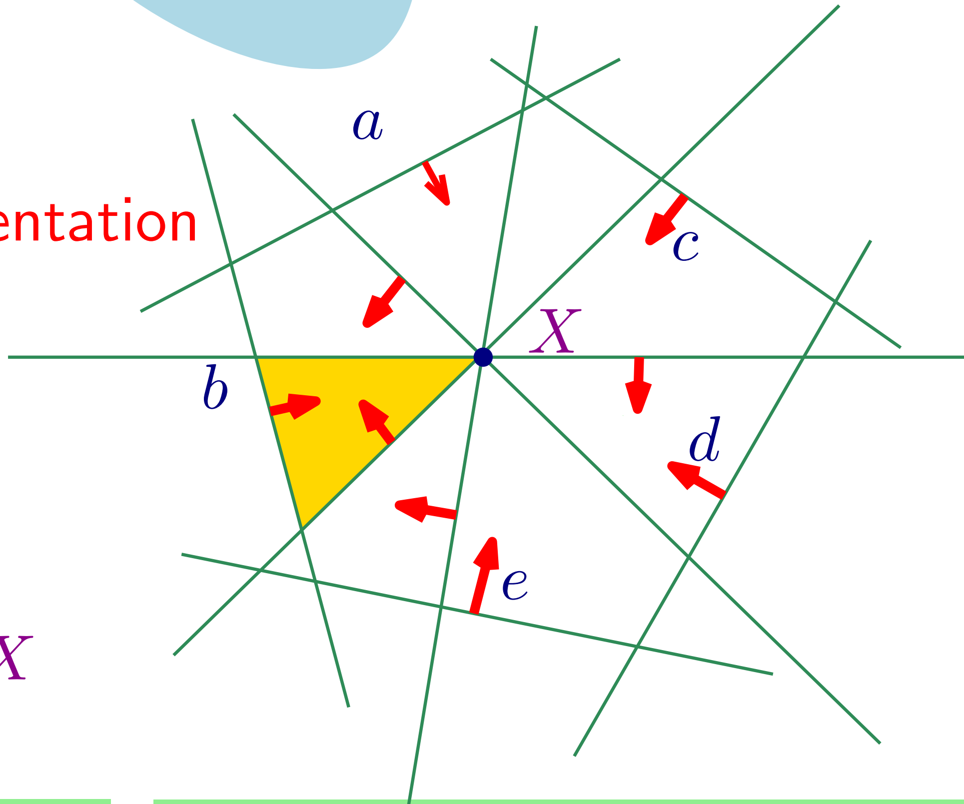
digraph $D = (V, E)$
minimal edge cut X
incidence matrix $I \in \{\pm 1, 0\}^{V \times E}$

$$
\begin{array}{r} \\ +1\cdot 1 \\ +1\cdot 2 \\ +1\cdot 3 \\ +1\cdot 4 \\ -1\cdot 5 \\ -1\cdot 6 \\ -1\cdot 7 \\ -1\cdot 8 \\ \vdots \end{array}
\begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & \cdots \end{array} \\
\begin{pmatrix}
+ & 0 & 0 & 0 & 0 & \cdots \\
0 & + & + & 0 & 0 & \cdots \\
0 & 0 & 0 & + & 0 & \cdots \\
0 & 0 & 0 & 0 & + & \cdots \\
- & 0 & 0 & 0 & 0 & \cdots \\
0 & - & 0 & 0 & 0 & \cdots \\
0 & 0 & - & - & 0 & \cdots \\
0 & 0 & 0 & 0 & - & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix} \end{array}
$$

$\mathrm{sgn}(\Sigma) = (+ \quad + \quad + \quad + \quad + \quad 0 \ldots 0) = X$



reorientation



max-dimensional cells
≅ acyclic orientations

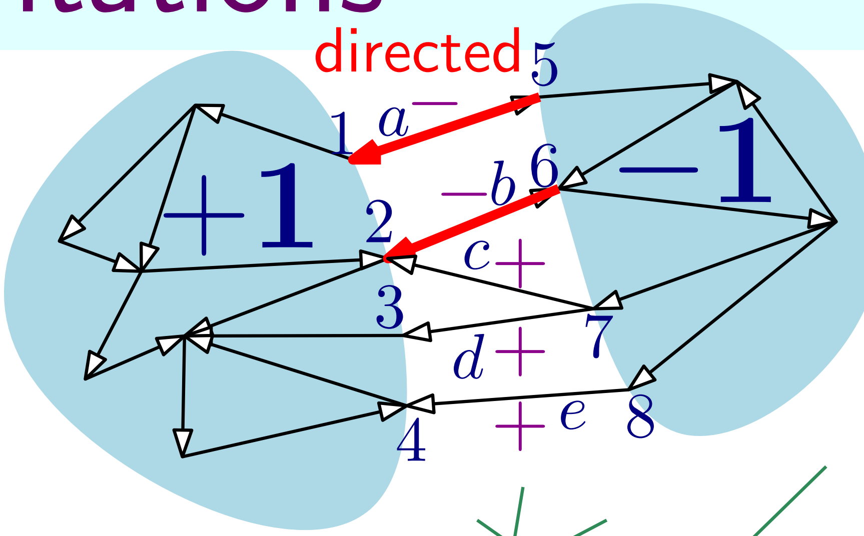
every edge in a directed cut
$\Longleftrightarrow$ acyclic orientation

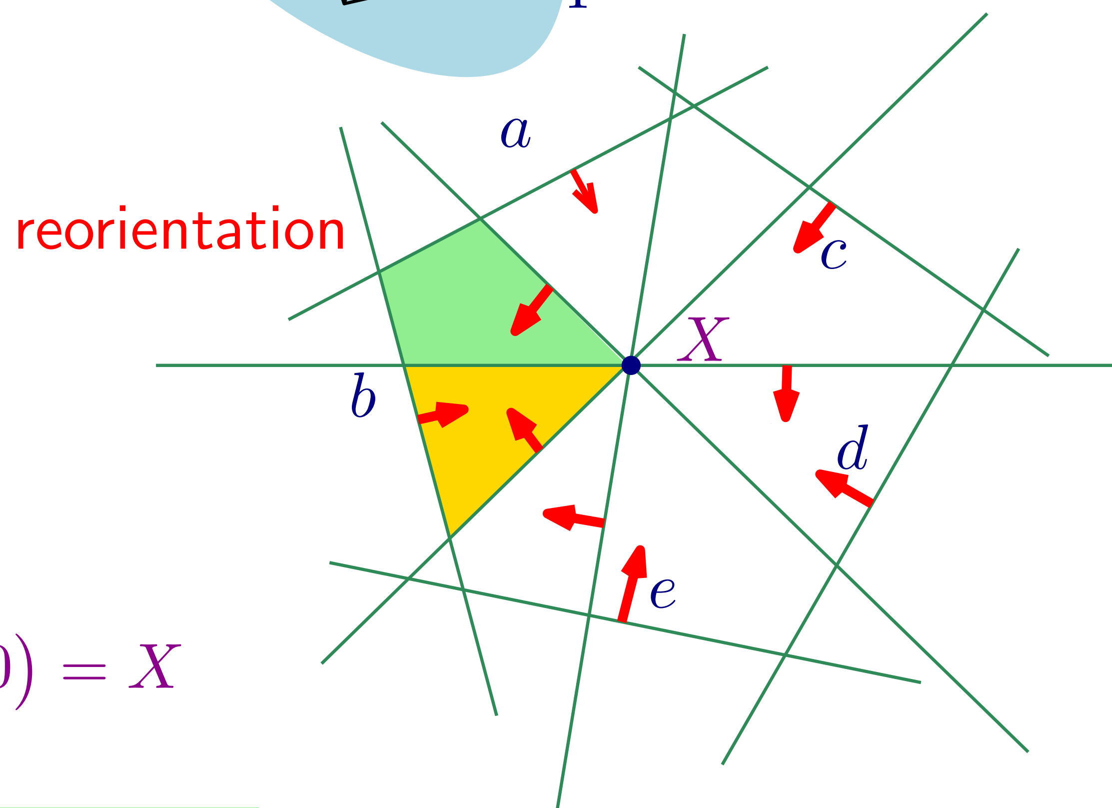
Acyclic orientations

digraph $D = (V, E)$
minimal edge cut X
incidence matrix $I \in \{\pm 1, 0\}^{V \times E}$

$$
\begin{array}{r} \\ +1\cdot 1 \\ +1\cdot 2 \\ +1\cdot 3 \\ +1\cdot 4 \\ -1\cdot 5 \\ -1\cdot 6 \\ -1\cdot 7 \\ -1\cdot 8 \\ \vdots \end{array}
\begin{array}{c} \begin{matrix} a & b & c & d & e & \cdots \end{matrix} \\
\begin{pmatrix}
+ & 0 & 0 & 0 & 0 & \cdots \\
0 & + & + & 0 & 0 & \cdots \\
0 & 0 & 0 & + & 0 & \cdots \\
0 & 0 & 0 & 0 & + & \cdots \\
- & 0 & 0 & 0 & 0 & \cdots \\
0 & - & 0 & 0 & 0 & \cdots \\
0 & 0 & - & - & 0 & \cdots \\
0 & 0 & 0 & 0 & - & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix}\end{array}
$$

$$\operatorname{sgn}(\Sigma) = (+ \quad + \quad + \quad + \quad + \quad 0 \ldots 0) = X$$





max-dimensional cells
$\cong$ acyclic orientations

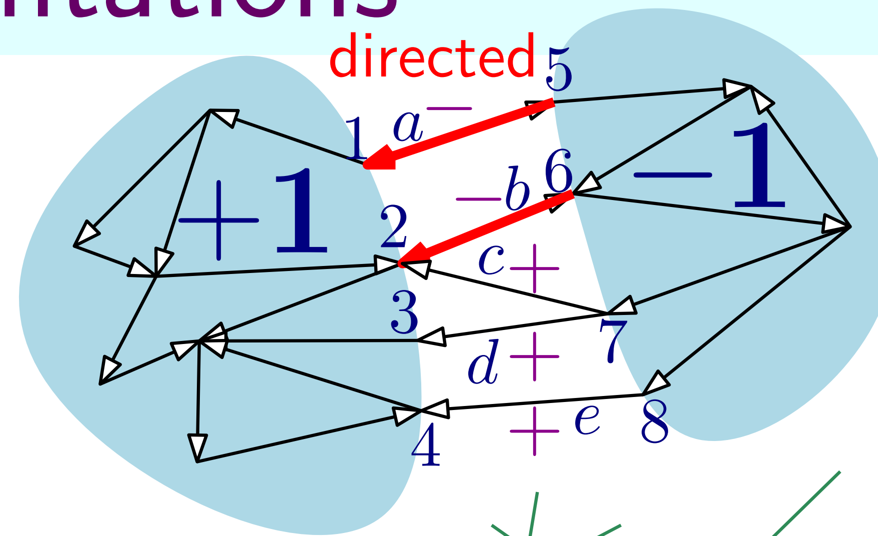
every edge in a directed cut
$\iff$ acyclic orientation

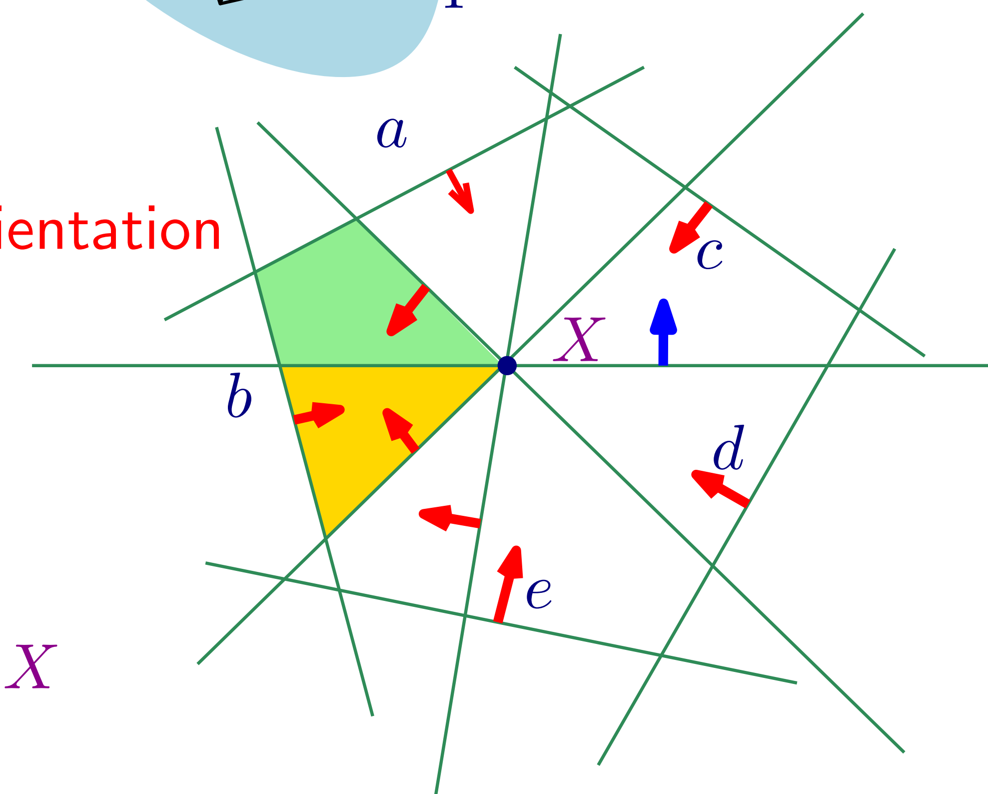
Acyclic orientations

digraph $D = (V, E)$
minimal edge cut X
incidence matrix $I \in \{\pm 1, 0\}^{V \times E}$

$$
\begin{array}{r}
\\
+1\cdot 1 \\ +1\cdot 2 \\ +1\cdot 3 \\ +1\cdot 4 \\ -1\cdot 5 \\ -1\cdot 6 \\ -1\cdot 7 \\ -1\cdot 8 \\ \vdots
\end{array}
\begin{array}{c}
\begin{array}{cccccc} a & b & c & d & e & \cdots \end{array} \\
\begin{pmatrix}
+ & 0 & 0 & 0 & 0 & \cdots \\
0 & + & + & 0 & 0 & \cdots \\
0 & 0 & 0 & + & 0 & \cdots \\
0 & 0 & 0 & 0 & + & \cdots \\
- & 0 & 0 & 0 & 0 & \cdots \\
0 & - & 0 & 0 & 0 & \cdots \\
0 & 0 & - & - & 0 & \cdots \\
0 & 0 & 0 & 0 & - & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix}
\end{array}
$$

$$\mathrm{sgn}(\Sigma) = (+ \quad + \quad + \quad + \quad + \quad 0 \ldots 0) = X$$





max-dimensional cells
$\cong$ acyclic orientations

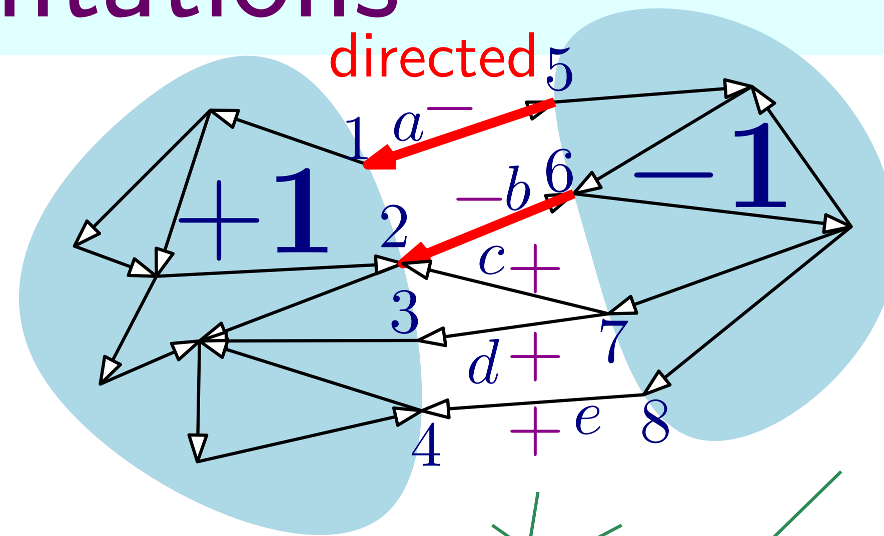
every edge in a directed cut
$\iff$ acyclic orientation

Acyclic orientations

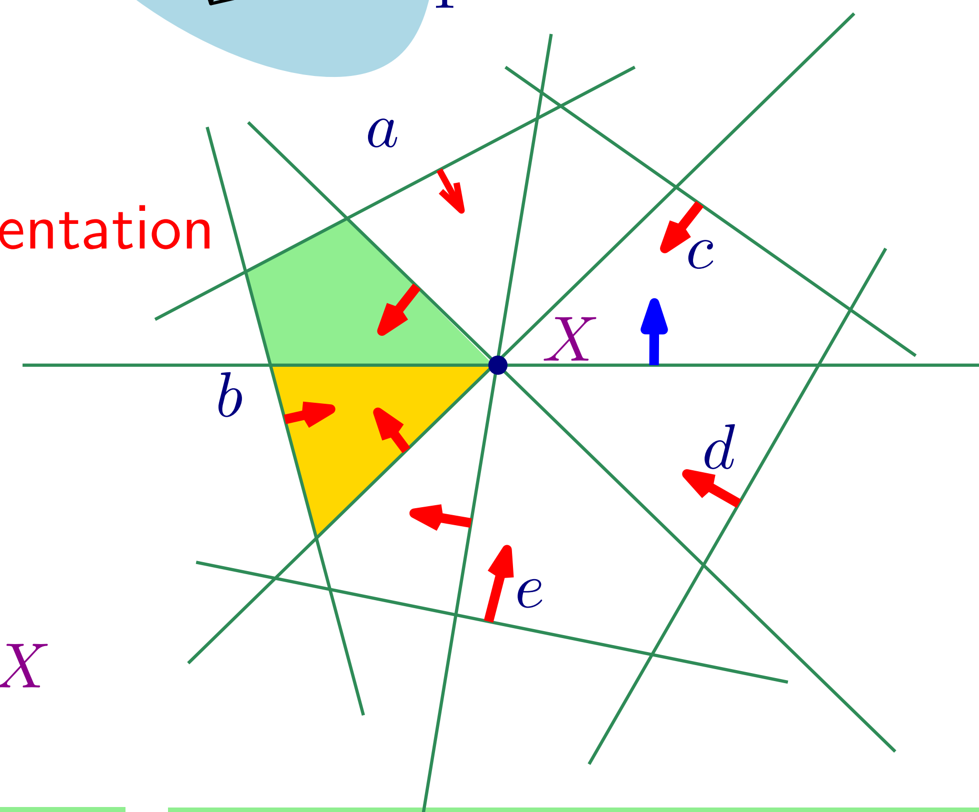
digraph $D = (V, E)$
minimal edge cut X
incidence matrix $I \in \{\pm 1, 0\}^{V \times E}$

$$
\begin{array}{r} \\ +1\cdot 1 \\ +1\cdot 2 \\ +1\cdot 3 \\ +1\cdot 4 \\ -1\cdot 5 \\ -1\cdot 6 \\ -1\cdot 7 \\ -1\cdot 8 \\ \vdots \end{array}
\begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & \cdots \end{array} \\
\left(\begin{array}{cccccc}
+ & 0 & 0 & 0 & 0 & \cdots \\
0 & + & + & 0 & 0 & \cdots \\
0 & 0 & 0 & + & 0 & \cdots \\
0 & 0 & 0 & 0 & + & \cdots \\
- & 0 & 0 & 0 & 0 & \cdots \\
0 & - & 0 & 0 & 0 & \cdots \\
0 & 0 & - & - & 0 & \cdots \\
0 & 0 & 0 & 0 & - & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{array} \right) \end{array}
$$

$\mathrm{sgn}(\Sigma) = (+ \quad + \quad + \quad + \quad + \quad 0 \ldots 0) = X$



reorientation



flip-graph on acyclic orientations $\cong$ region graph of arrangement

max-dimensional cells $\cong$ acyclic orientations

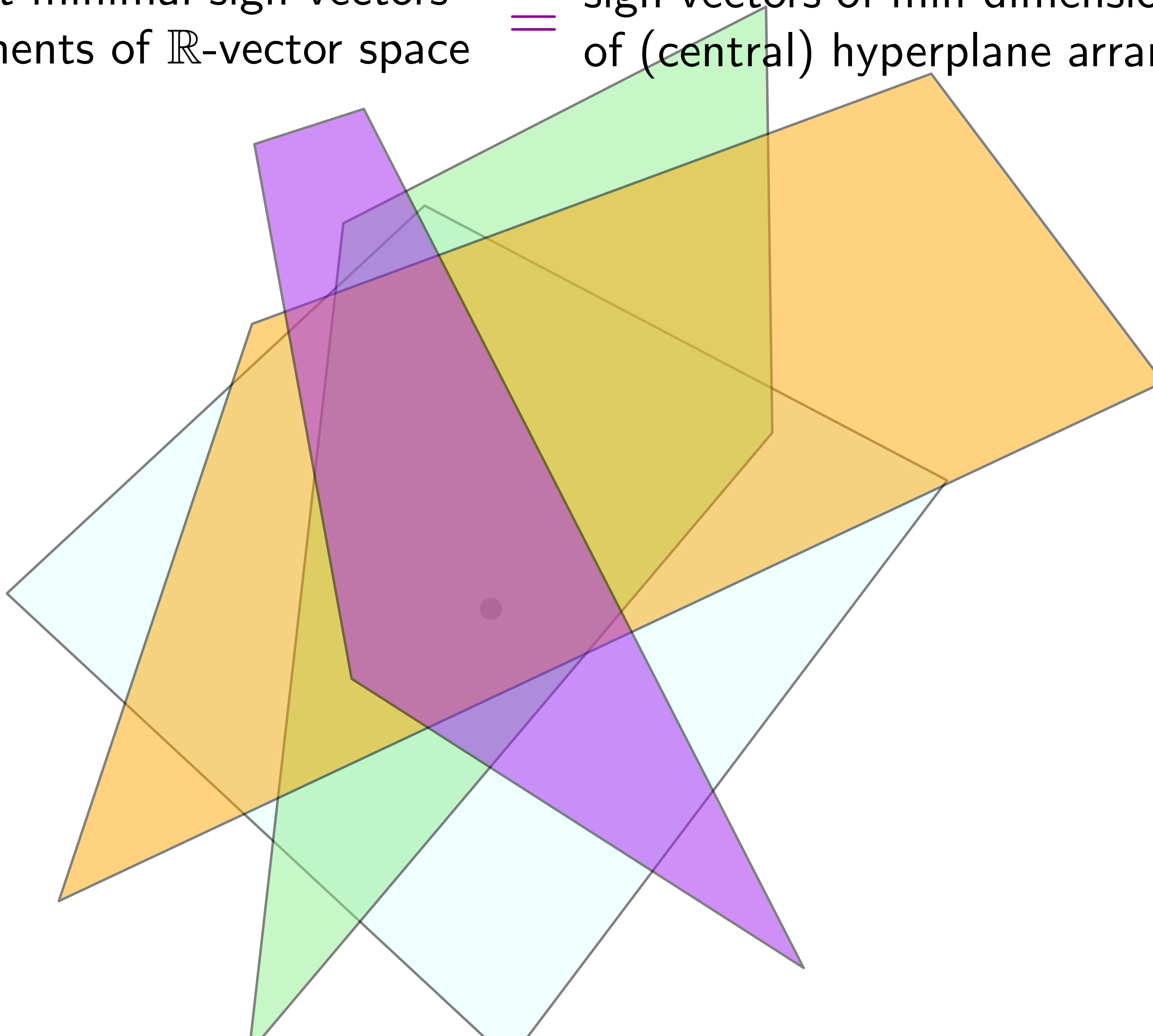
every edge in a directed cut $\iff$ acyclic orientation

Realizable oriented matroids

$\mathcal{C}^* :=$ support-minimal sign vectors of elements of $\mathbb{R}$-vector space $=$ sign vectors of min-dimensional cells of (central) hyperplane arrangement

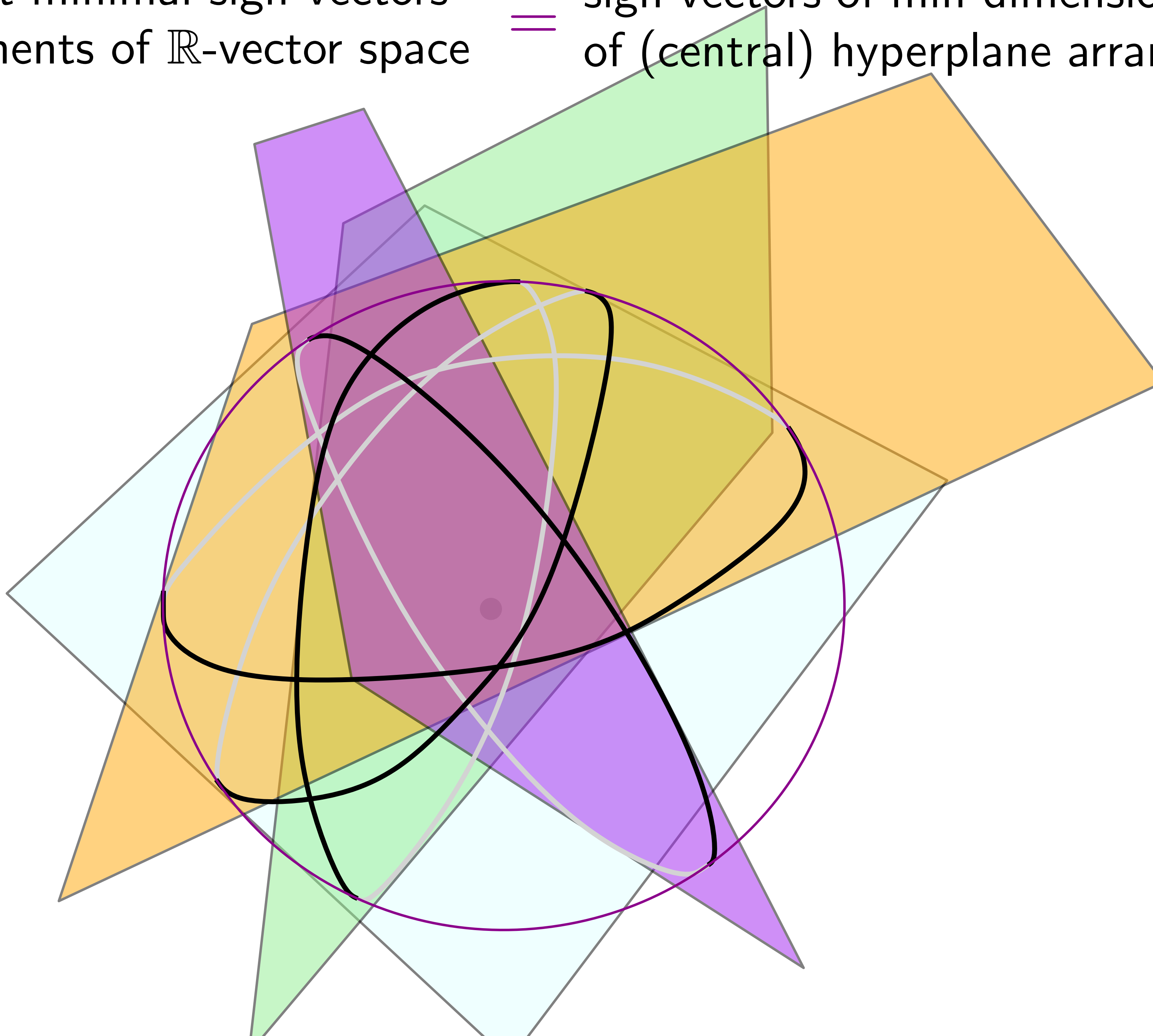
Realizable oriented matroids

$\mathcal{C}^* :=$ support-minimal sign vectors of elements of $\mathbb{R}$-vector space $=$ sign vectors of min-dimensional cells of (central) hyperplane arrangement



Realizable oriented matroids

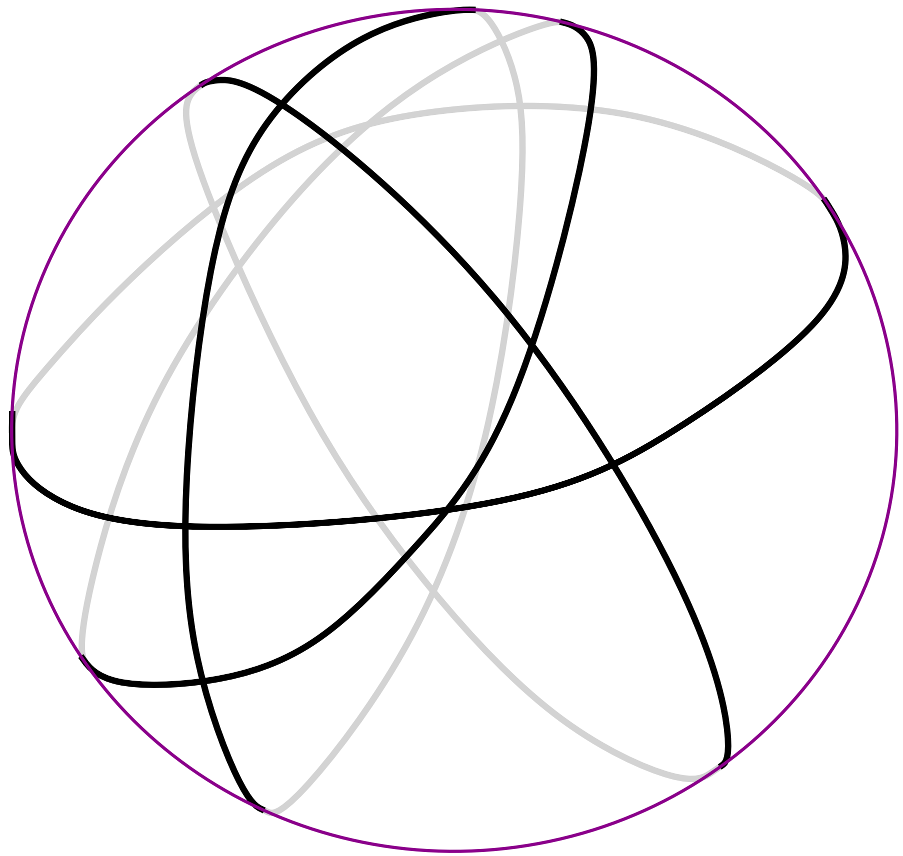
$\mathcal{C}^* :=$ support-minimal sign vectors of elements of $\mathbb{R}$-vector space $=$ sign vectors of min-dimensional cells of (central) hyperplane arrangement



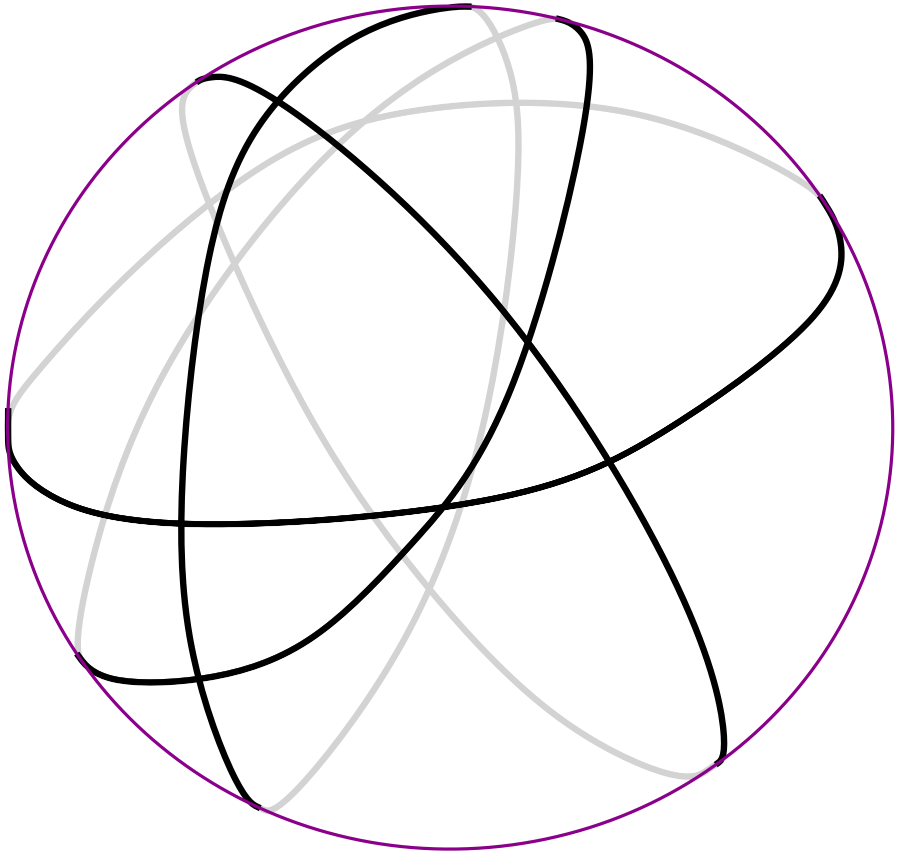
Realizable oriented matroids

$\mathcal{C}^* :=$ support-minimal sign vectors of elements of $\mathbb{R}$-vector space $=$ sign vectors of min-dimensional cells of (central) hyperplane arrangement

$=$ sign vectors of 0-dimensional cells of arrangement of great cycles on sphere



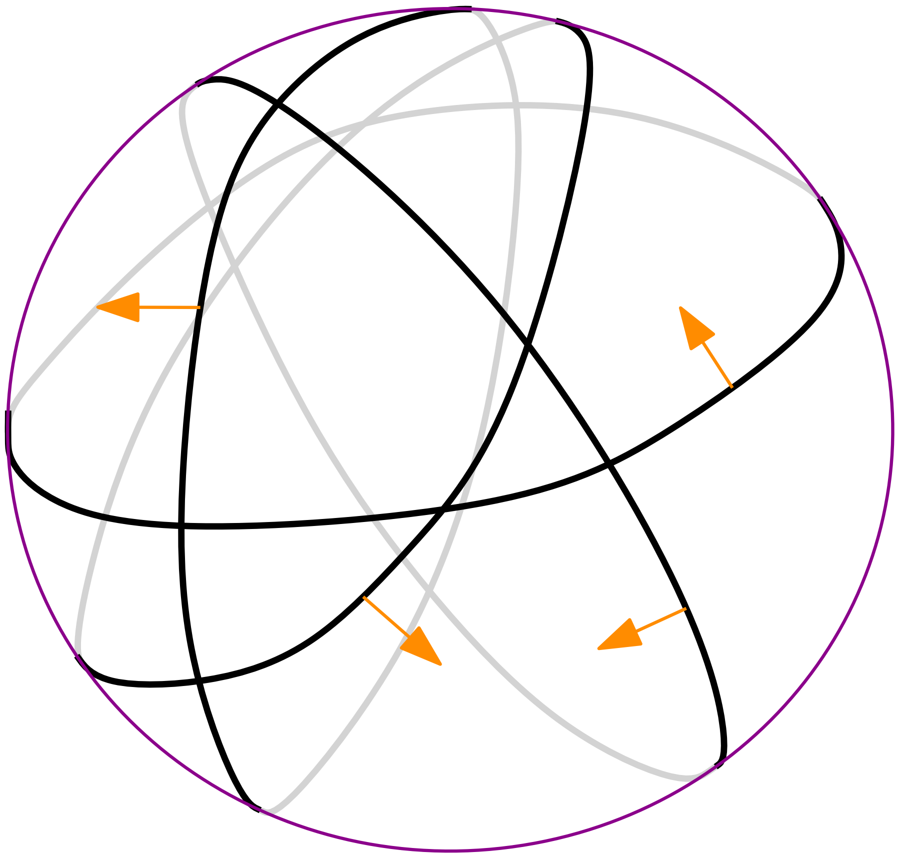
Oriented matroids



Thm[Folkman, Lawrence '78]

correspondence of **pseudo-sphere arrangements** and **oriented matroids**.

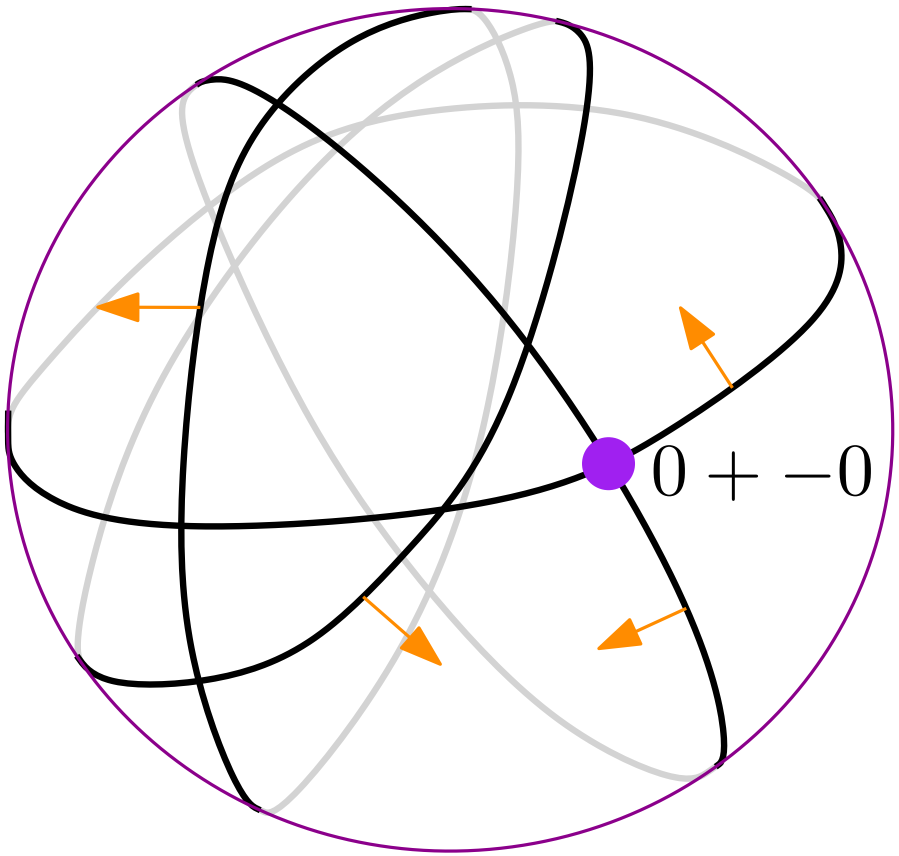
Oriented matroids



Thm[Folkman, Lawrence '78]

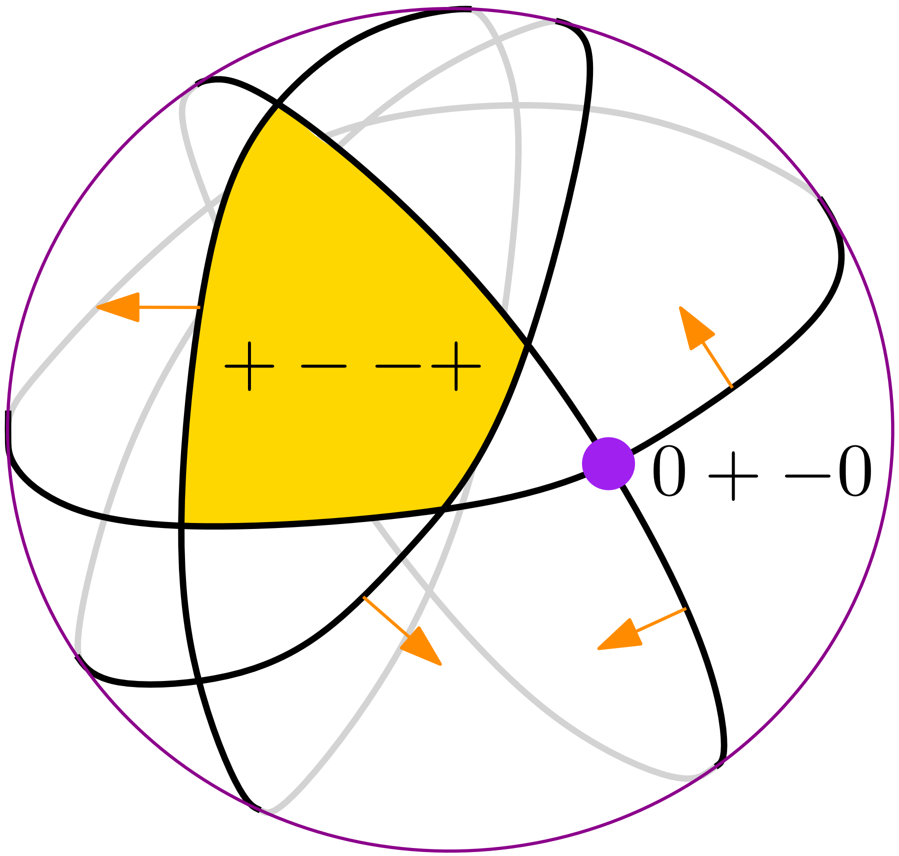
correspondence of **pseudo-sphere arrangements** and **oriented matroids**.

Oriented matroids



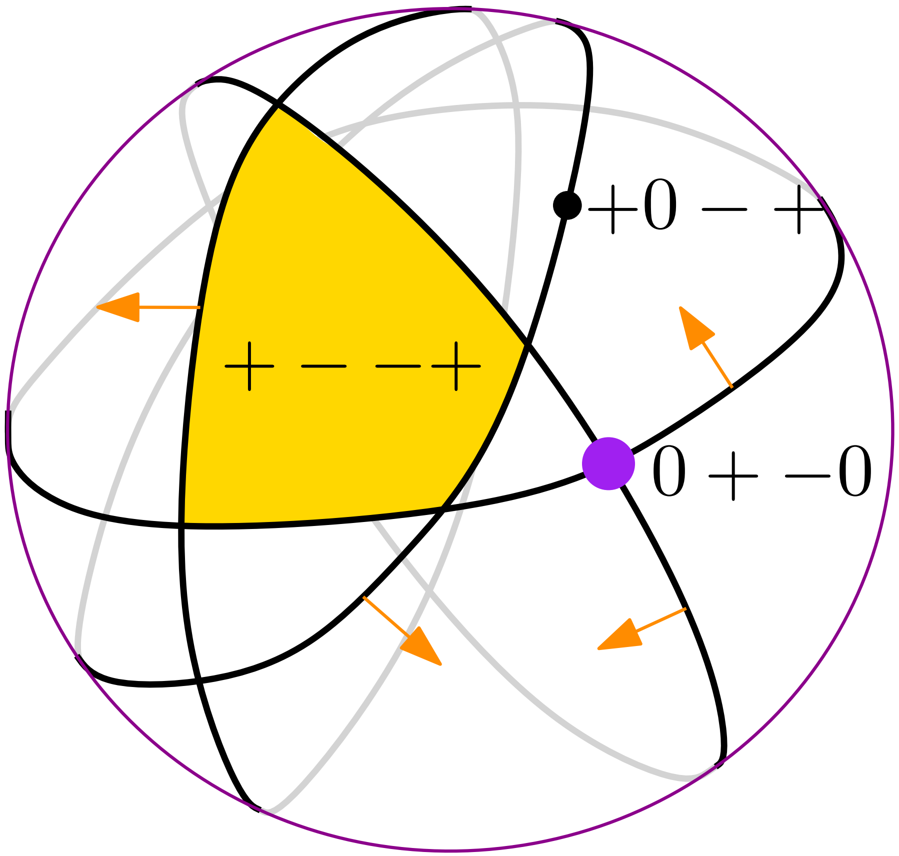
Thm[Folkman, Lawrence '78]
correspondence of **pseudo-sphere arrangements** and **oriented matroids**.

Oriented matroids



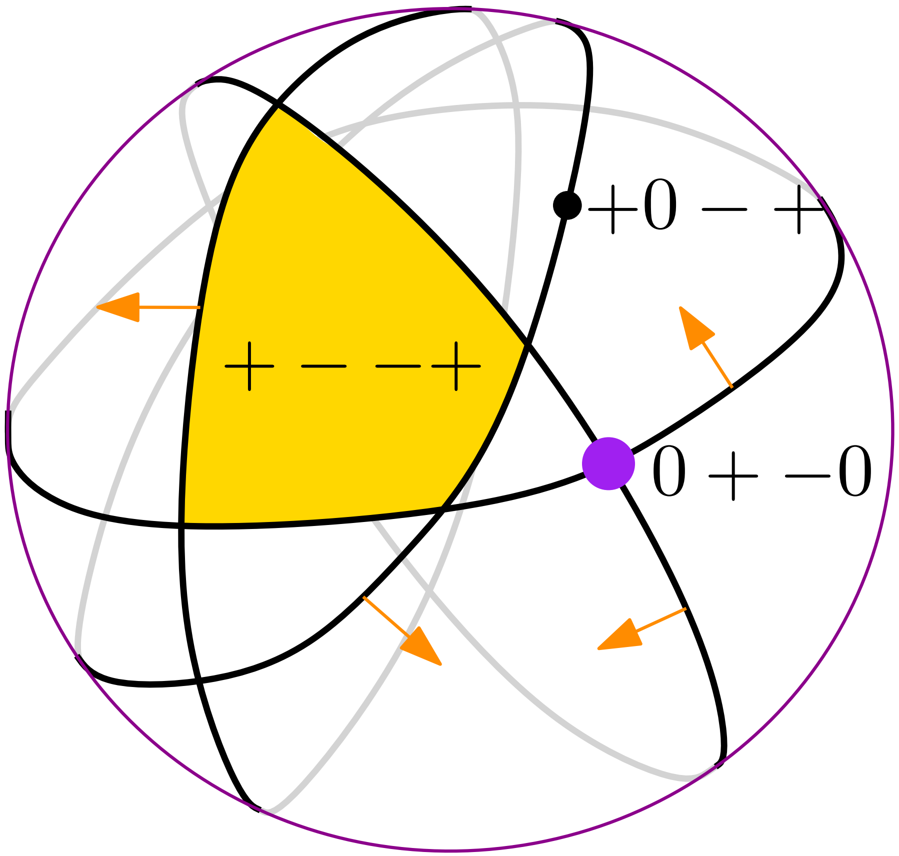
Thm[Folkman, Lawrence '78]
correspondence of **pseudo-sphere arrangements** and **oriented matroids**.

Oriented matroids



Thm[Folkman, Lawrence '78]
correspondence of **pseudo-sphere arrangements** and **oriented matroids**.

Oriented matroids



Thm[Folkman, Lawrence '78]
correspondence of **pseudo-sphere arrangements** and **oriented matroids**.

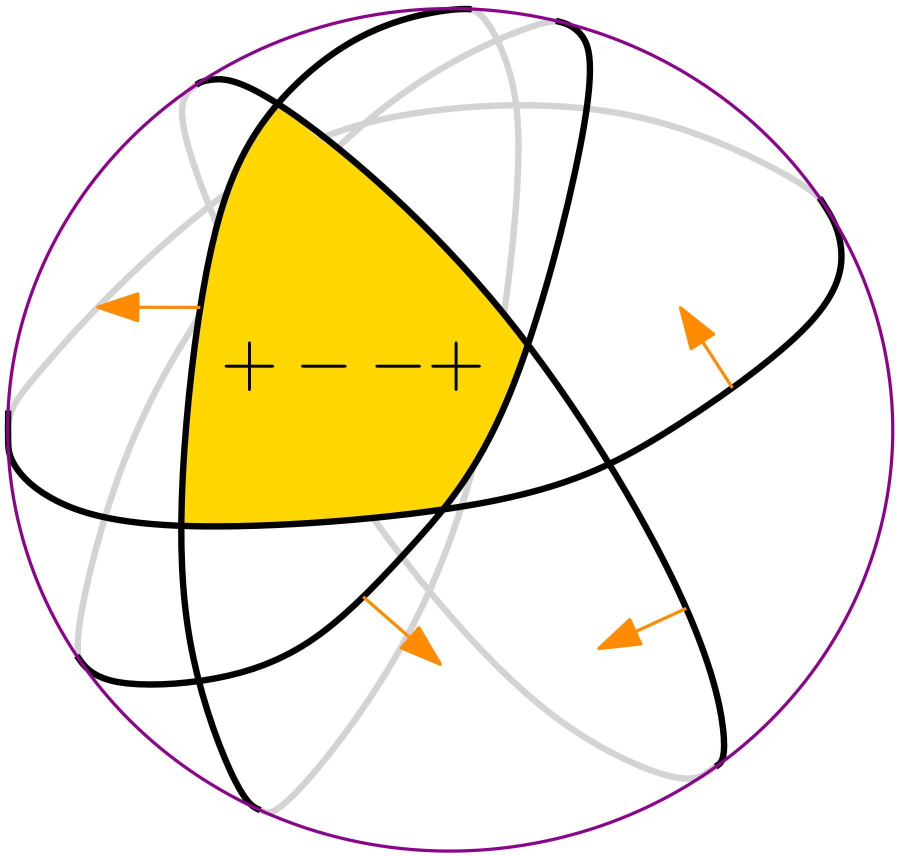
- Covector axioms: $(E, \mathcal{L})$ OM iff

(Z) $\mathbf{0} \in \mathcal{L}$

(FS) $\mathcal{L} \circ -\mathcal{L} \subseteq \mathcal{L}$

(SE) $\forall X, Y \in \mathcal{L}$ and $e \in S(X,Y) \exists Z \in \mathcal{L}$:
$Z_e = 0$ and $Z_f = X_f \circ Y_f$ for $f \notin S(X,Y)$.

Oriented matroids



topes $\mathcal{T}$ of $\mathcal{M}$=
maximal cells=
max. covectors

Thm[Folkman, Lawrence '78]
correspondence of **pseudo-sphere arrangements** and **oriented matroids**.

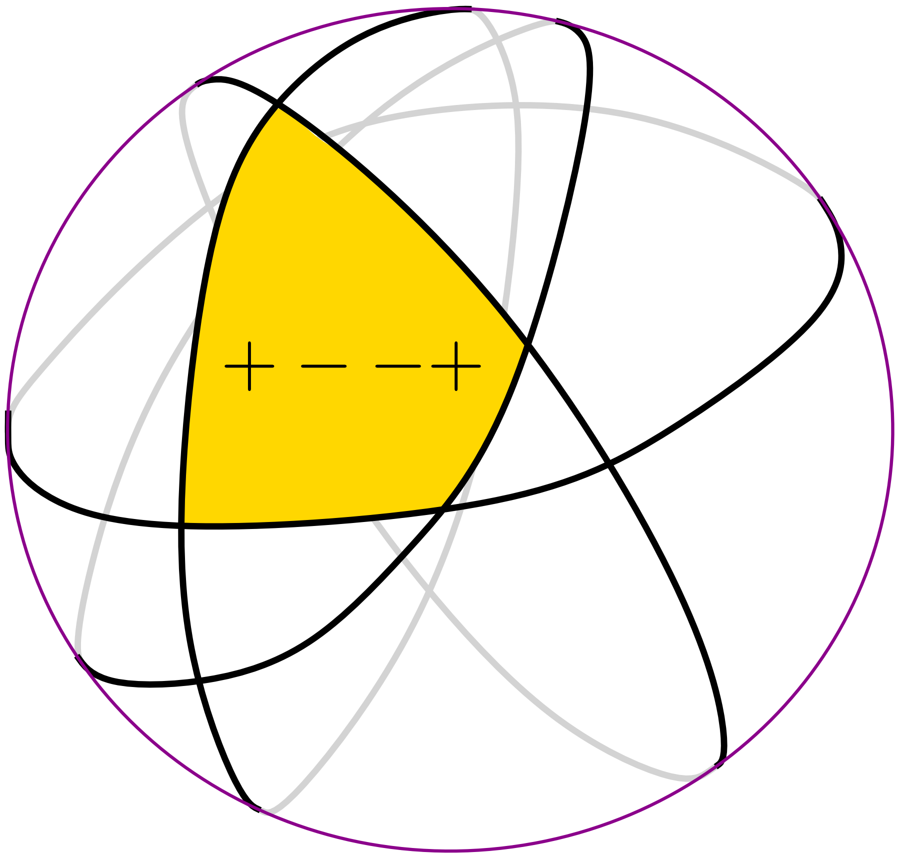
◦ Covector axioms: $(E,\mathcal{L})$ OM iff

(Z) $\mathbf{0} \in \mathcal{L}$

(FS) $\mathcal{L} \circ -\mathcal{L} \subseteq \mathcal{L}$

(SE) $\forall X, Y \in \mathcal{L}$ and $e \in S(X,Y) \exists Z \in \mathcal{L}$:
$Z_e = 0$ and $Z_f = X_f \circ Y_f$ for $f \notin S(X,Y)$.

Oriented matroids



Thm[Folkman, Lawrence '78]
correspondence of **pseudo-sphere arrangements** and **oriented matroids**.

topes $\mathcal{T}$ of $\mathcal{M}$=
maximal cells=
max. covectors

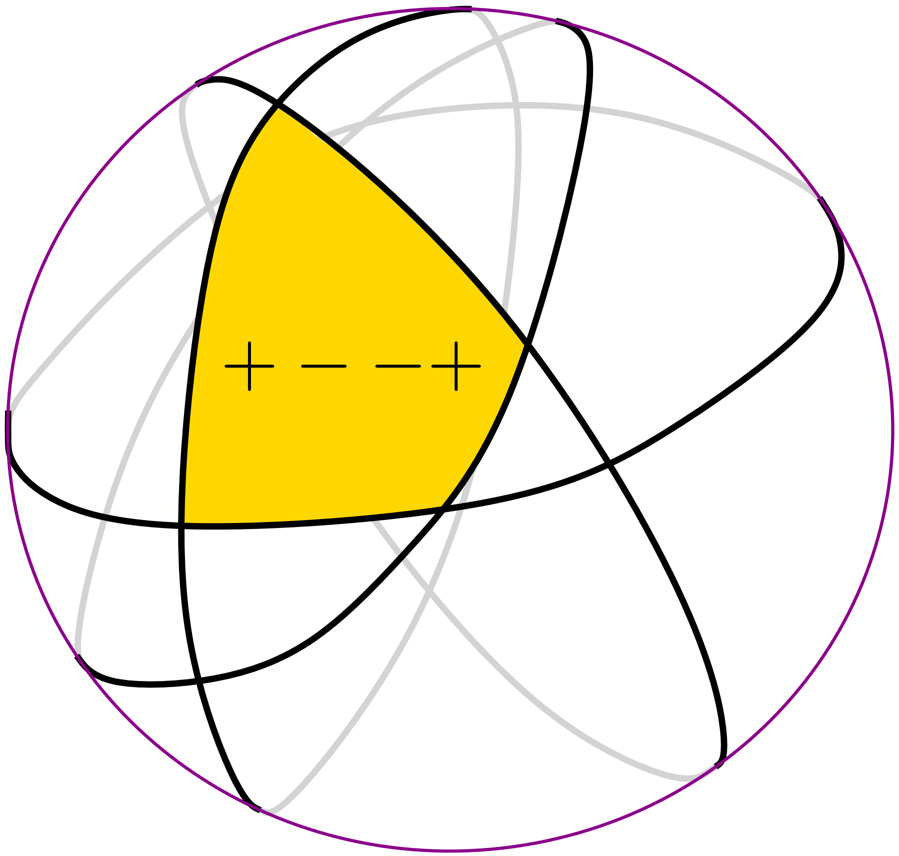
∘ Covector axioms: $(E, \mathcal{L})$ OM iff

(Z) $\mathbf{0} \in \mathcal{L}$

(FS) $\mathcal{L} \circ -\mathcal{L} \subseteq \mathcal{L}$

(SE) $\forall X, Y \in \mathcal{L}$ and $e \in S(X,Y) \exists Z \in \mathcal{L}$:
$Z_e = 0$ and $Z_f = X_f \circ Y_f$ for $f \notin S(X,Y)$.

Oriented matroids



Thm[Folkman, Lawrence '78]
correspondence of **pseudo-sphere arrangements** and **oriented matroids**.

topes $\mathcal{T}$ of $\mathcal{M}$= maximal cells= max. covectors

tope graph $G_{\mathcal{L}}$ =incidence graph= induced graph in Q_E

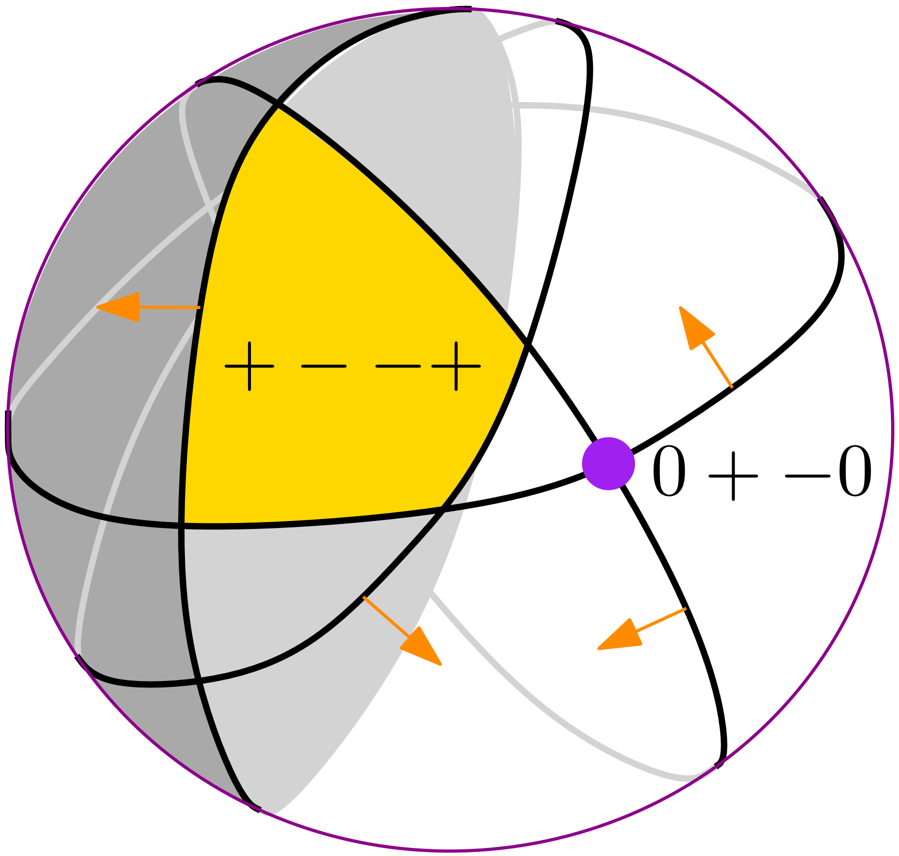
∘ Covector axioms: $(E, \mathcal{L})$ OM iff

(Z) $\mathbf{0} \in \mathcal{L}$

(FS) $\mathcal{L} \circ -\mathcal{L} \subseteq \mathcal{L}$

(SE) $\forall X, Y \in \mathcal{L}$ and $e \in S(X,Y) \exists Z \in \mathcal{L}$: $Z_e = 0$ and $Z_f = X_f \circ Y_f$ for $f \notin S(X,Y)$.

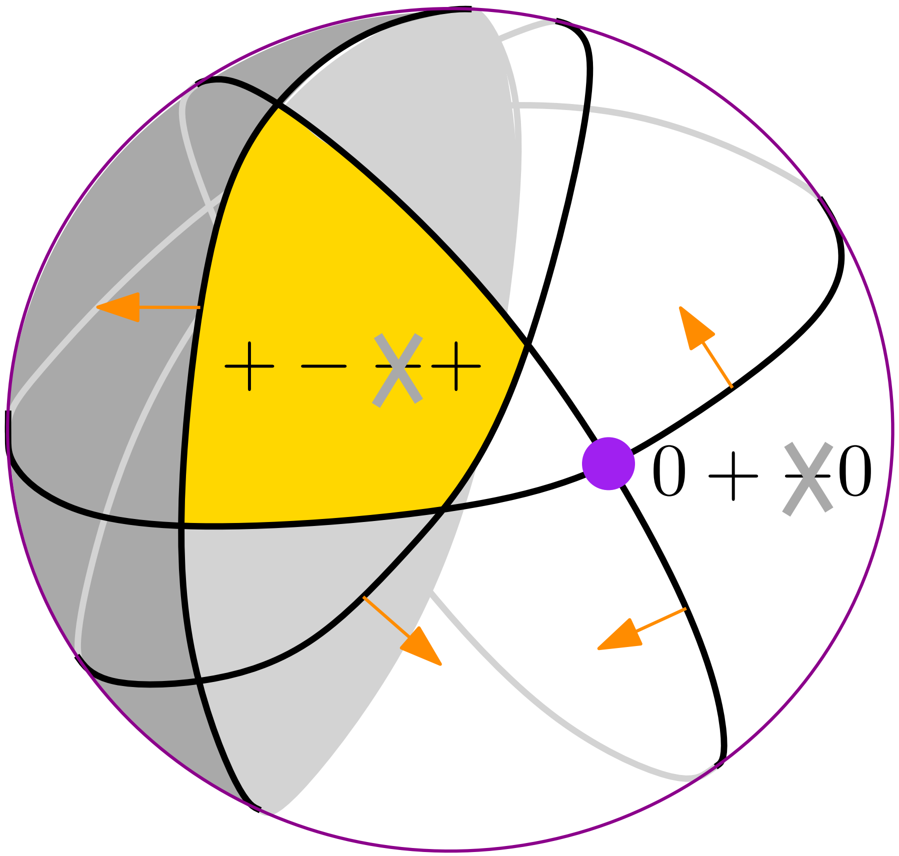
Affine oriented matroids



Thm[Karlander '92]

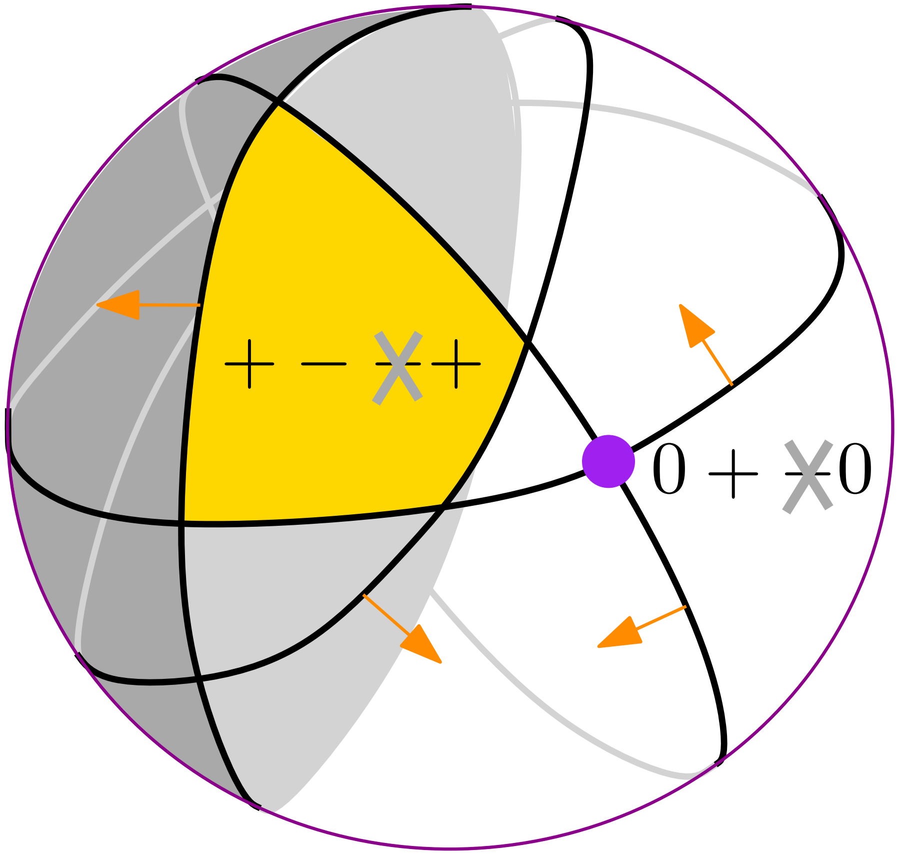
correspondence between **affine** arrangements of pseudospheres and **affine oriented matroids**.

Affine oriented matroids



Thm[Karlander '92]
correspondence between **affine** arrangements of pseudospheres and **affine oriented matroids**.

Affine oriented matroids



Thm[Karlander '92]
correspondence between **affine** arrangements of pseudospheres and **affine oriented matroids**.

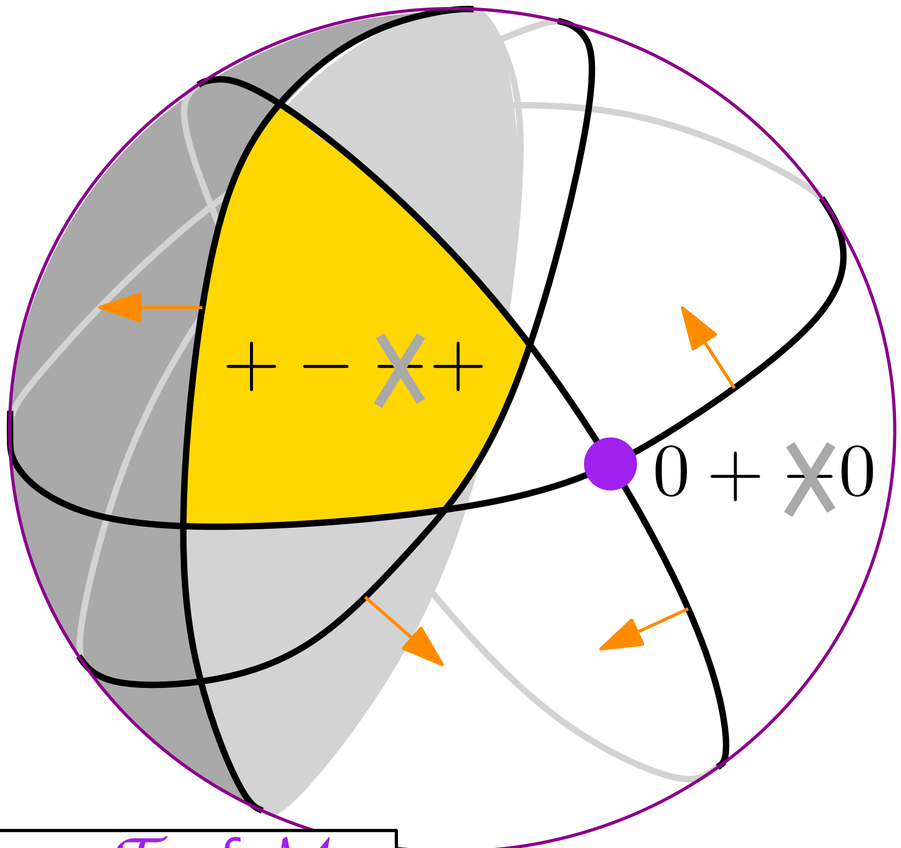
∘ Covector axioms: $(E,\mathcal{L})$ affine oriented matroid:

(A) *something lengthy*

(FS) $\mathcal{L} \circ -\mathcal{L} \subseteq \mathcal{L}$

(SE) $\forall X, Y \in \mathcal{L}$ and $e \in S(X,Y) \exists Z \in \mathcal{L}$:
$Z_e = 0$ and $Z_f = X_f \circ Y_f$ for $f \notin S(X,Y)$.

Affine oriented matroids



Thm[Karlander '92]
correspondence between **affine** arrangements of pseudospheres and **affine oriented matroids**.

topes $\mathcal{T}$ of $\mathcal{M}$= maximal cells= max. covectors

tope graph $G_{\mathcal{L}}$ =incidence graph= induced graph in Q_E

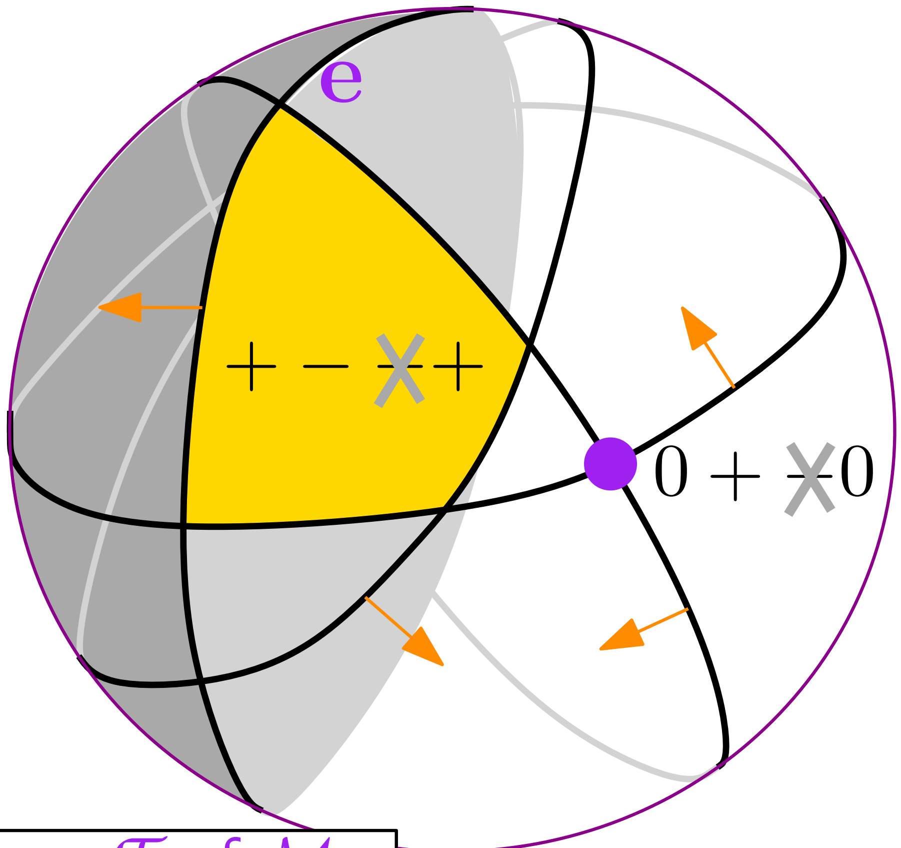
- Covector axioms: $(E, \mathcal{L})$ affine oriented matroid:

(A) *something lengthy*

(FS) $\mathcal{L} \circ -\mathcal{L} \subseteq \mathcal{L}$

(SE) $\forall X, Y \in \mathcal{L}$ and $e \in S(X, Y) \exists Z \in \mathcal{L}$:
$Z_e = 0$ and $Z_f = X_f \circ Y_f$ for $f \notin S(X, Y)$.

Affine oriented matroids



Thm[Karlander '92]
correspondence between **affine** arrangements of pseudospheres and **affine oriented matroids**.

digraph example:
topes $\mathcal{T} \cong$ acyclic orientations with edge **e**'s orientation fixed

topes $\mathcal{T}$ of $\mathcal{M}$= maximal cells= max. covectors

tope graph $G_{\mathcal{L}}$ =incidence graph= induced graph in Q_E

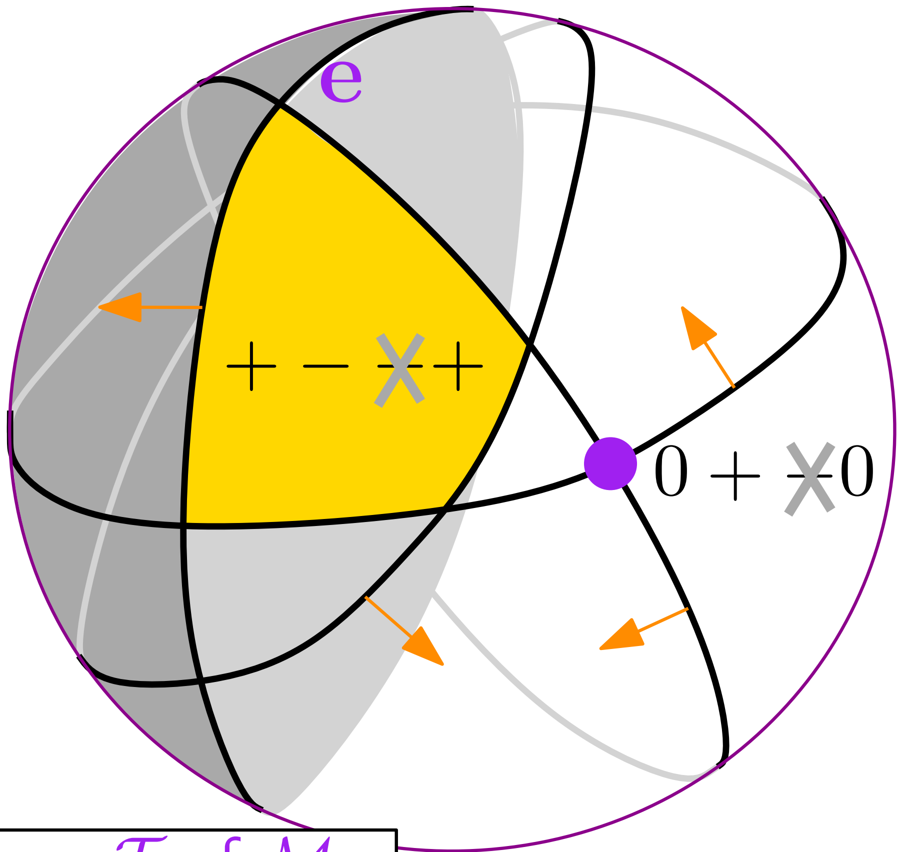
∘ Covector axioms: $(E, \mathcal{L})$ affine oriented matroid:

(A) *something lengthy*

(FS) $\mathcal{L} \circ -\mathcal{L} \subseteq \mathcal{L}$

(SE) $\forall X, Y \in \mathcal{L}$ and $e \in S(X,Y) \exists Z \in \mathcal{L}$: $Z_e = 0$ and $Z_f = X_f \circ Y_f$ for $f \notin S(X,Y)$.

Affine oriented matroids



topes $\mathcal{T}$ of $\mathcal{M}$=
maximal cells=
max. covectors

tope graph $G_{\mathcal{L}}$
=incidence graph=
induced graph in Q_E

Thm[Karlander '92]
correspondence between **affine** arrangements of pseudospheres and **affine oriented matroids**.

digraph example:
topes $\mathcal{T} \cong$ acyclic orientations with edge **e**'s orientation fixed

Bandelt, Chepoi, K '15:

why not fix more?

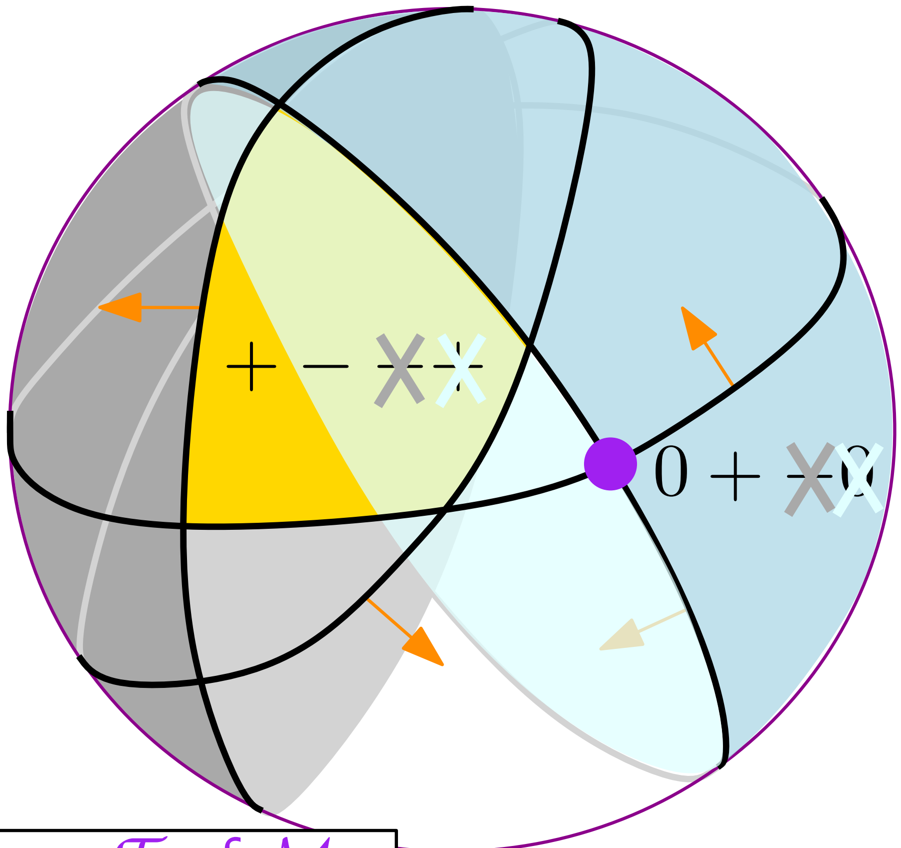
∘ Covector axioms: $(E, \mathcal{L})$ affine oriented matroid:

(A) *something lengthy*

(FS) $\mathcal{L} \circ -\mathcal{L} \subseteq \mathcal{L}$

(SE) $\forall X, Y \in \mathcal{L}$ and $e \in S(X,Y) \exists Z \in \mathcal{L}$:
$Z_e = 0$ and $Z_f = X_f \circ Y_f$ for $f \notin S(X,Y)$.

Complexes of oriented matroids



topes $\mathcal{T}$ of $\mathcal{M}$= maximal cells= max. covectors

tope graph $G_{\mathcal{L}}$ =incidence graph= induced graph in Q_E

Bandelt, Chepoi, K '15:

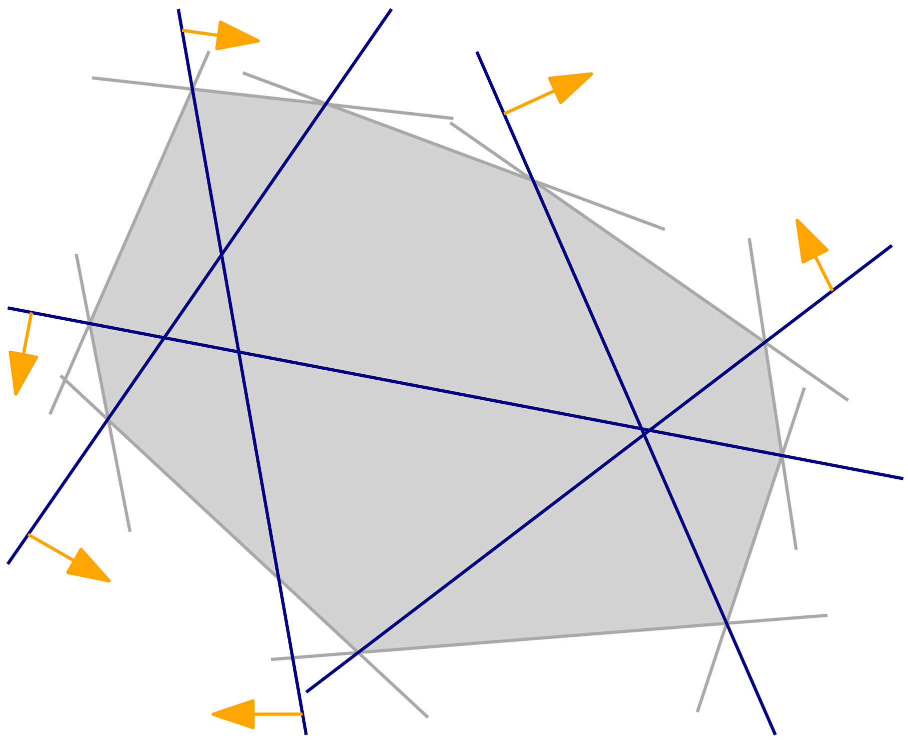
why not fix more?

- Covector axioms: $(E, \mathcal{L})$ COM iff

(FS) $\mathcal{L} \circ -\mathcal{L} \subseteq \mathcal{L}$

(SE) $\forall X, Y \in \mathcal{L}$ and $e \in S(X,Y) \exists Z \in \mathcal{L}$:
$Z_e = 0$ and $Z_f = X_f \circ Y_f$ for $f \notin S(X,Y)$.

Complexes of oriented matroids



Def[Bandelt, Chepoi, K '15]
realizable COM = sign systems from arrangement of open halfspaces and hyperplanes.

topes $\mathcal{T}$ of $\mathcal{M}$= maximal cells= max. covectors

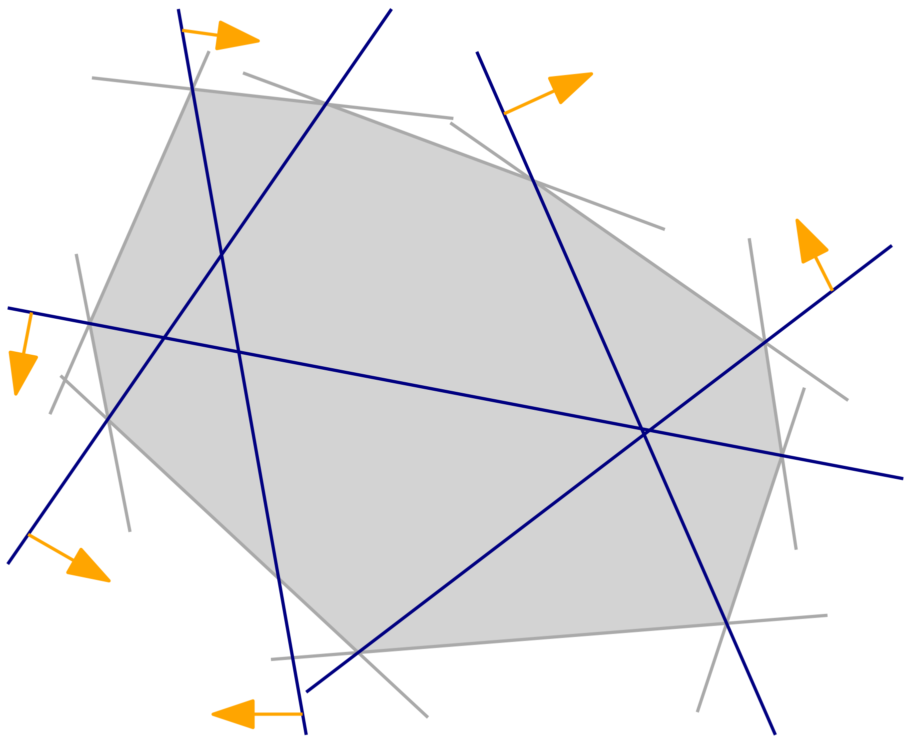
tope graph $G_{\mathcal{L}}$ =incidence graph= induced graph in Q_E

◦ Covector axioms: $(E, \mathcal{L})$ COM iff

(FS) $\mathcal{L} \circ -\mathcal{L} \subseteq \mathcal{L}$

(SE) $\forall X, Y \in \mathcal{L}$ and $e \in S(X,Y) \exists Z \in \mathcal{L}$:
$Z_e = 0$ and $Z_f = X_f \circ Y_f$ for $f \notin S(X,Y)$.

Complexes of oriented matroids



Def[Bandelt, Chepoi, K '15]
realizable COM = sign systems from arrangement of open halfspaces and hyperplanes.

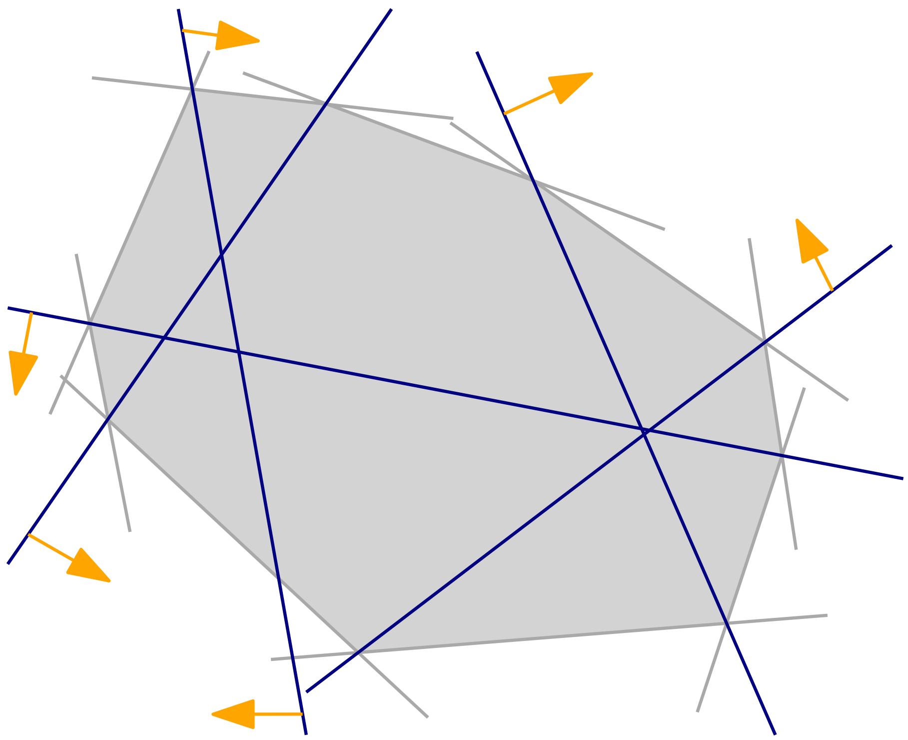
(FS) $\begin{pmatrix} 0 \\ + \\ - \\ + \end{pmatrix} \circ (-\begin{pmatrix} + \\ + \\ + \\ + \end{pmatrix}) = \begin{pmatrix} - \\ + \\ - \\ + \end{pmatrix}$

topes $\mathcal{T}$ of $\mathcal{M}$= maximal cells= max. covectors

tope graph $G_{\mathcal{L}}$ =incidence graph= induced graph in Q_E

◦ Covector axioms: $(E, \mathcal{L})$ COM iff
(FS) $\mathcal{L} \circ -\mathcal{L} \subseteq \mathcal{L}$
(SE) $\forall X, Y \in \mathcal{L}$ and $e \in S(X,Y) \exists Z \in \mathcal{L}$:
$Z_e = 0$ and $Z_f = X_f \circ Y_f$ for $f \notin S(X,Y)$.

Complexes of oriented matroids



Def[Bandelt, Chepoi, K '15]
realizable COM $=$ sign systems from arrangement of open halfspaces and hyperplanes.

(FS) $\begin{pmatrix} 0 \\ + \\ - \\ + \end{pmatrix} \circ \begin{pmatrix} - \\ - \\ - \\ - \end{pmatrix} = \begin{pmatrix} - \\ + \\ - \\ + \end{pmatrix}$

topes $\mathcal{T}$ of $\mathcal{M}=$ maximal cells$=$ max. covectors

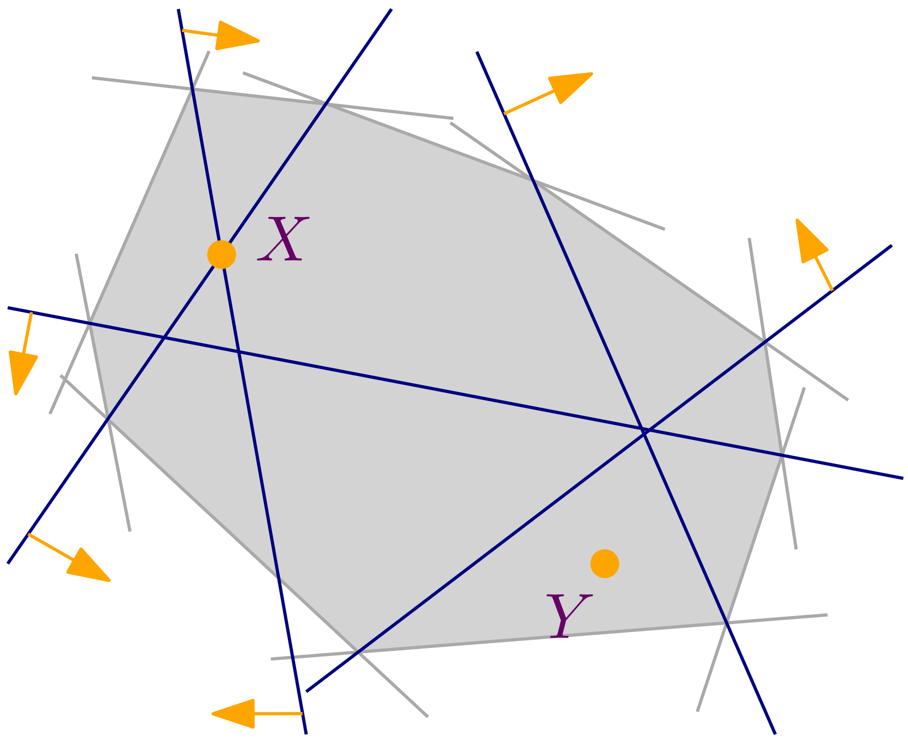
tope graph $G_{\mathcal{L}}$ $=$incidence graph$=$ induced graph in Q_E

◦ Covector axioms: $(E, \mathcal{L})$ COM iff

(FS) $\mathcal{L} \circ -\mathcal{L} \subseteq \mathcal{L}$

(SE) $\forall X, Y \in \mathcal{L}$ and $e \in S(X,Y) \exists Z \in \mathcal{L}$: $Z_e = 0$ and $Z_f = X_f \circ Y_f$ for $f \notin S(X,Y)$.

Complexes of oriented matroids



Def[Bandelt, Chepoi, K '15]
realizable COM $=$ sign systems from arrangement of open halfspaces and hyperplanes.

(FS)
$$\begin{pmatrix} 0 \\ + \\ - \\ + \end{pmatrix} \circ \begin{pmatrix} - \\ - \\ - \\ - \end{pmatrix} = \begin{pmatrix} - \\ + \\ - \\ + \end{pmatrix}$$

topes $\mathcal{T}$ of $\mathcal{M}=$ maximal cells$=$ max. covectors

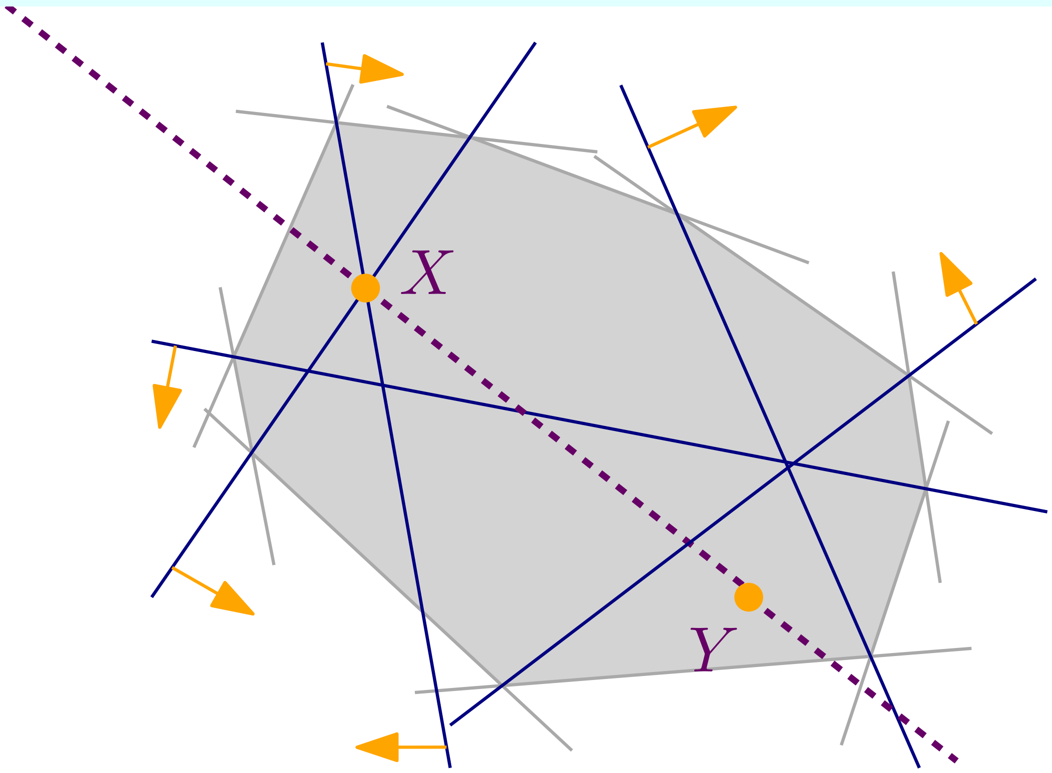
tope graph $G_{\mathcal{L}}$ $=$incidence graph$=$ induced graph in Q_E

- Covector axioms: $(E,\mathcal{L})$ COM iff

(FS) $\mathcal{L} \circ -\mathcal{L} \subseteq \mathcal{L}$

(SE) $\forall X, Y \in \mathcal{L}$ and $e \in S(X,Y) \exists Z \in \mathcal{L}$:
$Z_e = 0$ and $Z_f = X_f \circ Y_f$ for $f \notin S(X,Y)$.

Complexes of oriented matroids



Def[Bandelt, Chepoi, K '15]
realizable COM = sign systems from arrangement of open halfspaces and hyperplanes.

$$\text{(FS)} \quad \begin{pmatrix} 0 \\ + \\ - \\ + \end{pmatrix} \circ \begin{pmatrix} - \\ - \\ - \\ - \end{pmatrix} = \begin{pmatrix} - \\ + \\ - \\ + \end{pmatrix}$$

topes $\mathcal{T}$ of $\mathcal{M}$= maximal cells= max. covectors

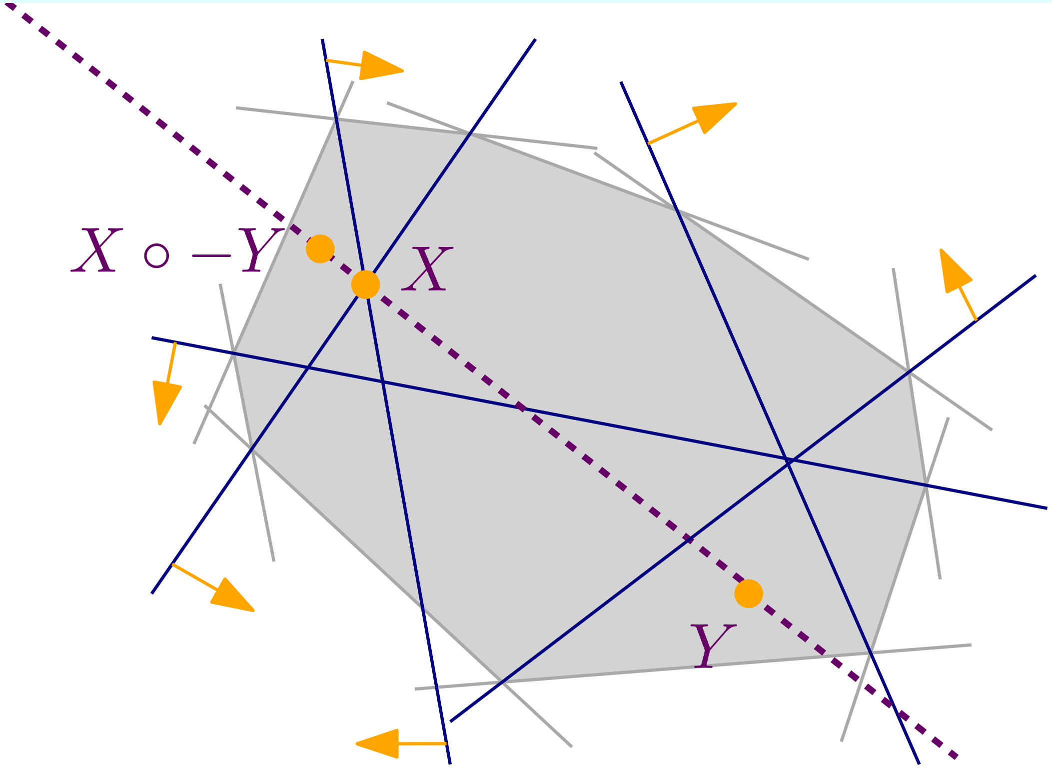
tope graph $G_{\mathcal{L}}$ =incidence graph= induced graph in Q_E

◦ Covector axioms: $(E, \mathcal{L})$ COM iff

(FS) $\mathcal{L} \circ -\mathcal{L} \subseteq \mathcal{L}$

(SE) $\forall X, Y \in \mathcal{L}$ and $e \in S(X,Y) \exists Z \in \mathcal{L}$:
$Z_e = 0$ and $Z_f = X_f \circ Y_f$ for $f \notin S(X,Y)$.

Complexes of oriented matroids



Def[Bandelt, Chepoi, K '15]
realizable COM = sign systems from arrangement of open halfspaces and hyperplanes.

(FS) $\begin{pmatrix} 0 \\ + \\ - \\ + \end{pmatrix} \circ \begin{pmatrix} - \\ - \\ - \\ - \end{pmatrix} = \begin{pmatrix} - \\ + \\ - \\ + \end{pmatrix}$

topes $\mathcal{T}$ of $\mathcal{M}$= maximal cells= max. covectors

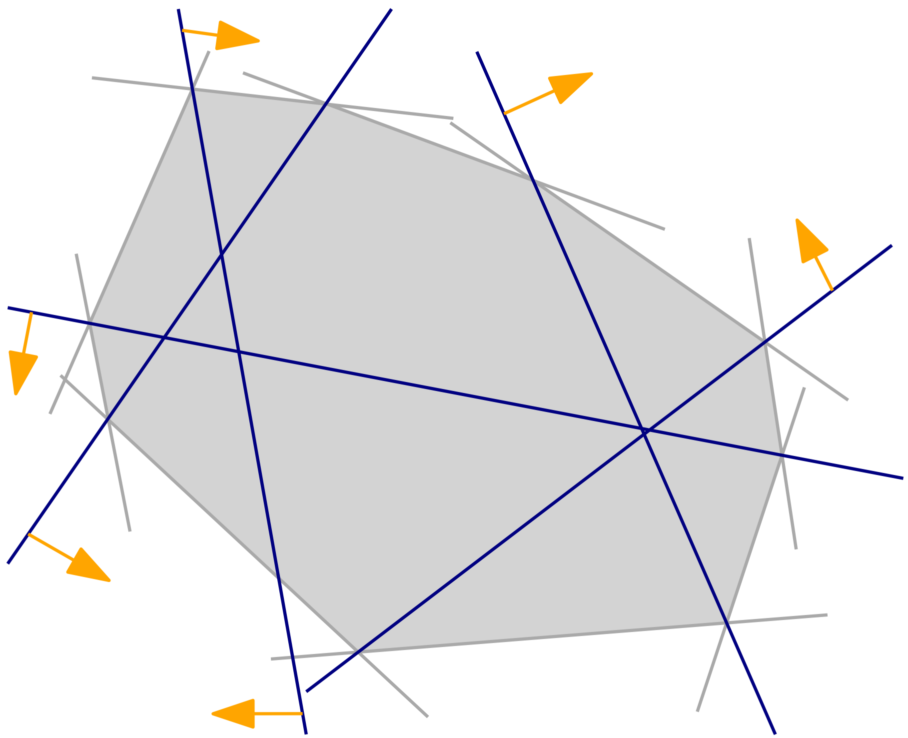
tope graph $G_{\mathcal{L}}$ =incidence graph= induced graph in Q_E

- Covector axioms: $(E, \mathcal{L})$ COM iff

(FS) $\mathcal{L} \circ -\mathcal{L} \subseteq \mathcal{L}$

(SE) $\forall X, Y \in \mathcal{L}$ and $e \in S(X,Y) \exists Z \in \mathcal{L}$: $Z_e = 0$ and $Z_f = X_f \circ Y_f$ for $f \notin S(X,Y)$.

Complexes of oriented matroids



Def[Bandelt, Chepoi, K '15]
realizable COM $=$ sign systems from arrangement of open halfspaces and hyperplanes.

(SE) $\begin{pmatrix} 0 \\ + \\ - \\ + \end{pmatrix}, \begin{pmatrix} - \\ - \\ - \\ - \end{pmatrix}$

topes $\mathcal{T}$ of $\mathcal{M}=$ maximal cells$=$ max. covectors

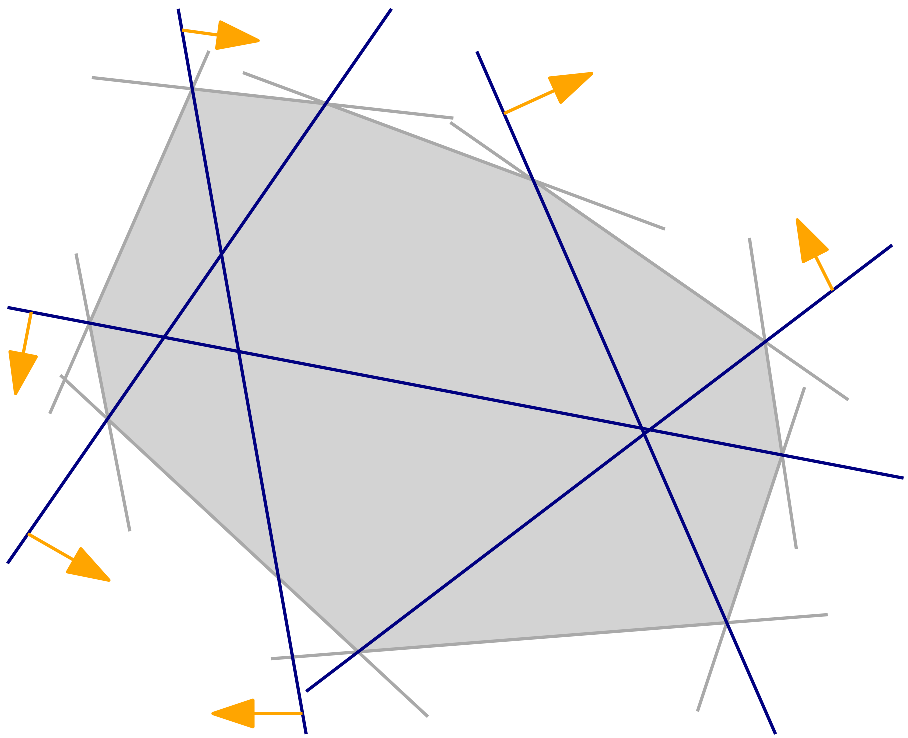
tope graph $G_{\mathcal{L}}$ $=$incidence graph$=$ induced graph in Q_E

∘ Covector axioms: $(E, \mathcal{L})$ COM iff

(FS) $\mathcal{L} \circ -\mathcal{L} \subseteq \mathcal{L}$

(SE) $\forall X, Y \in \mathcal{L}$ and $e \in S(X,Y) \exists Z \in \mathcal{L}$: $Z_e = 0$ and $Z_f = X_f \circ Y_f$ for $f \notin S(X,Y)$.

Complexes of oriented matroids



Def[Bandelt, Chepoi, K '15]
realizable COM = sign systems from arrangement of open halfspaces and hyperplanes.

topes $\mathcal{T}$ of $\mathcal{M}$= maximal cells= max. covectors

(SE) $e \begin{pmatrix} 0 \\ + \\ - \\ + \end{pmatrix}, \begin{pmatrix} - \\ - \\ - \\ - \end{pmatrix}$

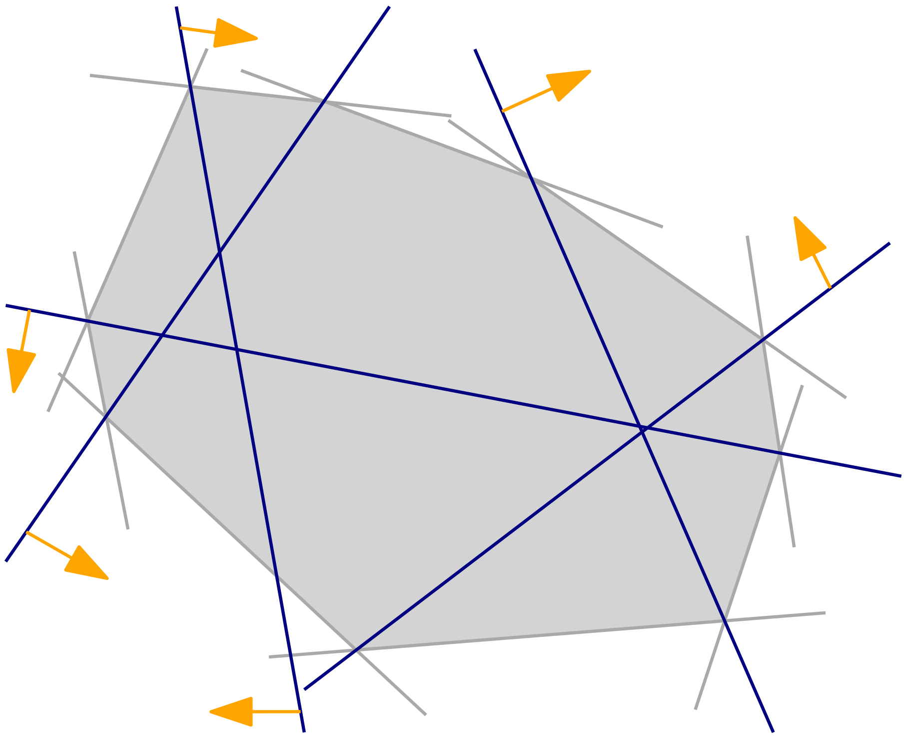
tope graph $G_{\mathcal{L}}$ =incidence graph= induced graph in Q_E

◦ Covector axioms: $(E, \mathcal{L})$ COM iff

(FS) $\mathcal{L} \circ -\mathcal{L} \subseteq \mathcal{L}$

(SE) $\forall X, Y \in \mathcal{L}$ and $e \in S(X,Y) \exists Z \in \mathcal{L}$: $Z_e = 0$ and $Z_f = X_f \circ Y_f$ for $f \notin S(X,Y)$.

Complexes of oriented matroids



Def[Bandelt, Chepoi, K '15]
realizable COM $=$ sign systems from arrangement of open halfspaces and hyperplanes.

topes $\mathcal{T}$ of $\mathcal{M}=$ maximal cells$=$ max. covectors

(SE) e $\begin{pmatrix} 0 \\ + \\ - \\ + \end{pmatrix}, \begin{pmatrix} - \\ - \\ - \\ - \end{pmatrix} \rightsquigarrow \begin{pmatrix} - \\ 0 \\ - \\ ? \end{pmatrix}$

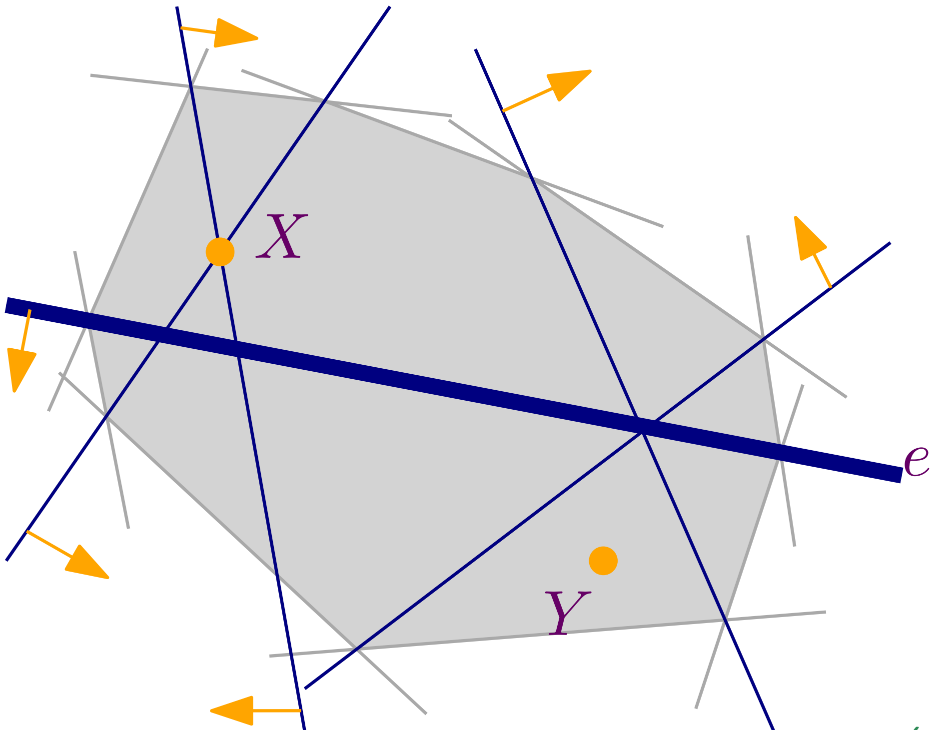
tope graph $G_{\mathcal{L}}$ $=$incidence graph$=$ induced graph in Q_E

◦ Covector axioms: $(E, \mathcal{L})$ COM iff

(FS) $\mathcal{L} \circ -\mathcal{L} \subseteq \mathcal{L}$

(SE) $\forall X, Y \in \mathcal{L}$ and $e \in S(X,Y) \exists Z \in \mathcal{L}$: $Z_e = 0$ and $Z_f = X_f \circ Y_f$ for $f \notin S(X,Y)$.

Complexes of oriented matroids



Def[Bandelt, Chepoi, K '15]
realizable COM = sign systems from arrangement of open halfspaces and hyperplanes.

topes $\mathcal{T}$ of $\mathcal{M}$= maximal cells= max. covectors

(SE) $e \quad \begin{pmatrix} 0 \\ + \\ - \\ + \end{pmatrix}, \begin{pmatrix} - \\ - \\ - \\ - \end{pmatrix} \quad \rightsquigarrow \quad \begin{pmatrix} - \\ 0 \\ - \\ ? \end{pmatrix}$

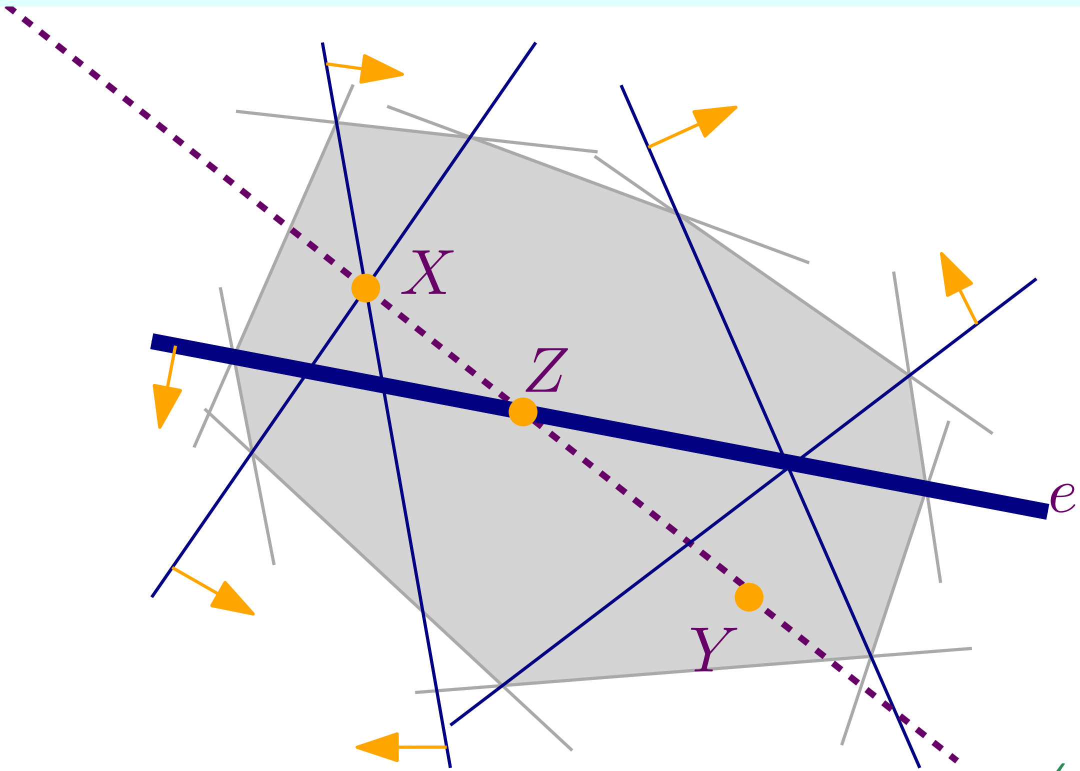
tope graph $G_{\mathcal{L}}$ =incidence graph= induced graph in Q_E

∘ Covector axioms: $(E, \mathcal{L})$ COM iff

(FS) $\mathcal{L} \circ -\mathcal{L} \subseteq \mathcal{L}$

(SE) $\forall X, Y \in \mathcal{L}$ and $e \in S(X,Y) \exists Z \in \mathcal{L}$: $Z_e = 0$ and $Z_f = X_f \circ Y_f$ for $f \notin S(X,Y)$.

Complexes of oriented matroids



Def[Bandelt, Chepoi, K '15]

realizable COM = sign systems from arrangement of open halfspaces and hyperplanes.

topes $\mathcal{T}$ of $\mathcal{M}$= maximal cells= max. covectors

tope graph $G_{\mathcal{L}}$ =incidence graph= induced graph in Q_E

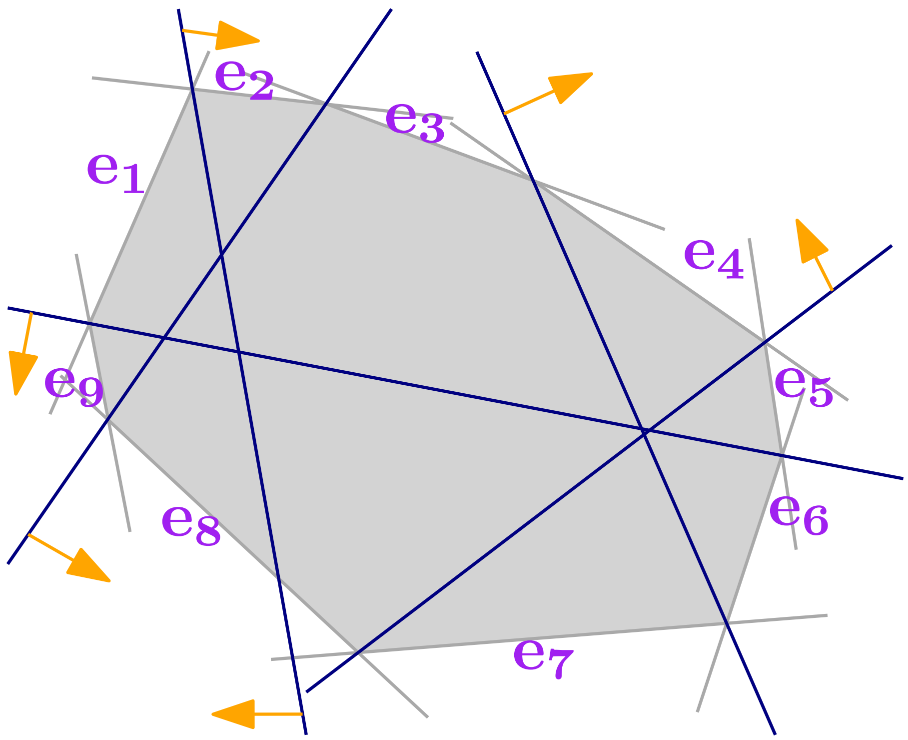
(SE) $e \begin{pmatrix} 0 \\ + \\ - \\ + \end{pmatrix}, \begin{pmatrix} - \\ - \\ - \\ - \end{pmatrix} \rightsquigarrow \begin{pmatrix} - \\ 0 \\ - \\ ? \end{pmatrix}$

◦ Covector axioms: $(E, \mathcal{L})$ COM iff

(FS) $\mathcal{L} \circ -\mathcal{L} \subseteq \mathcal{L}$

(SE) $\forall X, Y \in \mathcal{L}$ and $e \in S(X,Y) \exists Z \in \mathcal{L}$:
$Z_e = 0$ and $Z_f = X_f \circ Y_f$ for $f \notin S(X,Y)$.

Complexes of oriented matroids



Def[Bandelt, Chepoi, K '15]
realizable COM = sign systems from arrangement of open halfspaces and hyperplanes.

digraph example:
topes $\mathcal{T} \cong$ acyclic orientations with edges **E**'s orientation fixed

topes $\mathcal{T}$ of $\mathcal{M}$= maximal cells= max. covectors

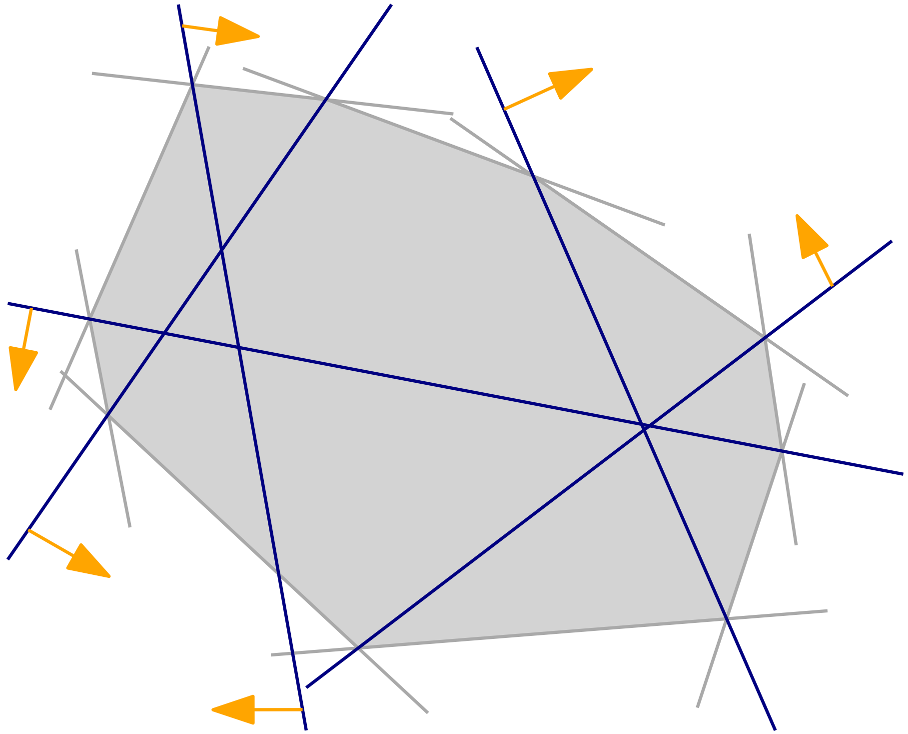
tope graph $G_{\mathcal{L}}$ =incidence graph= induced graph in Q_E

◦ Covector axioms: $(E, \mathcal{L})$ COM iff

(FS) $\mathcal{L} \circ -\mathcal{L} \subseteq \mathcal{L}$

(SE) $\forall X, Y \in \mathcal{L}$ and $e \in S(X,Y) \exists Z \in \mathcal{L}$:
$Z_e = 0$ and $Z_f = X_f \circ Y_f$ for $f \notin S(X,Y)$.

Complexes of oriented matroids



Def[Bandelt, Chepoi, K '15]
realizable COM $=$ sign systems from arrangement of open halfspaces and hyperplanes.

topes $\mathcal{T} \cong$ acyclic orientations of a **mixed graph**

topes $\mathcal{T}$ of $\mathcal{M}=$ maximal cells$=$ max. covectors

tope graph $G_{\mathcal{L}}$ $=$incidence graph$=$ induced graph in Q_E

◦ Covector axioms: $(E, \mathcal{L})$ COM iff

(FS) $\mathcal{L} \circ -\mathcal{L} \subseteq \mathcal{L}$

(SE) $\forall X, Y \in \mathcal{L}$ and $e \in S(X,Y) \exists Z \in \mathcal{L}$: $Z_e = 0$ and $Z_f = X_f \circ Y_f$ for $f \notin S(X,Y)$.

A common generalization

- Covector axioms: $(E, \mathcal{L})$ COM iff

(FS) $\mathcal{L} \circ -\mathcal{L} \subseteq \mathcal{L}$

(SE) $\forall X, Y \in \mathcal{L}$ and $e \in S(X,Y) \exists Z \in \mathcal{L}$:
$Z_e = 0$ and $Z_f = X_f \circ Y_f$ for $f \notin S(X,Y)$.

- Covector axioms: $(E, \mathcal{L})$ oriented matroid:

(Z) $\emptyset \in \mathcal{L}$

(FS) $\mathcal{L} \circ -\mathcal{L} \subseteq \mathcal{L}$

(SE) $\forall X, Y \in \mathcal{L}$ and $e \in S(X,Y) \exists Z \in \mathcal{L}$:
$Z_e = 0$ and $Z_f = X_f \circ Y_f$ for $f \notin S(X,Y)$.

- Covector axioms: $(E, \mathcal{L})$ affine oriented matroid:

(A) *something lengthy*

(FS) $\mathcal{L} \circ -\mathcal{L} \subseteq \mathcal{L}$

(SE) $\forall X, Y \in \mathcal{L}$ and $e \in S(X,Y) \exists Z \in \mathcal{L}$:
$Z_e = 0$ and $Z_f = X_f \circ Y_f$ for $f \notin S(X,Y)$.

A common generalization

◦ Covector axioms: $(E,\mathcal{L})$ COM iff

(FS) $\mathcal{L} \circ -\mathcal{L} \subseteq \mathcal{L}$

(SE) $\forall X,Y \in \mathcal{L}$ and $e \in S(X,Y) \exists Z \in \mathcal{L}$:
$Z_e = 0$ and $Z_f = X_f \circ Y_f$ for $f \notin S(X,Y)$.

◦ Covector axioms: $(E,\mathcal{L})$ oriented matroid:

(Z) $\emptyset \in \mathcal{L}$

(FS) $\mathcal{L} \circ -\mathcal{L} \subseteq \mathcal{L}$

(SE) $\forall X,Y \in \mathcal{L}$ and $e \in S(X, Y$ ...
$Z_e = 0$ and $Z_f = X$ ... (X,Y).

◦ Covector axio... oriented matroid:

(A) *somet*...

(FS)

... and $e \in S(X,Y) \exists Z \in \mathcal{L}$:

... $= 0$ and $Z_f = X_f \circ Y_f$ for $f \notin S(X,Y)$.

tope graphs partial cubes and determine $\mathcal{L}$

Partial cubes and partial cube minors

G partial cube $:\Leftrightarrow$ G isometric subgraph of hypercube

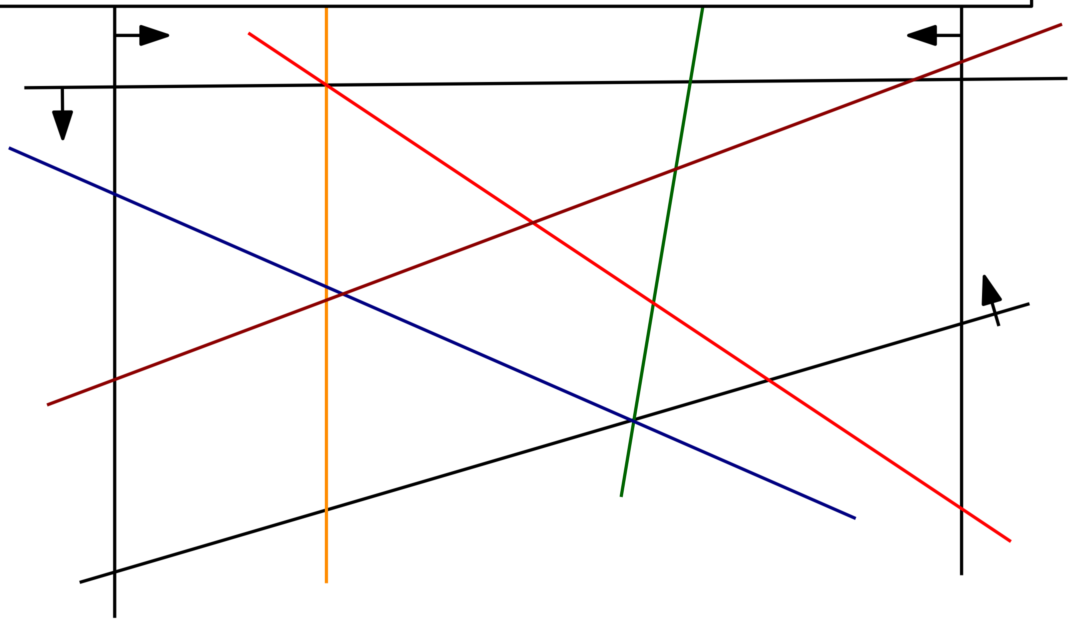
$G \subseteq Q^n$ such that $d_G(v,w) = d_{Q^n}(v,w) \forall v, w \in G$

Partial cubes and partial cube minors

G partial cube $:\Leftrightarrow$ G isometric subgraph of hypercube

$G \subseteq Q^n$ such that $d_G(v,w) = d_{Q^n}(v,w) \forall v,w \in G$

tope graph of realizable COM
(arrangement of half and hyperplanes)

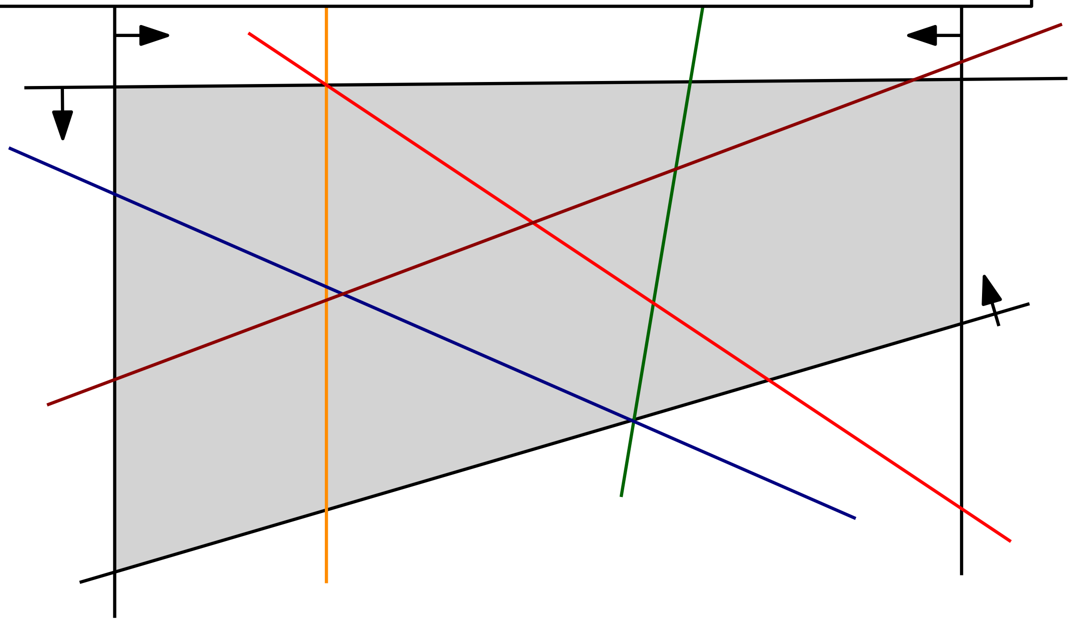


Partial cubes and partial cube minors

G partial cube $:\Leftrightarrow$ G isometric subgraph of hypercube

$G \subseteq Q^n$ such that $d_G(v,w) = d_{Q^n}(v,w) \forall v,w \in G$

tope graph of realizable COM
(arrangement of half and hyperplanes)

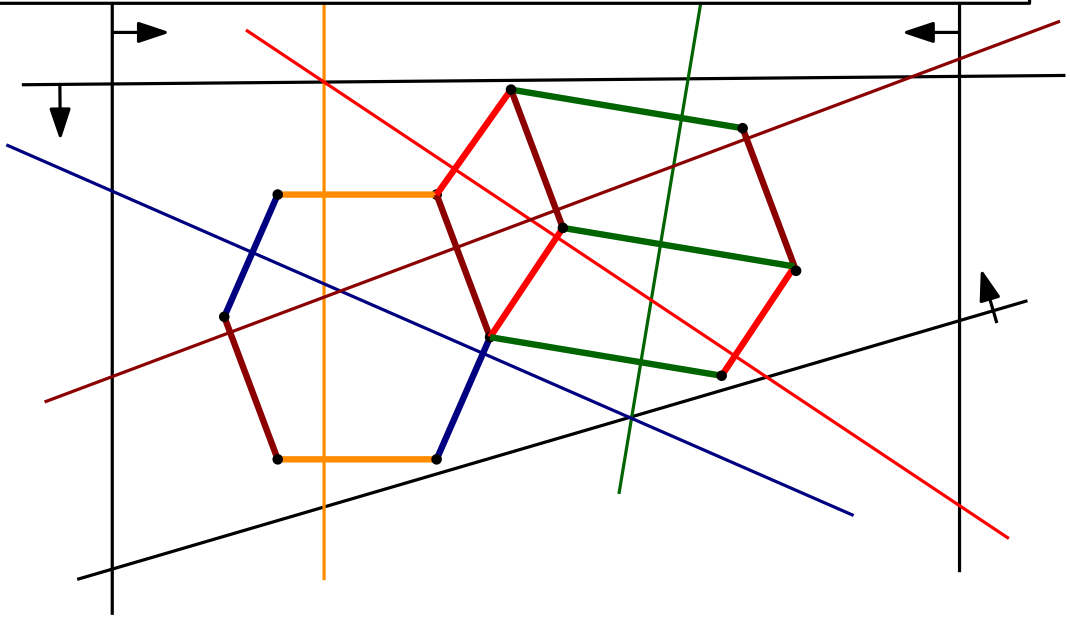


Partial cubes and partial cube minors

G partial cube $:\Leftrightarrow$ G isometric subgraph of hypercube

$G \subseteq Q^n$ such that $d_G(v,w) = d_{Q^n}(v,w) \forall v,w \in G$

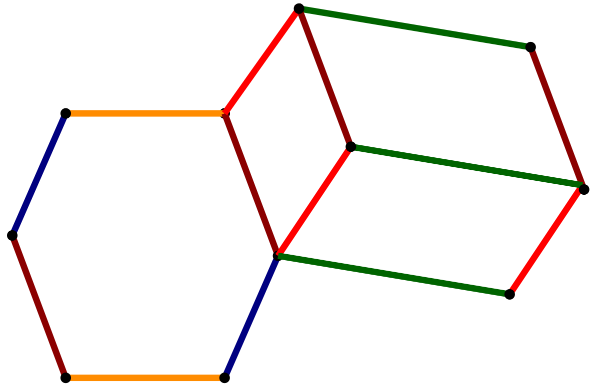
tope graph of realizable COM
(arrangement of half and hyperplanes)



Partial cubes and partial cube minors

G partial cube $:\Leftrightarrow$ G isometric subgraph of hypercube

$G \subseteq Q^n$ such that $d_G(v,w) = d_{Q^n}(v,w) \forall v,w \in G$

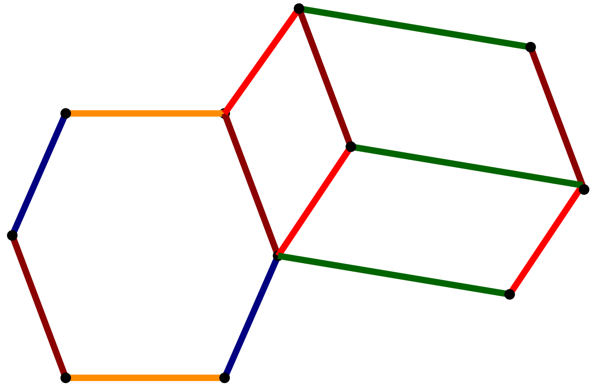


edges of partial cube naturally
partitioned into minimal cuts $\mathcal{C}$

Partial cubes and partial cube minors

G partial cube $:\Leftrightarrow$ G isometric subgraph of hypercube

$G \subseteq Q^n$ such that $d_G(v,w) = d_{Q^n}(v,w) \forall v,w \in G$



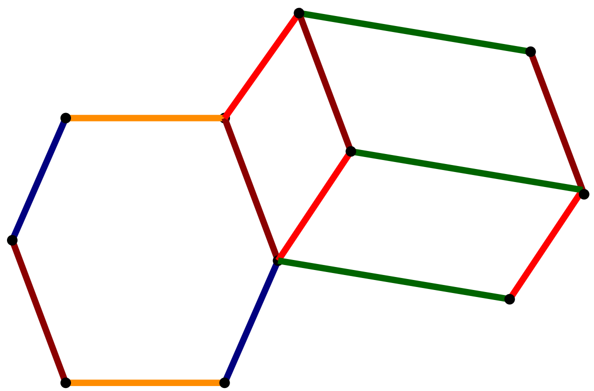
edges of partial cube naturally
partitioned into minimal cuts $\mathcal{C}$
$\rightsquigarrow$ minor-relation

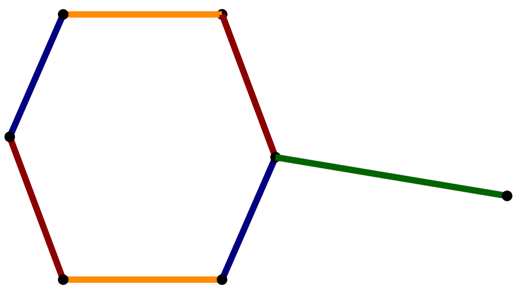
Partial cubes and partial cube minors

G partial cube $:\Leftrightarrow$ G isometric subgraph of hypercube

$G \subseteq Q^n$ such that $d_G(v,w) = d_{Q^n}(v,w) \forall v, w \in G$

restriction to a side of a cut



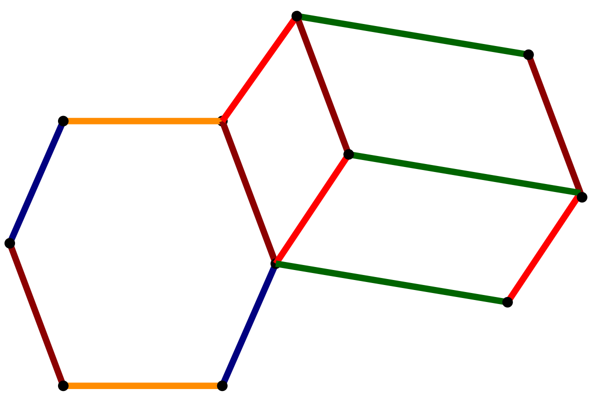


edges of partial cube naturally
partitioned into minimal cuts $\mathcal{C}$
$\rightsquigarrow$ minor-relation

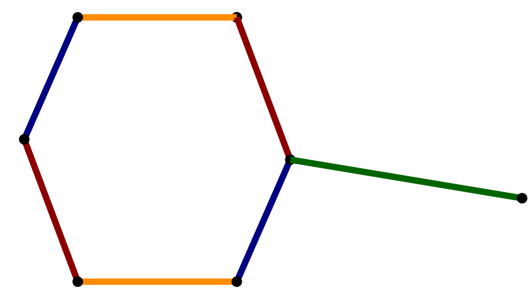
Partial cubes and partial cube minors

G partial cube $:\Leftrightarrow$ G isometric subgraph of hypercube

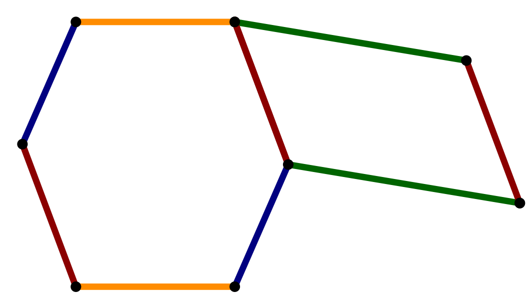
$G \subseteq Q^n$ such that $d_G(v,w) = d_{Q^n}(v,w) \forall v,w \in G$



restriction to a side of a cut



contraction of a cut



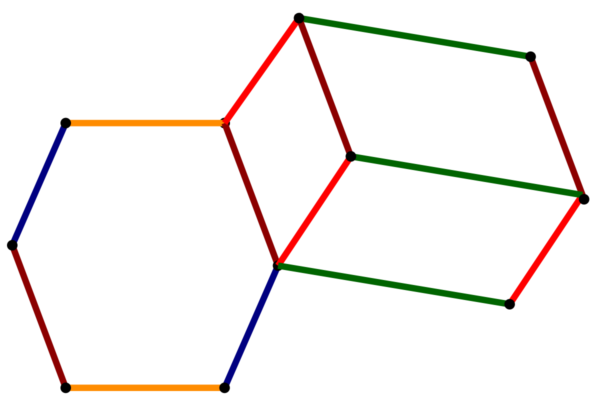
edges of partial cube naturally partitioned into minimal cuts $\mathcal{C}$
$\rightsquigarrow$ minor-relation

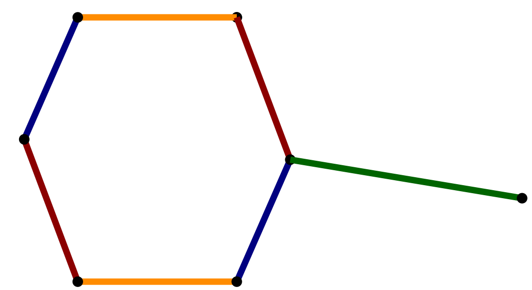
Partial cubes and partial cube minors

G partial cube $:\Leftrightarrow G$ isometric subgraph of hypercube

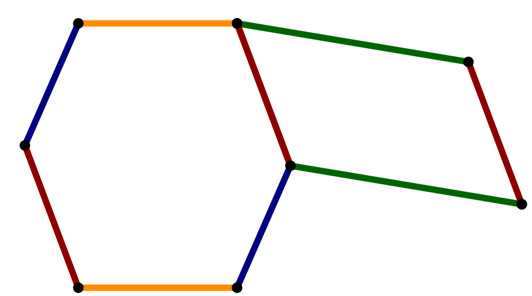
$G \subseteq Q^n$ such that $d_G(v,w) = d_{Q^n}(v,w) \forall v,w \in G$

restriction to a side of a cut





contraction of a cut



edges of partial cube naturally partitioned into minimal cuts $\mathcal{C}$

$\rightsquigarrow$ minor-relation

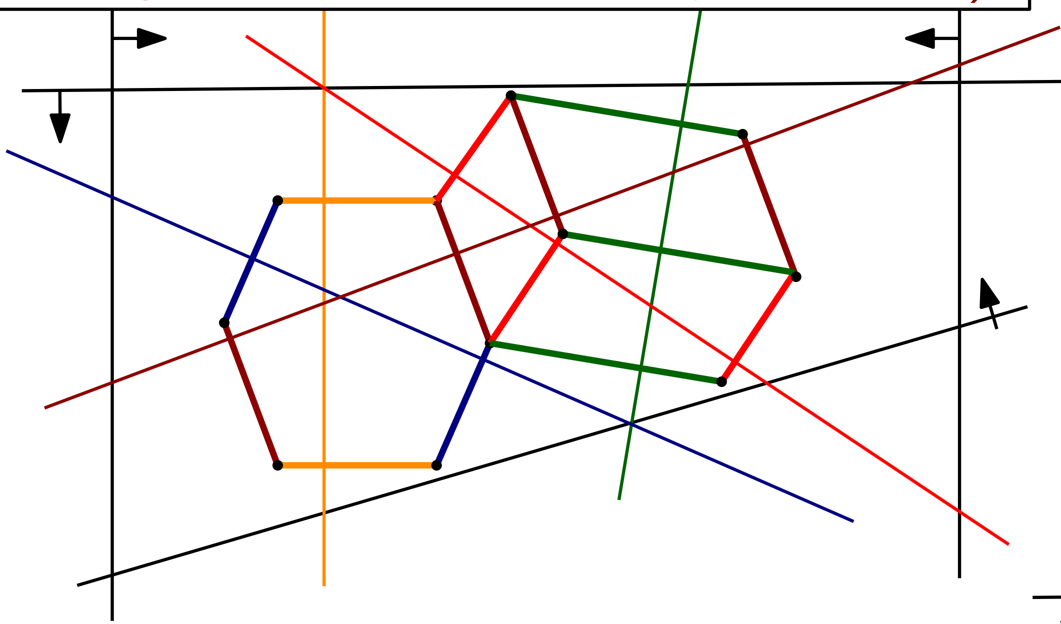
$\rightsquigarrow$ yields new partial cube

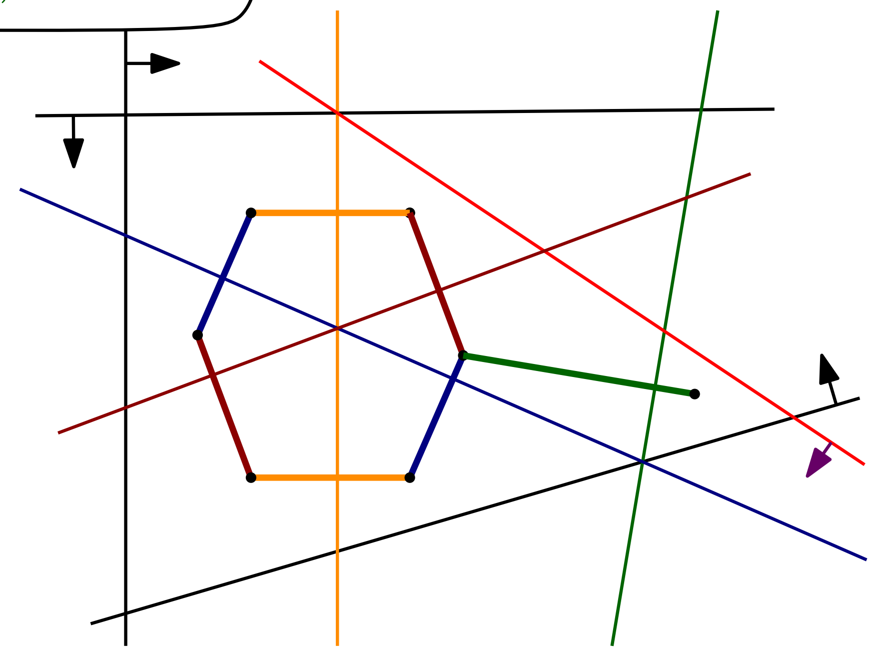
Partial cubes and partial cube minors

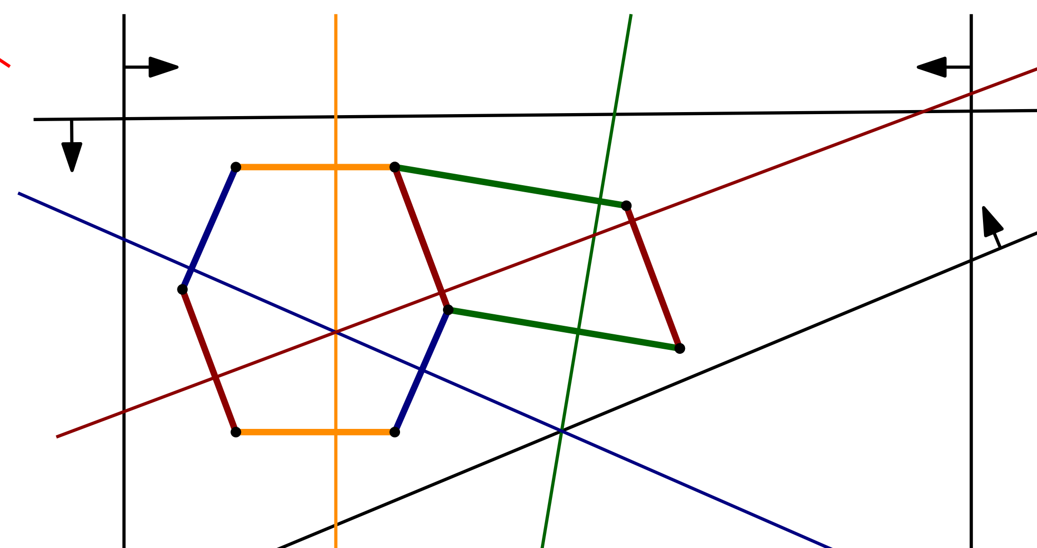
G partial cube $:\Leftrightarrow$ G isometric subgraph of hypercube

$G \subseteq Q^n$ such that $d_G(v,w) = d_{Q^n}(v,w) \forall v,w \in G$

tope graph of realizable COM
(arrangement of half and hyperplanes)





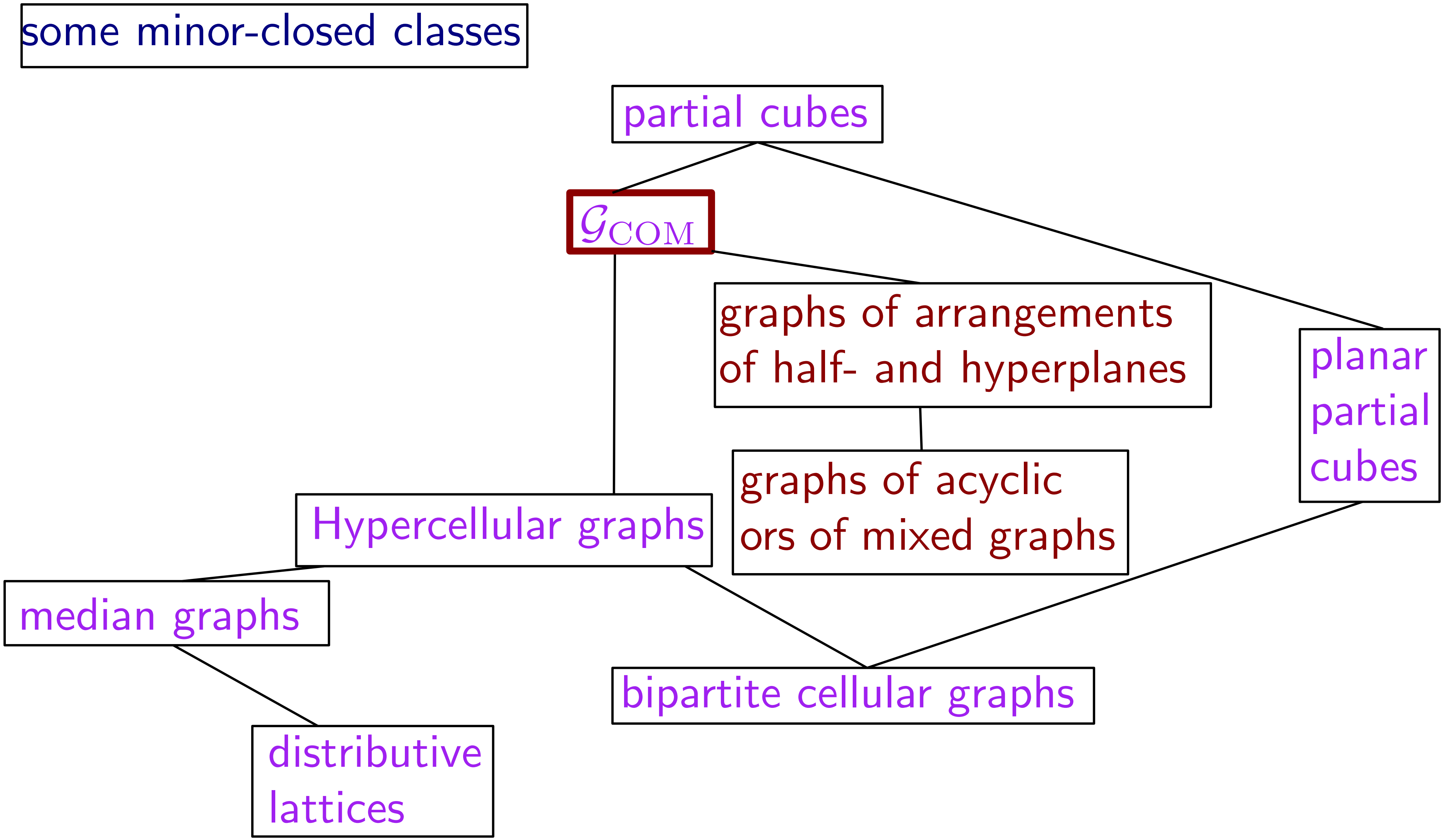


edges of partial cube naturally partitioned into minimal cuts $\mathcal{C}$

$\rightsquigarrow$ minor-relation

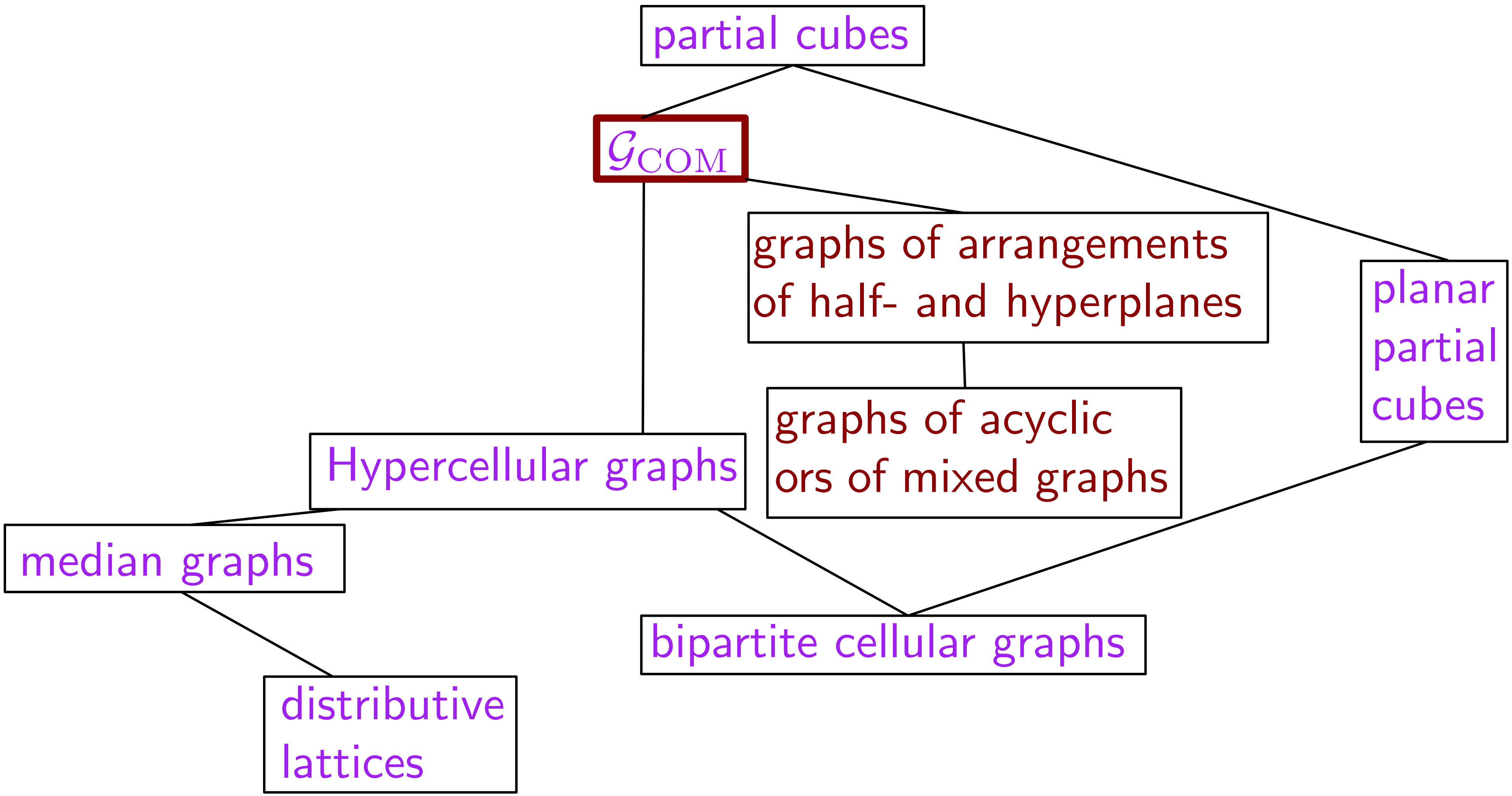
$\rightsquigarrow$ yields new tope graph

Partial cube minors



Partial cube minors

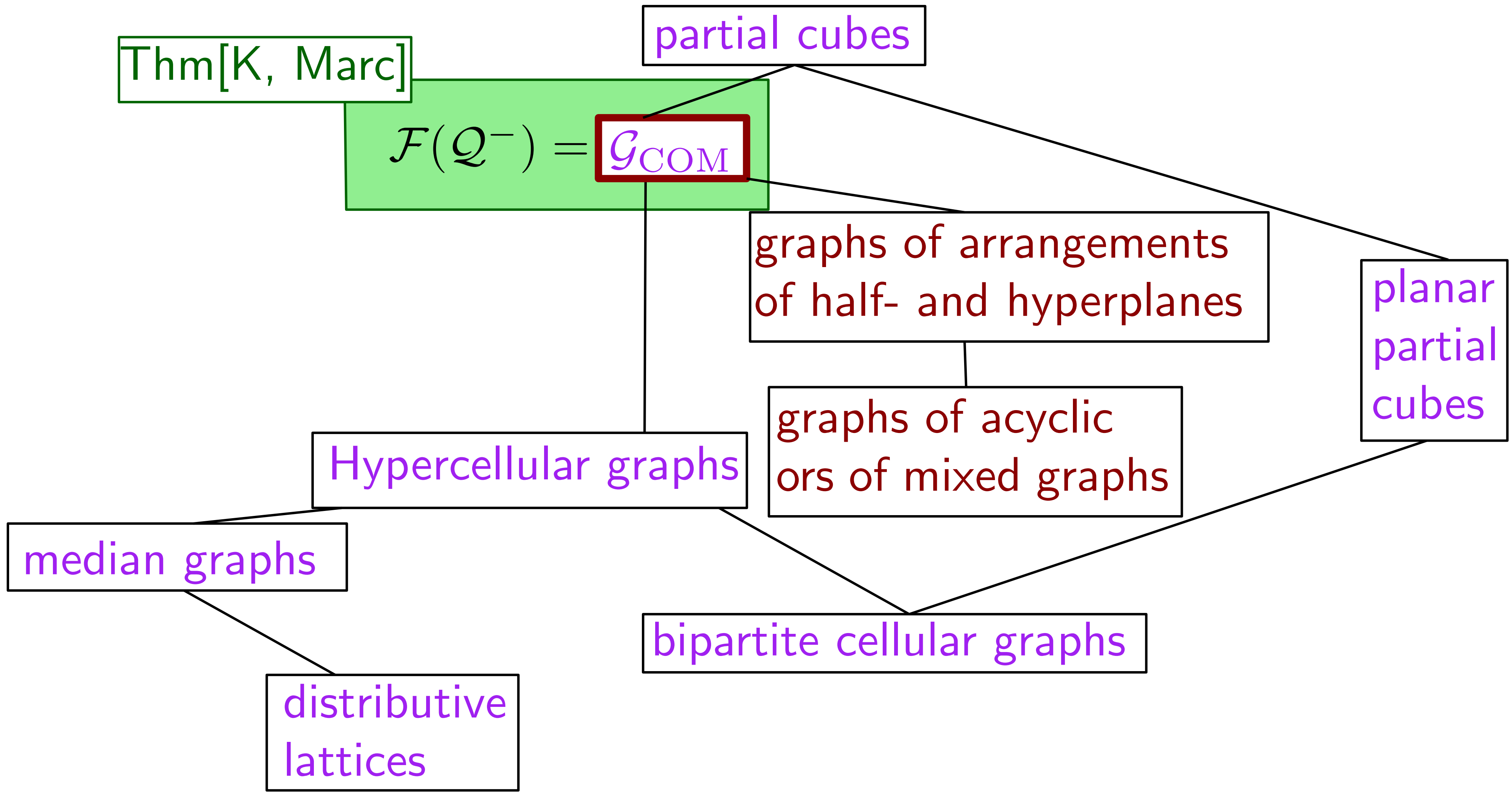
some minor-closed classes



each has a family of excluded minors

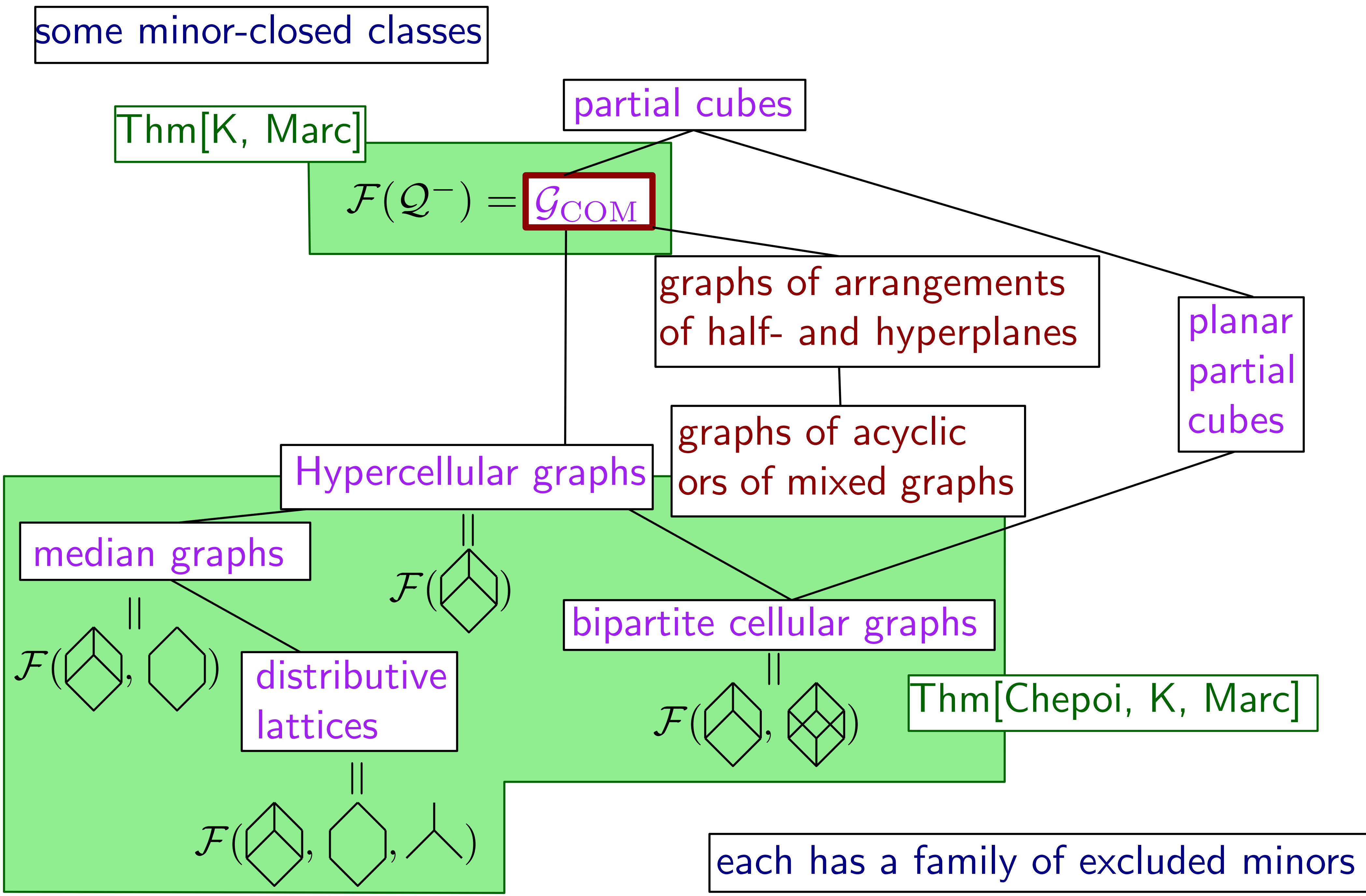
Partial cube minors

some minor-closed classes

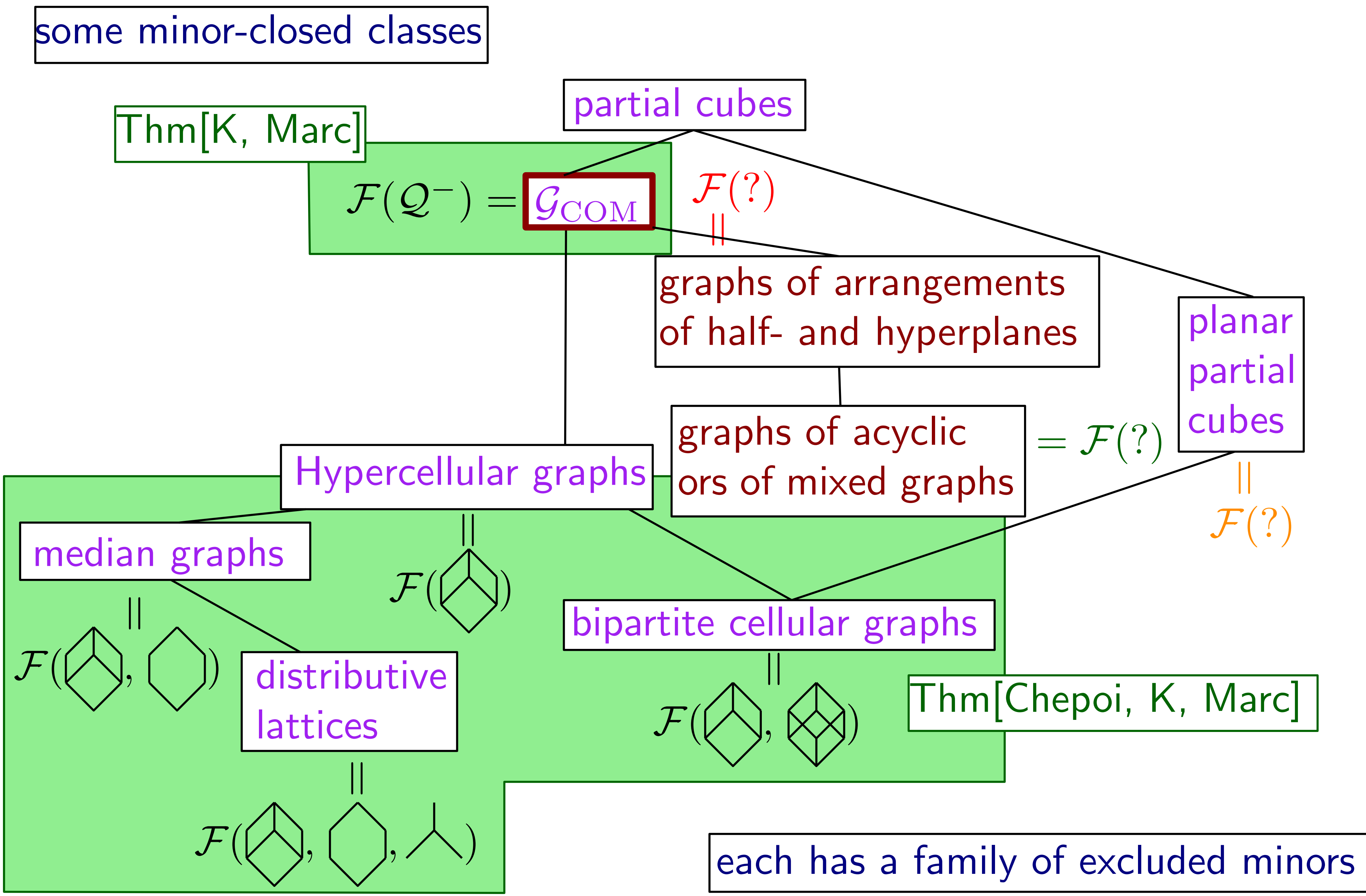


each has a family of excluded minors

Partial cube minors



Partial cube minors



From partial cubes to sign vectors

Let G partial cube, then $G' \subset G$ convex $\iff$ G' restriction of G

shortest paths between vertices of G' stay in G'

From partial cubes to sign vectors

Let G partial cube, then $G' \subset G$ convex $\iff$ G' restriction of G

shortest paths between vertices of G' stay in G'

intersection of halfspaces $X(G')$ containing G'

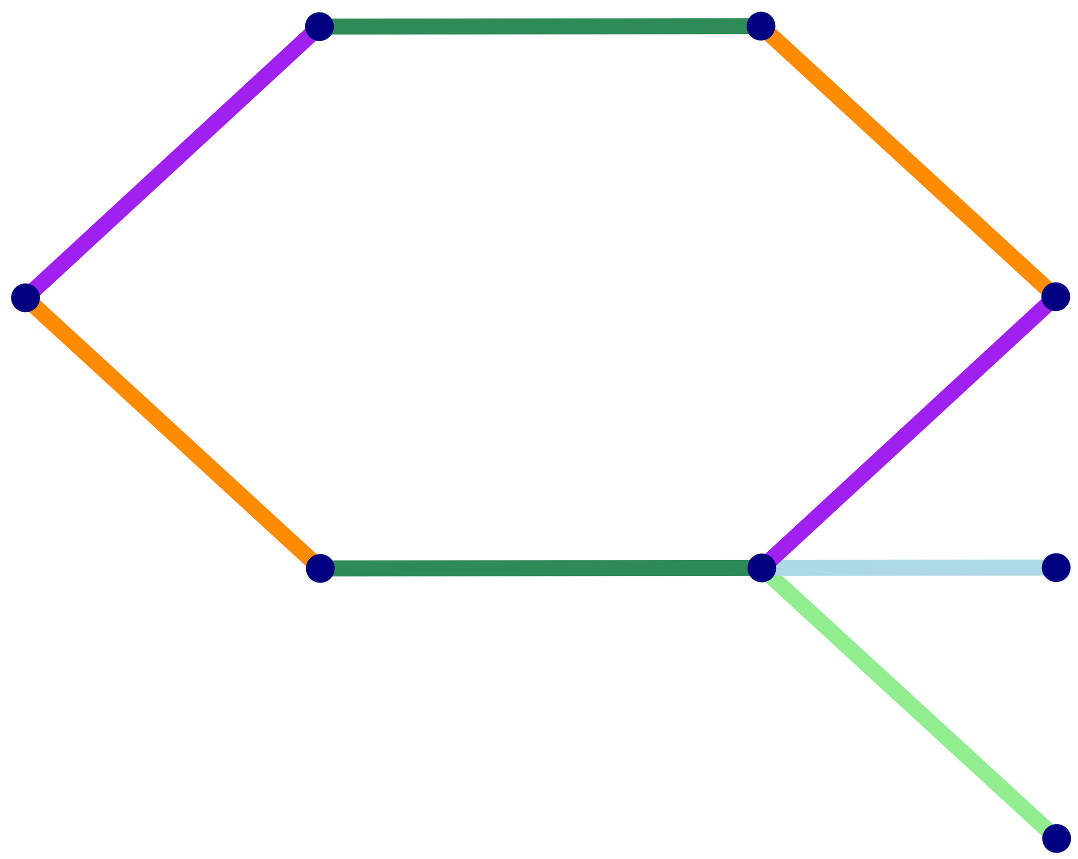
From partial cubes to sign vectors

Let G partial cube, then $G' \subset G$ convex $\iff$ G' restriction of G

shortest paths between vertices of G' stay in G'

intersection of halfspaces $X(G')$ containing G'

associate convex subgraph G' with sign vector $X(G')$



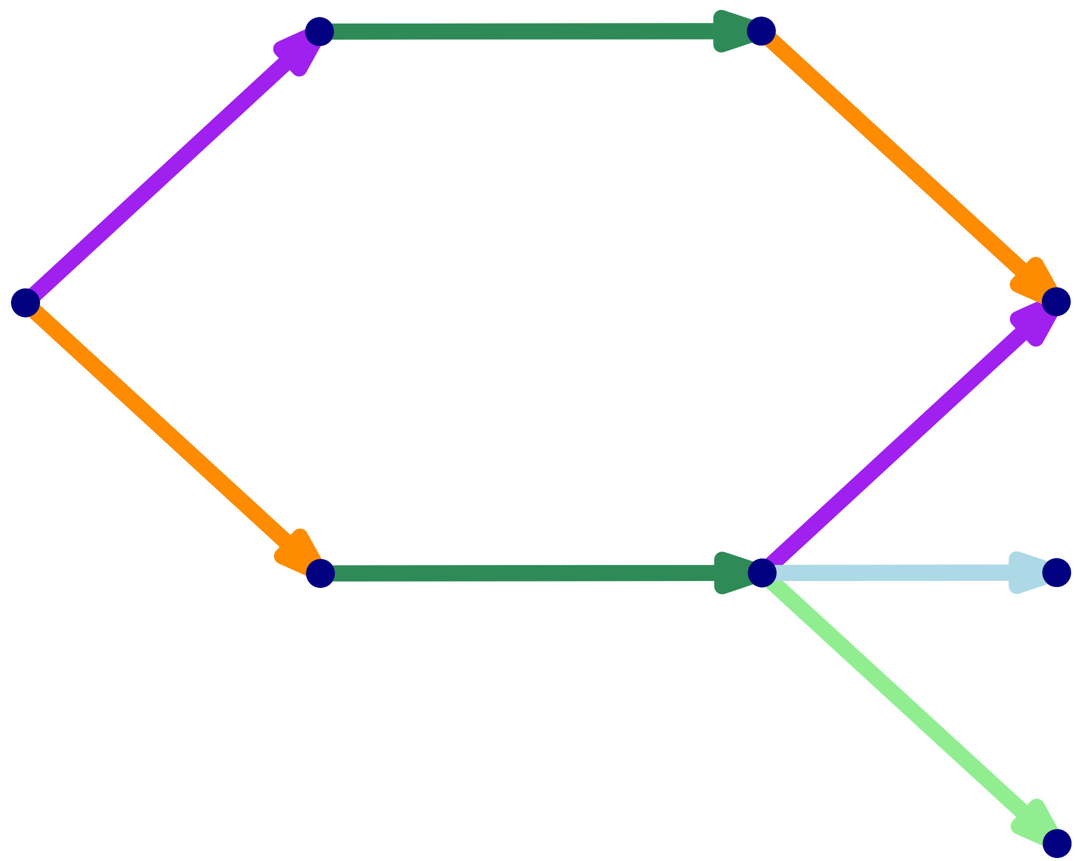
From partial cubes to sign vectors

Let G partial cube, then $G' \subset G$ convex $\iff$ G' restriction of G

shortest paths between vertices of G' stay in G'

intersection of halfspaces $X(G')$ containing G'

associate convex subgraph G' with sign vector $X(G')$



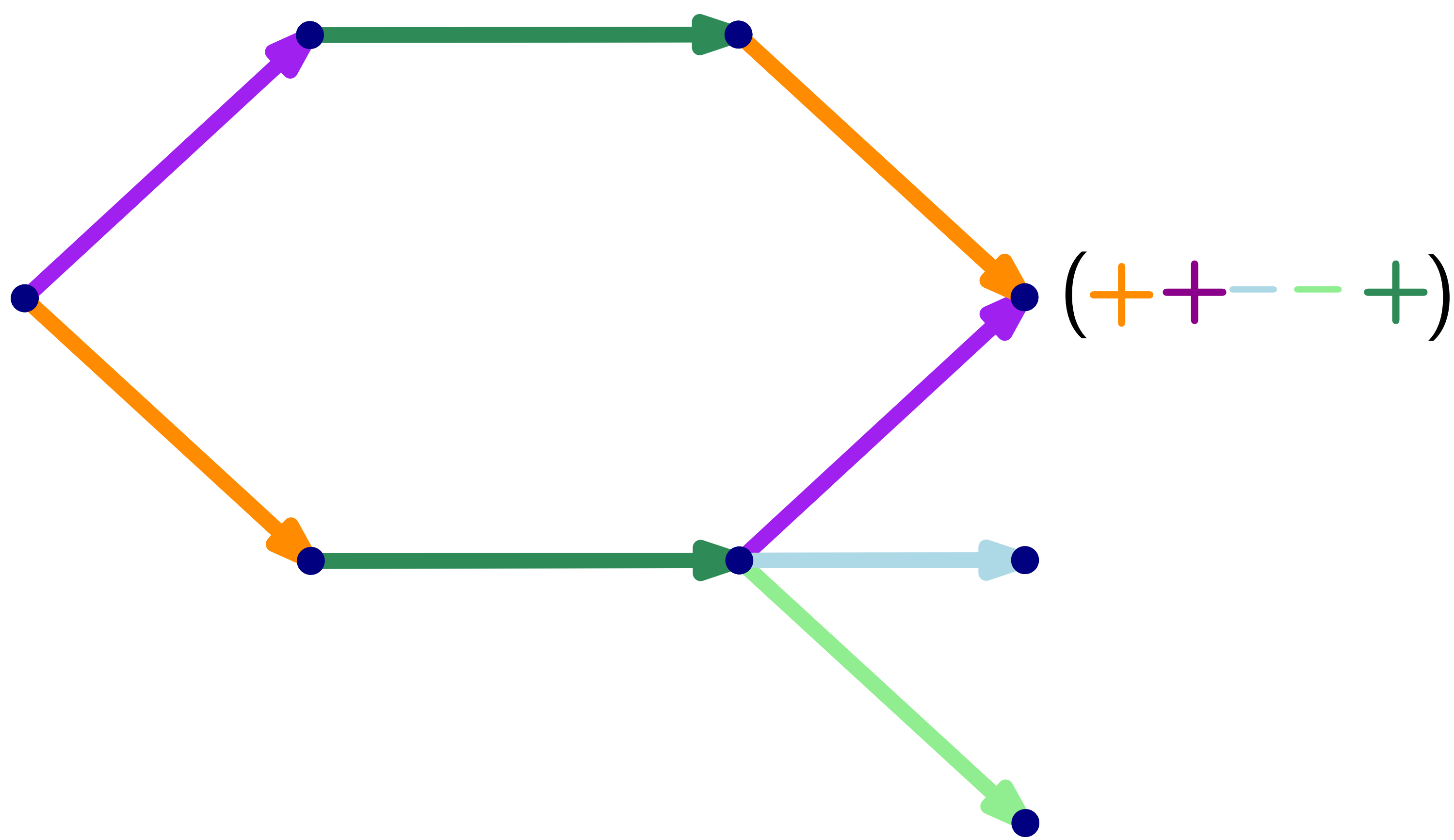
From partial cubes to sign vectors

Let G partial cube, then $G' \subset G$ convex $\iff$ G' restriction of G

shortest paths between vertices of G' stay in G'

intersection of halfspaces $X(G')$ containing G'

associate convex subgraph G' with sign vector $X(G')$



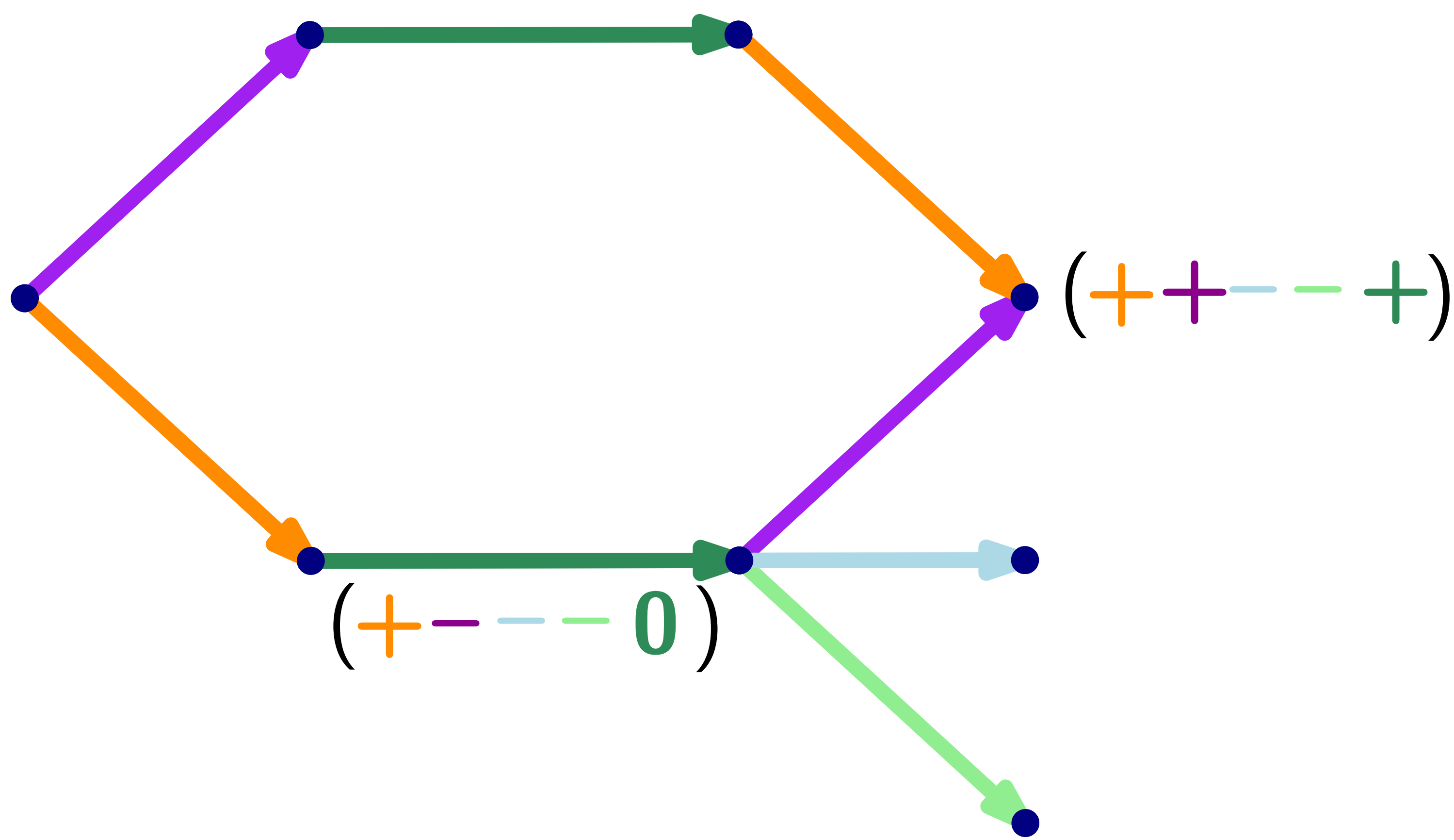
From partial cubes to sign vectors

Let G partial cube, then $G' \subset G$ convex $\iff$ G' restriction of G

shortest paths between vertices of G' stay in G'

intersection of halfspaces $X(G')$ containing G'

associate convex subgraph G' with sign vector $X(G')$



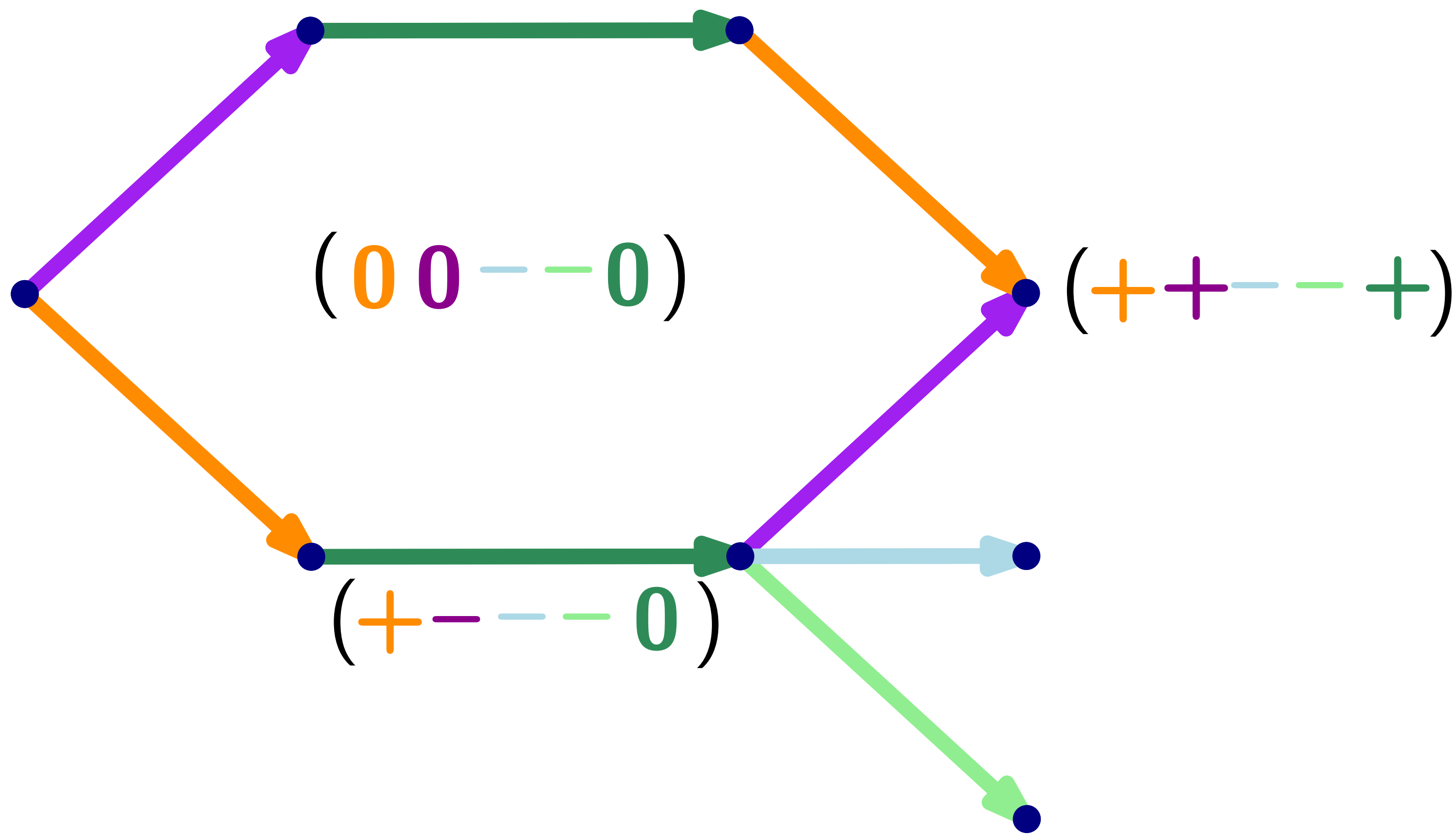
From partial cubes to sign vectors

Let G partial cube, then $G' \subset G$ convex $\iff$ G' restriction of G

shortest paths between vertices of G' stay in G'

intersection of halfspaces $X(G')$ containing G'

associate convex subgraph G' with sign vector $X(G')$



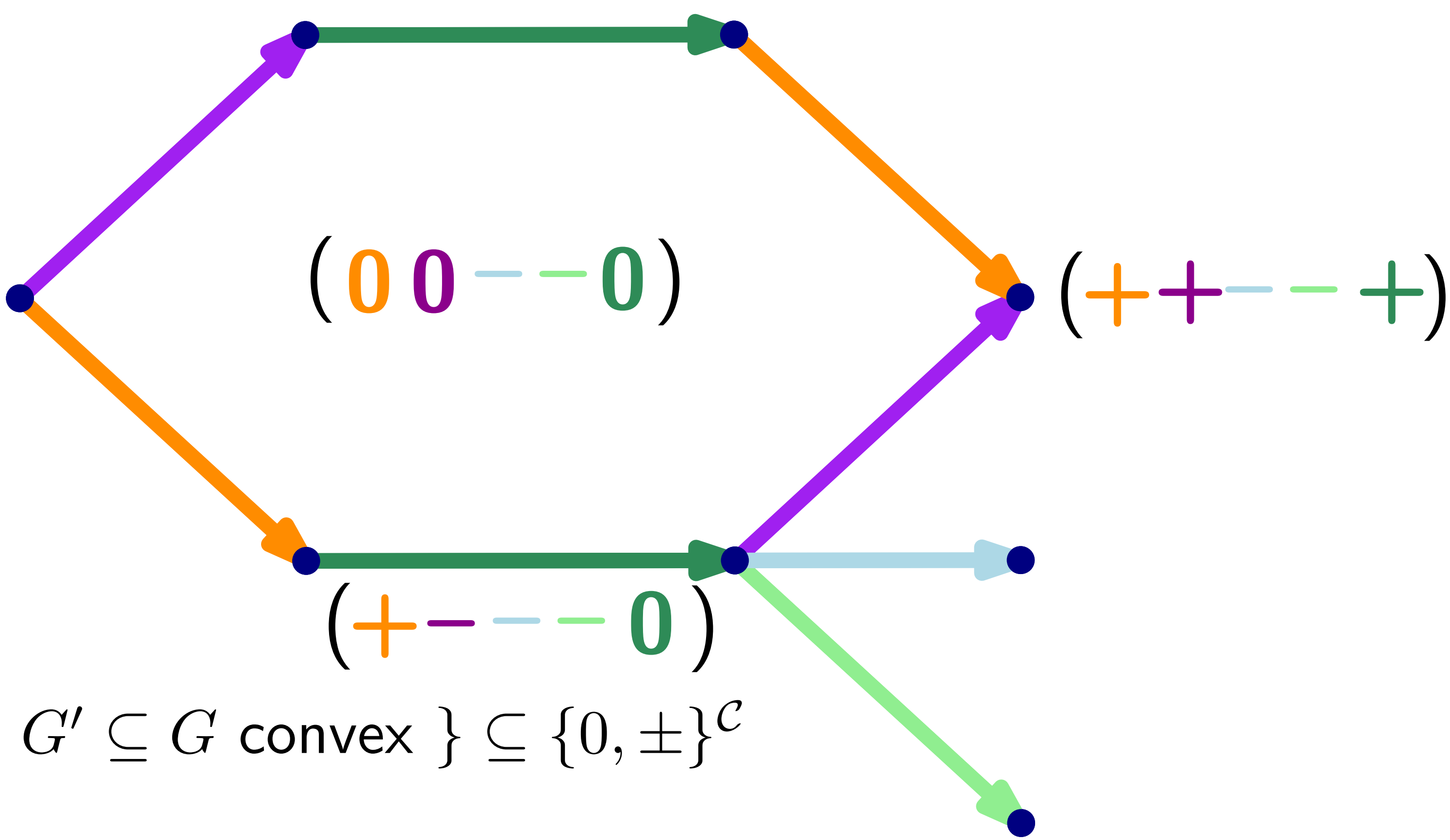
From partial cubes to sign vectors

Let G partial cube, then $G' \subset G$ convex $\iff$ G' restriction of G

shortest paths between vertices of G' stay in G'

intersection of halfspaces $X(G')$ containing G'

associate convex subgraph G' with sign vector $X(G')$



- $\mathcal{L} = \{X(G') \mid G' \subseteq G \text{ convex } \} \subseteq \{0, \pm\}^{\mathcal{C}}$

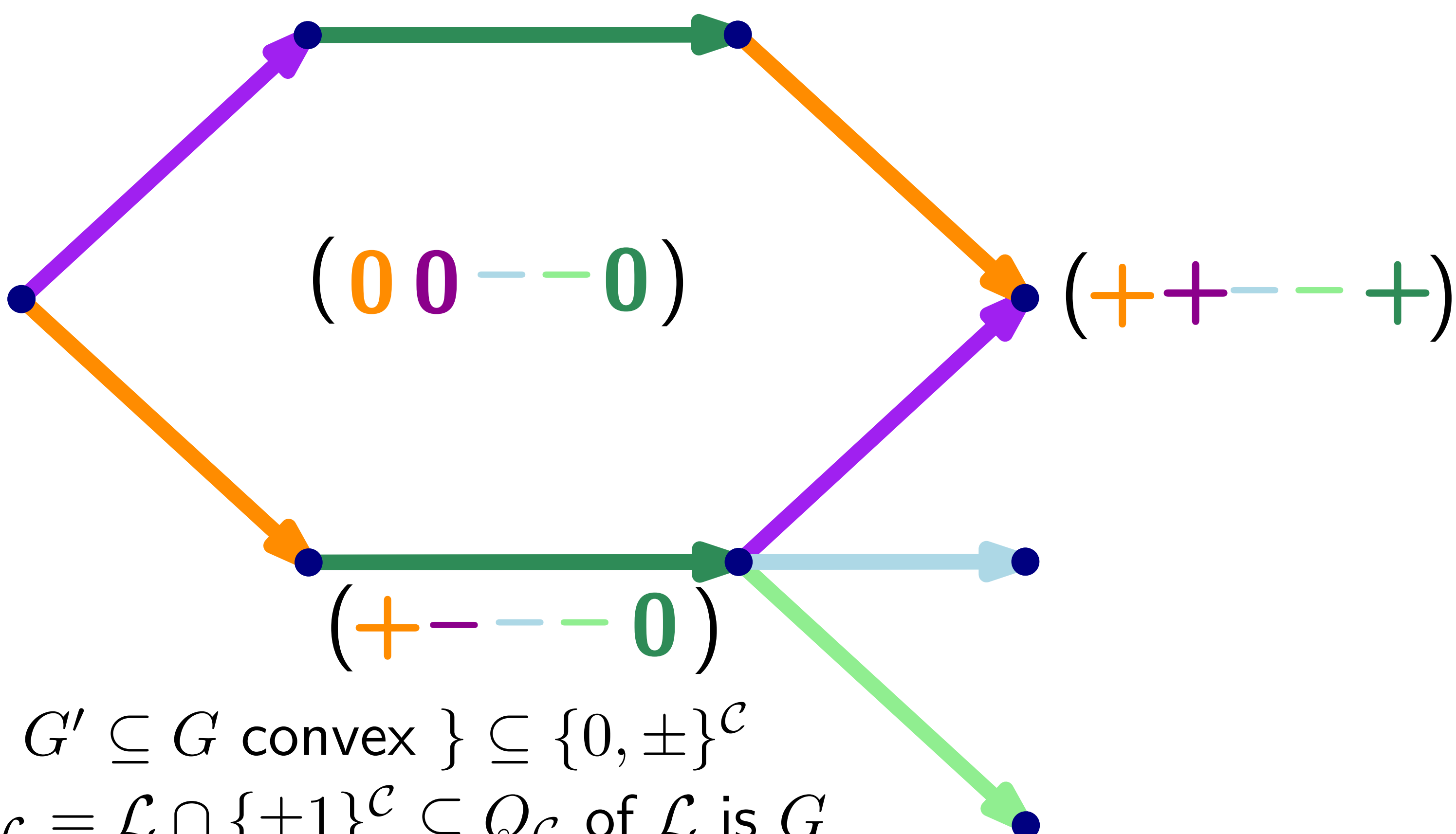
From partial cubes to sign vectors

Let G partial cube, then $G' \subset G$ convex $\iff$ G' restriction of G

shortest paths between vertices of G' stay in G'

intersection of halfspaces $X(G')$ containing G'

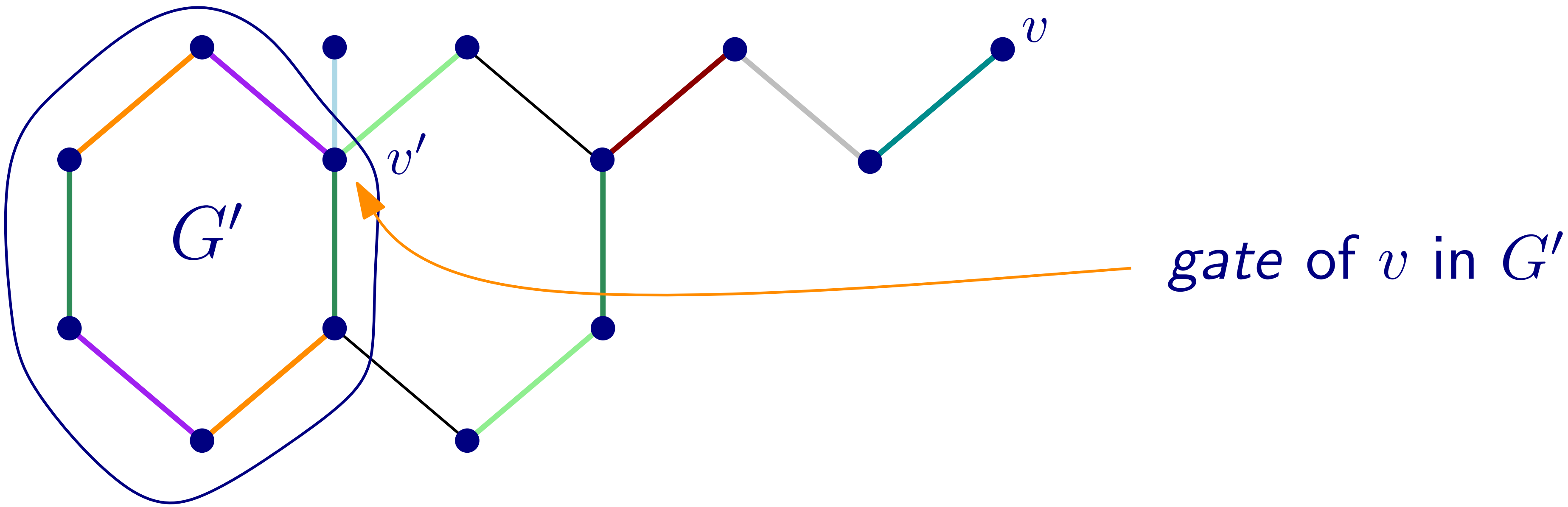
associate convex subgraph G' with sign vector $X(G')$



- $\mathcal{L} = \{X(G') \mid G' \subseteq G \text{ convex } \} \subseteq \{0, \pm\}^{\mathcal{C}}$
- tope graph $G_{\mathcal{L}} = \mathcal{L} \cap \{\pm 1\}^{\mathcal{C}} \subseteq Q_{\mathcal{C}}$ of $\mathcal{L}$ is G

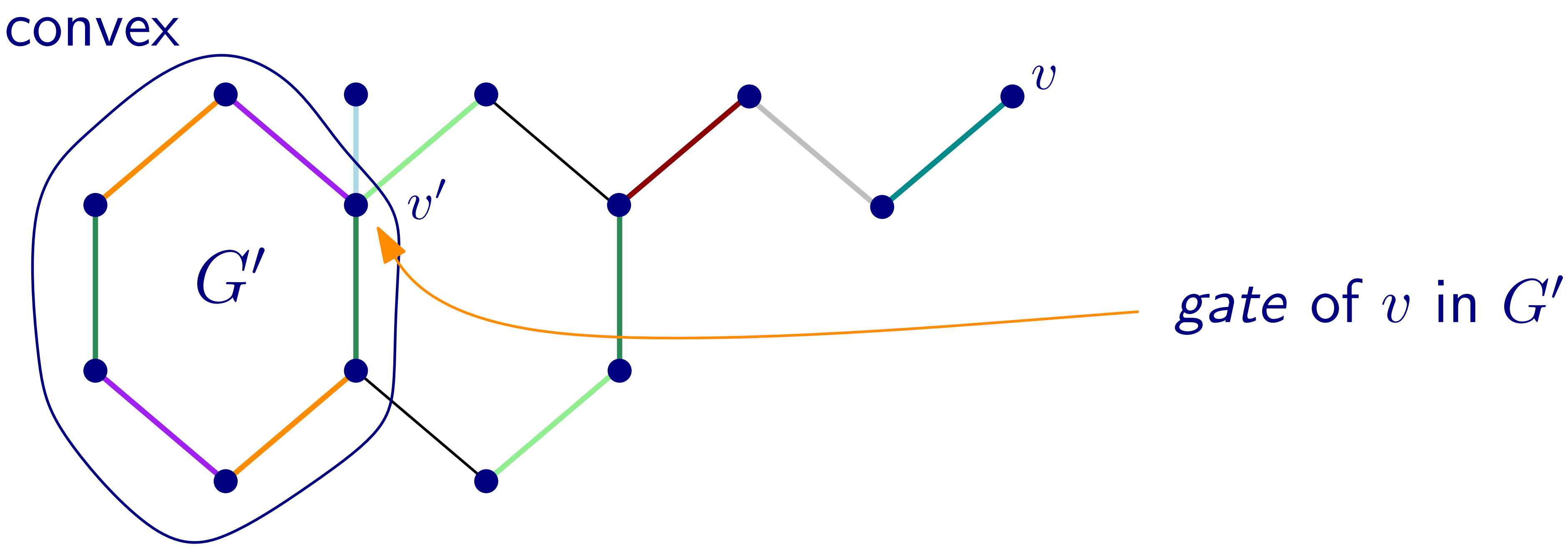
Gated subgraphs

$G' \subseteq G$ *gated* if $\forall v \in G$ $\exists v' \in G'$ s.th $\forall w \in G'$ there is a shortest (v, w)-path through v'



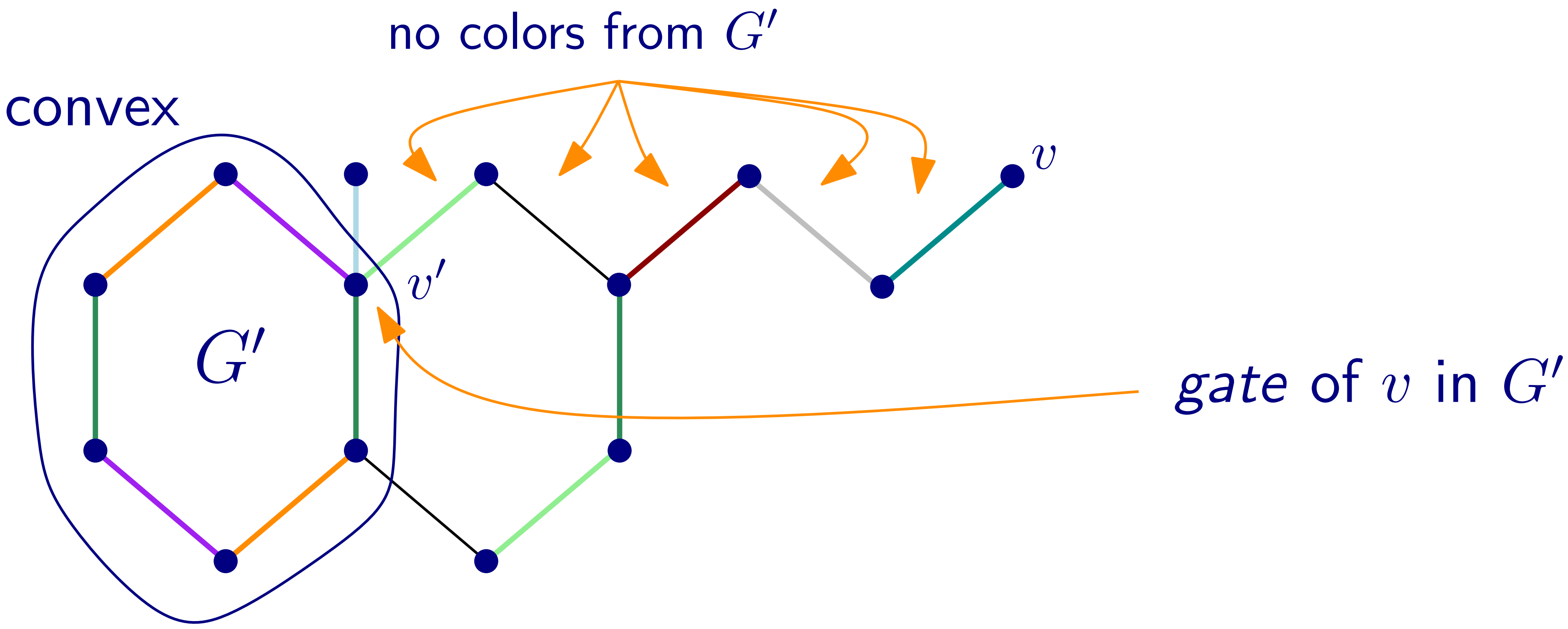
Gated subgraphs

$G' \subseteq G$ *gated* if $\forall v \in G$ $\exists v' \in G'$ s.th $\forall w \in G'$ there is a shortest (v, w)-path through v'



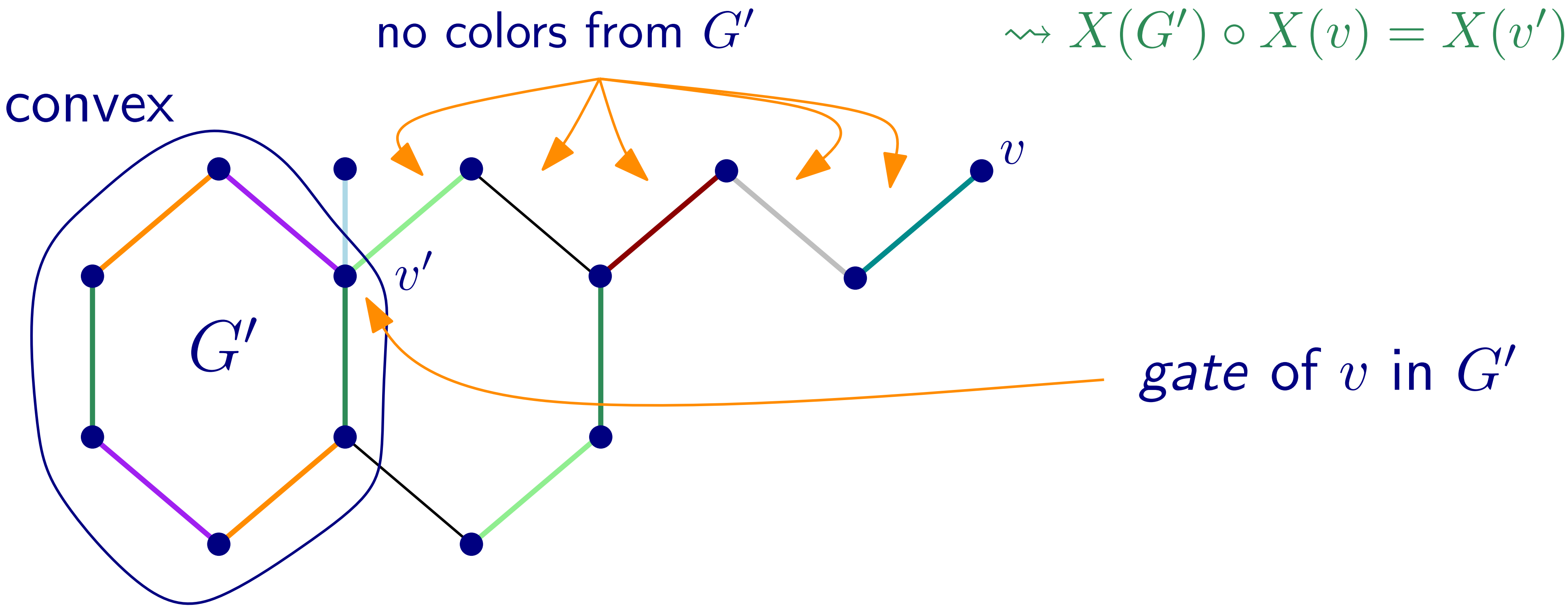
Gated subgraphs

$G' \subseteq G$ *gated* if $\forall v \in G\ \exists v' \in G'$ s.th $\forall w \in G'$ there is a shortest (v, w)-path through v'



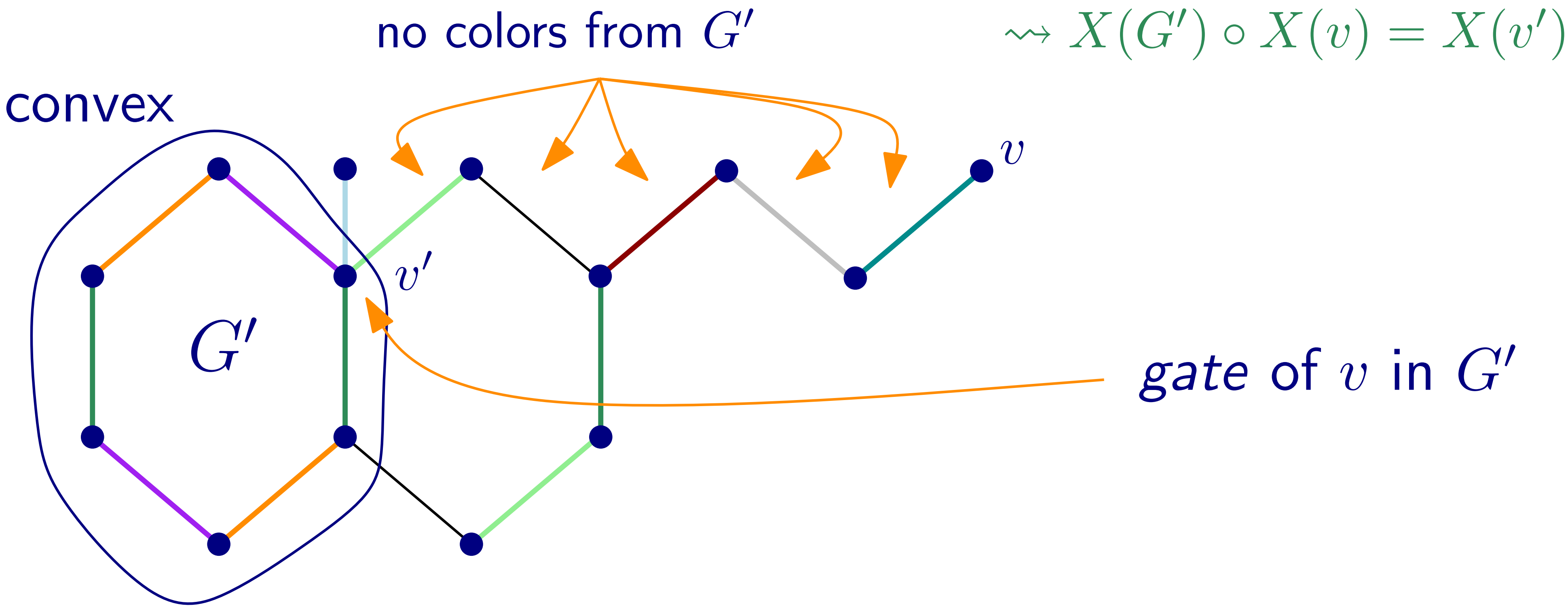
Gated subgraphs

$G' \subseteq G$ *gated* if $\forall v \in G$ $\exists v' \in G'$ s.th $\forall w \in G'$ there is a shortest (v, w)-path through v'



Gated subgraphs

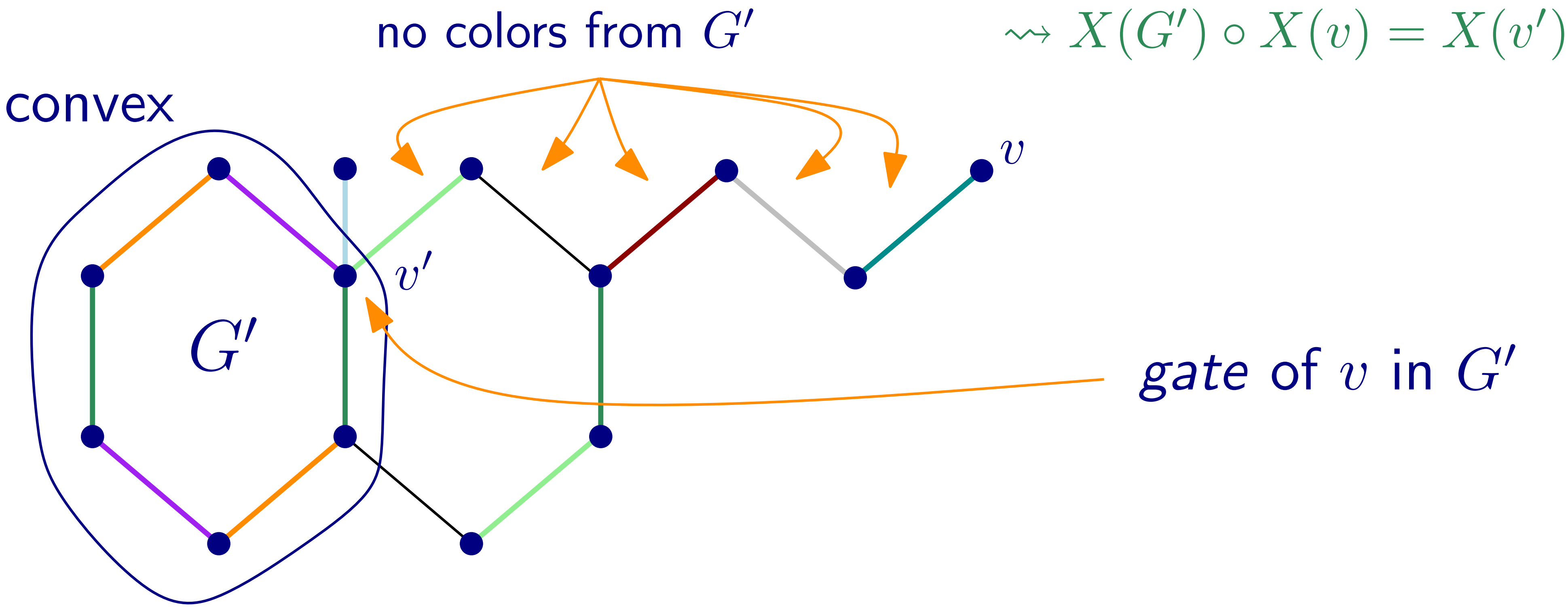
$G' \subseteq G$ *gated* if $\forall v \in G\ \exists v' \in G'$ s.th $\forall w \in G'$ there is a shortest (v, w)-path through v'



- $\mathcal{L} = \{X(G') \mid G' \subseteq G \text{ gated }\} \subseteq \{0, \pm\}^{\mathcal{C}}$

Gated subgraphs

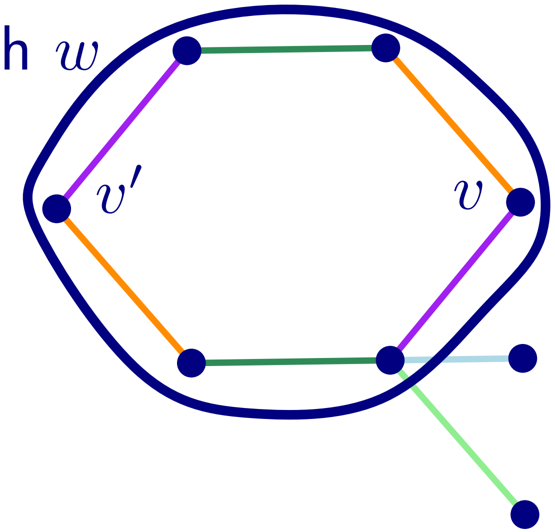
$G' \subseteq G$ *gated* if $\forall v \in G\ \exists v' \in G'$ s.th $\forall w \in G'$ there is a shortest (v, w)-path through v'



- $\mathcal{L} = \{X(G') \mid G' \subseteq G \text{ gated }\} \subseteq \{0, \pm\}^{\mathcal{C}}$
- $\mathcal{L}$ has $\mathcal{L} \circ \mathcal{L} \subseteq \mathcal{L}$ while $G_{\mathcal{L}} = G$

Antipodal gated subgraphs

G' *antipodal* if $\forall v \in G'$ $\exists v' \in G'$ s. th. $\forall w \in G'$ there is a shortest (v, v')-path through w ((antipodal $\Rightarrow$ convex)

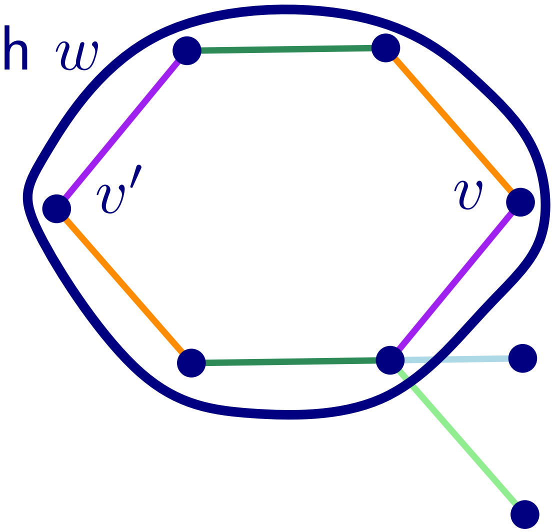


Antipodal gated subgraphs

G' *antipodal* if $\forall v \in G'$ $\exists v' \in G'$ s. th. $\forall w \in G'$ there is a shortest (v, v')-path through w

((antipodal $\Rightarrow$ convex)

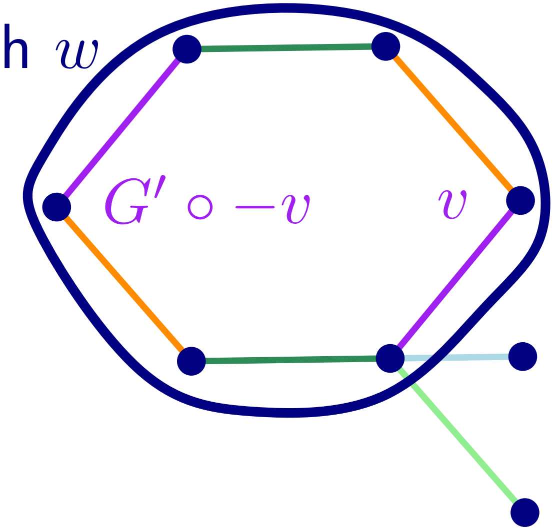
$\Leftrightarrow X(v') = X(G') \circ -X(v)$



Antipodal gated subgraphs

G' *antipodal* if $\forall v \in G'$ $\exists v' \in G'$ s. th. $\forall w \in G'$ there is a shortest (v, v')-path through w ((antipodal $\Rightarrow$ convex)

$\Leftrightarrow X(v') = X(G') \circ -X(v)$

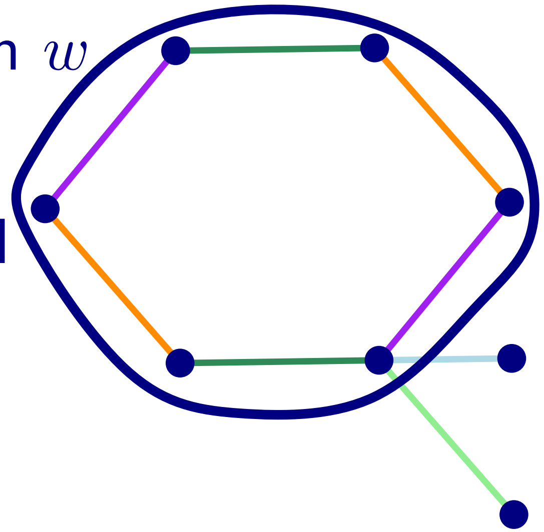


Antipodal gated subgraphs

G' *antipodal* if $\forall v \in G'$ $\exists v' \in G'$ s. th. $\forall w \in G'$ there is a shortest (v, v')-path through w ((antipodal $\Rightarrow$ convex)

$\Leftrightarrow X(v') = X(G') \circ -X(v)$

antipodal and gated

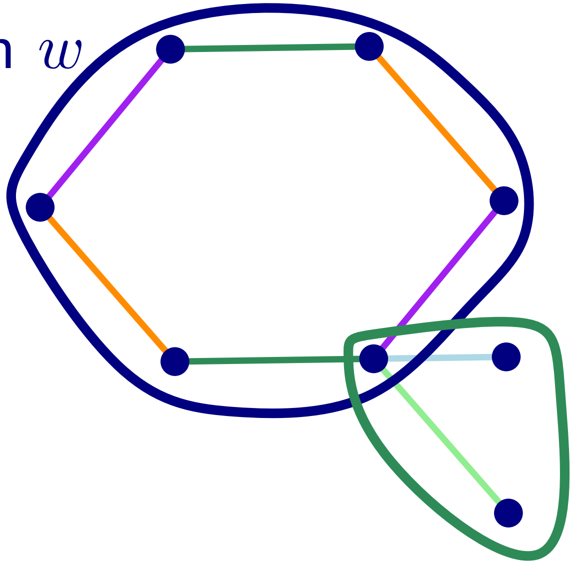


Antipodal gated subgraphs

G' *antipodal* if $\forall v \in G'$ $\exists v' \in G'$ s. th. $\forall w \in G'$ there is a shortest (v, v')-path through w

((antipodal $\Rightarrow$ convex)

$\Leftrightarrow X(v') = X(G') \circ -X(v)$



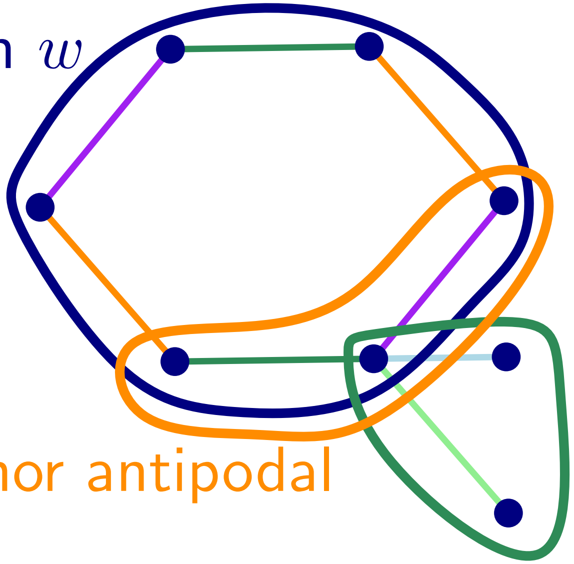
Antipodal gated subgraphs

G' *antipodal* if $\forall v \in G'$ $\exists v' \in G'$ s. th. $\forall w \in G'$ there is a shortest (v, v')-path through w

((antipodal $\Rightarrow$ convex)

$\Leftrightarrow X(v') = X(G') \circ -X(v)$

antipodal and gated



gated and not antipodal

convex, not gated nor antipodal

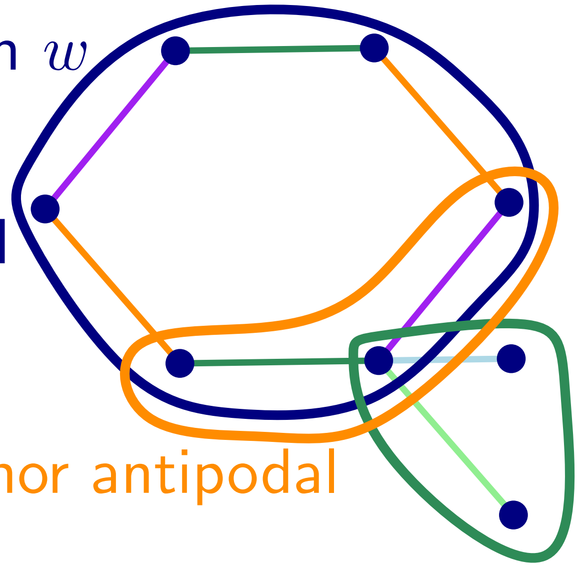
Antipodal gated subgraphs

G' *antipodal* if $\forall v \in G'\ \exists v' \in G'$ s. th. $\forall w \in G'$ there is a shortest (v, v')-path through w

((antipodal $\Rightarrow$ convex)

$\Leftrightarrow X(v') = X(G') \circ -X(v)$

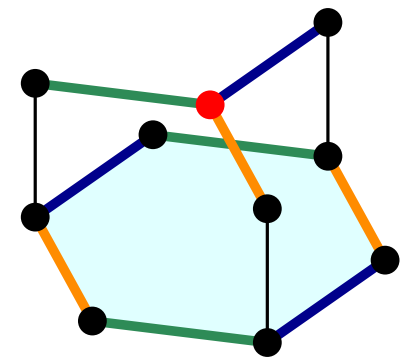
antipodal and gated



gated and not antipodal

convex, not gated nor antipodal

antipodal and *not* gated



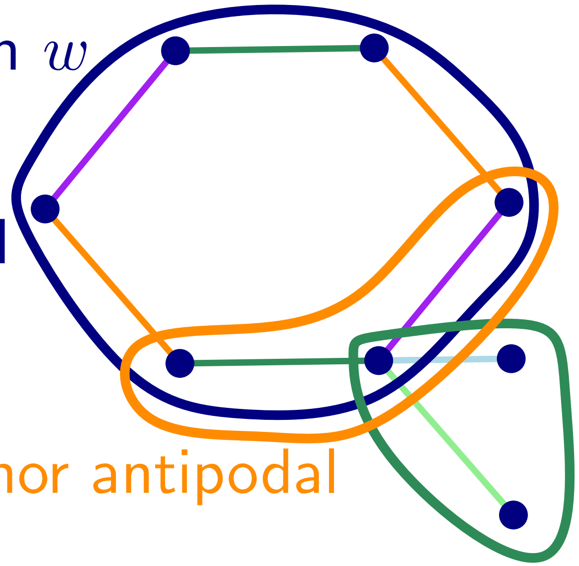
Antipodal gated subgraphs

G' *antipodal* if $\forall v \in G'$ $\exists v' \in G'$ s. th. $\forall w \in G'$ there is a shortest (v, v')-path through w

((antipodal $\Rightarrow$ convex)

$\Leftrightarrow X(v') = X(G') \circ -X(v)$

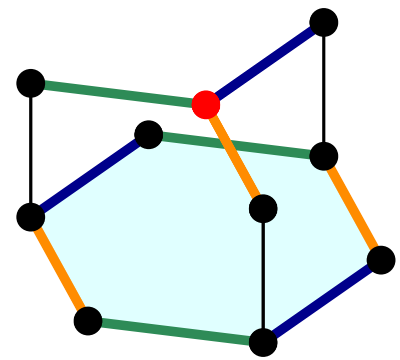
antipodal and gated



gated and not antipodal

convex, not gated nor antipodal

antipodal and *not* gated



G' antipodal and gated $\Leftrightarrow$ every v has gate with antipode in G' $\Leftrightarrow X(G') \circ -X(v) \in \{X(v') \mid v' \in V\}$

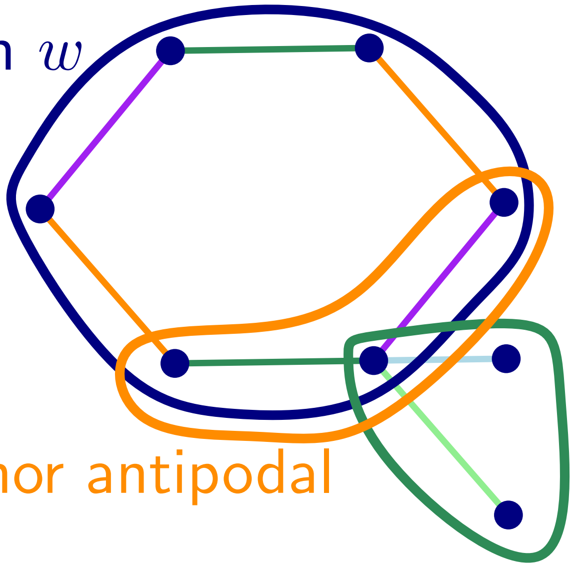
Antipodal gated subgraphs

G' *antipodal* if $\forall v \in G'$ $\exists v' \in G'$ s. th. $\forall w \in G'$ there is a shortest (v, v')-path through w

((antipodal $\Rightarrow$ convex)

$\Leftrightarrow X(v') = X(G') \circ -X(v)$

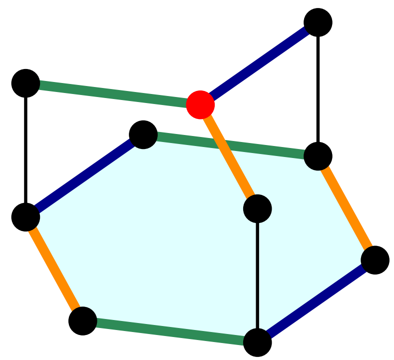
antipodal and gated



gated and not antipodal

convex, not gated nor antipodal

antipodal and *not* gated



G' antipodal and gated
$\Leftrightarrow$ every v has gate with antipode in G'
$\Leftrightarrow X(G') \circ -X(v) \in \{X(v') \mid v' \in V\}$

- $\mathcal{L} = \{X(G') \mid G' \subseteq G$ antipodal and gated $\} \subseteq \{0, \pm\}^{\mathcal{C}}$

  (FS) $\mathcal{L} \circ -\mathcal{L} \subseteq \mathcal{L}$

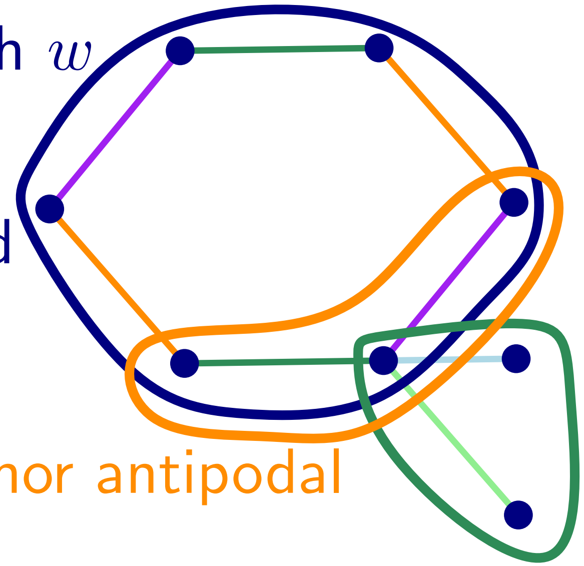
Antipodal gated subgraphs

G' *antipodal* if $\forall v \in G'$ $\exists v' \in G'$ s. th. $\forall w \in G'$ there is a shortest (v, v')-path through w

((antipodal $\Rightarrow$ convex)

$\Leftrightarrow X(v') = X(G') \circ -X(v)$

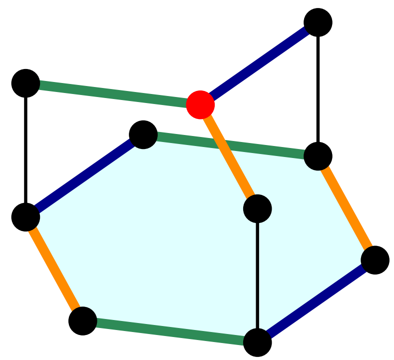
antipodal and gated



gated and not antipodal

convex, not gated nor antipodal

antipodal and *not* gated



G' antipodal and gated
$\Leftrightarrow$ every v has gate with antipode in G'
$\Leftrightarrow X(G') \circ -X(v) \in \{X(v') \mid v' \in V\}$

- $\mathcal{L} = \{X(G') \mid G' \subseteq G$ antipodal and gated $\} \subseteq \{0, \pm\}^{\mathcal{C}}$
  (FS) $\mathcal{L} \circ -\mathcal{L} \subseteq \mathcal{L}$

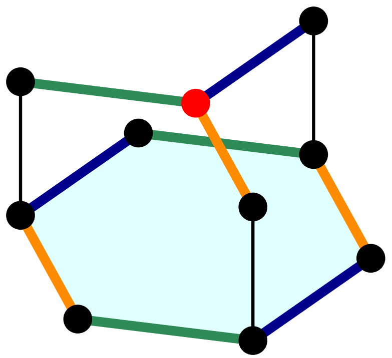
$\rightsquigarrow G$ tope graph of COM
$\Longrightarrow$ antipodal subgraphs gated

Antipodal gated partial cubes and $\mathcal{Q}^-$

AG$= \{G$ partial cube$|$ all antipodal subgraphs gated$\}$

Antipodal gated partial cubes and $\mathcal{Q}^-$

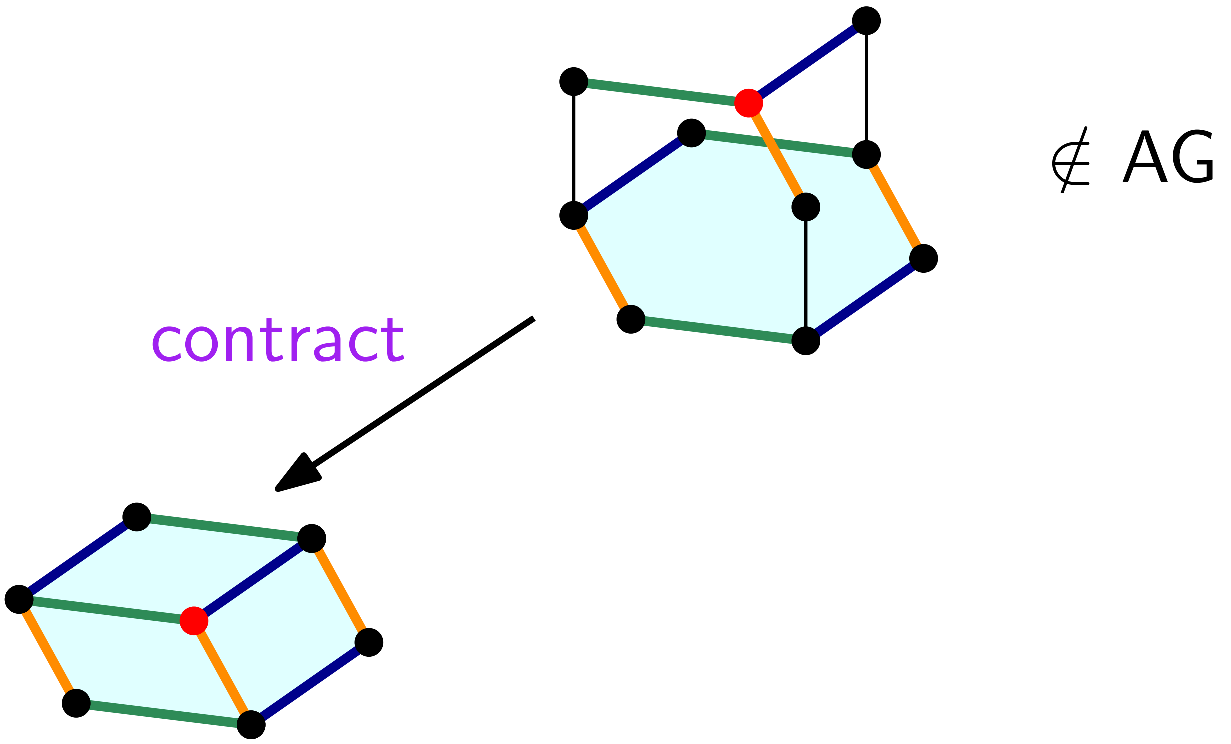
AG$=\{G$ partial cube$|$ all antipodal subgraphs gated$\}$



$\notin$ AG

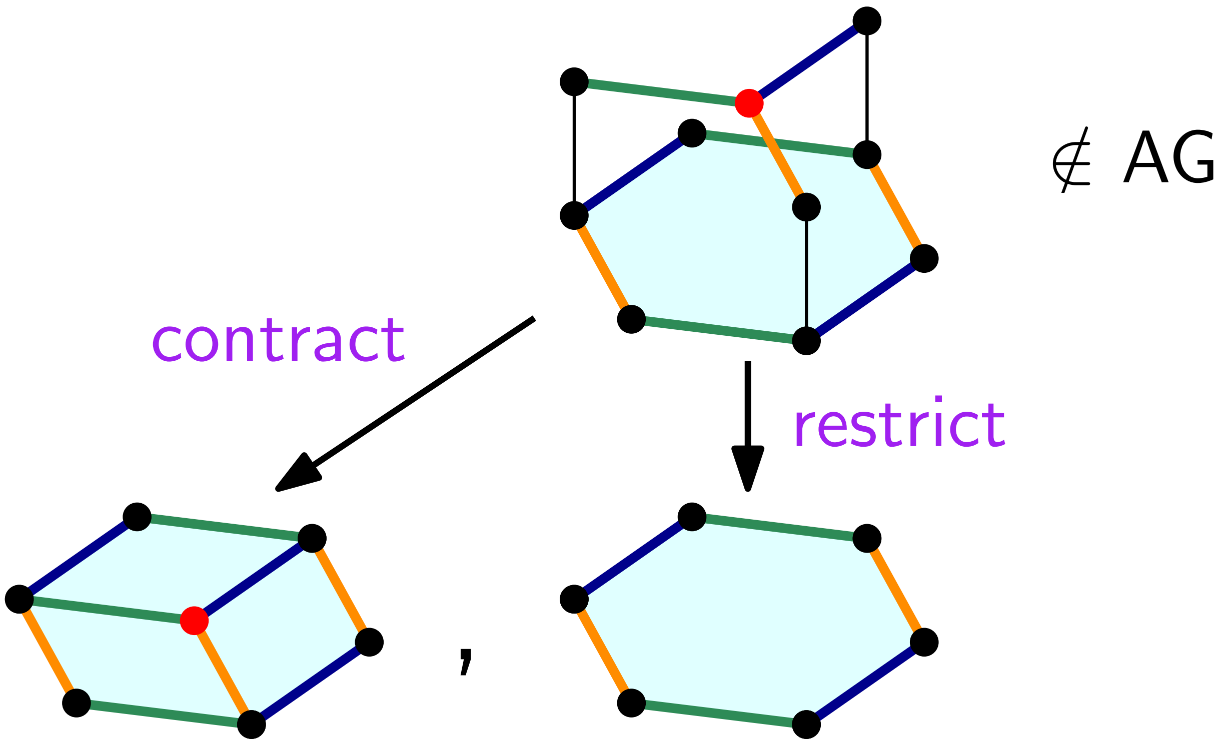
Antipodal gated partial cubes and $\mathcal{Q}^-$

AG$=\{G$ partial cube$|$ all antipodal subgraphs gated$\}$



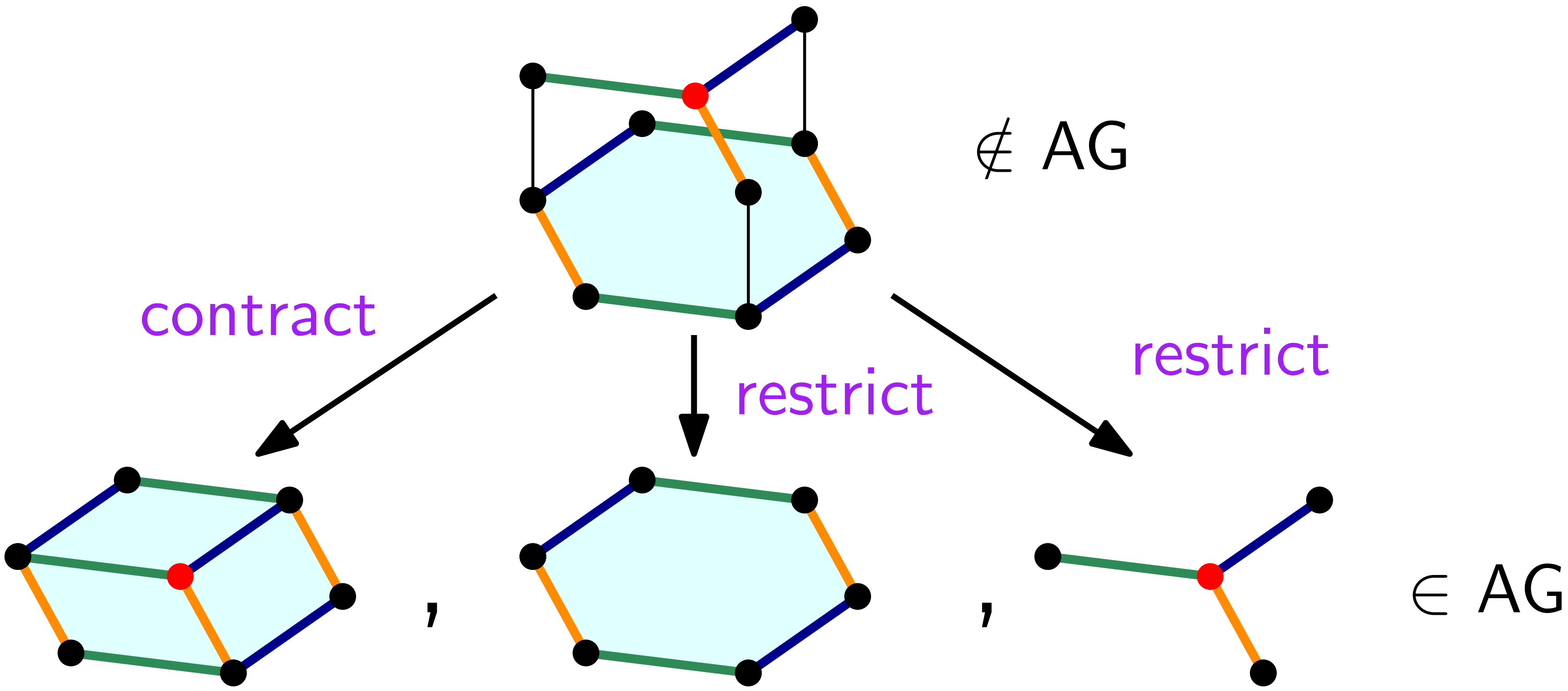
Antipodal gated partial cubes and $\mathcal{Q}^-$

AG$=\{G$ partial cube$|$ all antipodal subgraphs gated$\}$



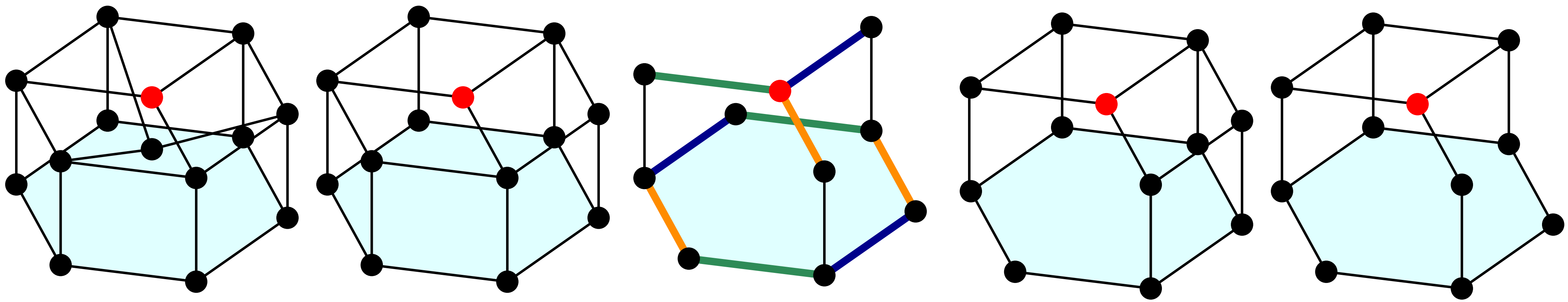
Antipodal gated partial cubes and $\mathcal{Q}^-$

AG= $\{G$ partial cube$|$ all antipodal subgraphs gated$\}$



Antipodal gated partial cubes and $\mathcal{Q}^-$

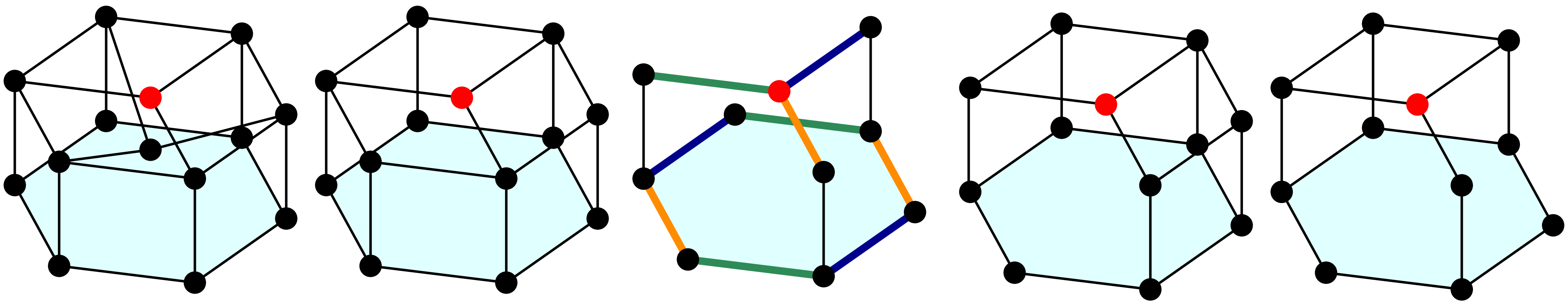
AG$=\{G$ partial cube$|$ all antipodal subgraphs gated$\}$



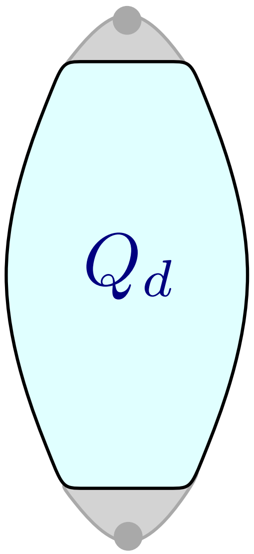
all these are minor-minimally non AG

Antipodal gated partial cubes and $\mathcal{Q}^-$

AG$=\{G$ partial cube$|$ all antipodal subgraphs gated$\}$

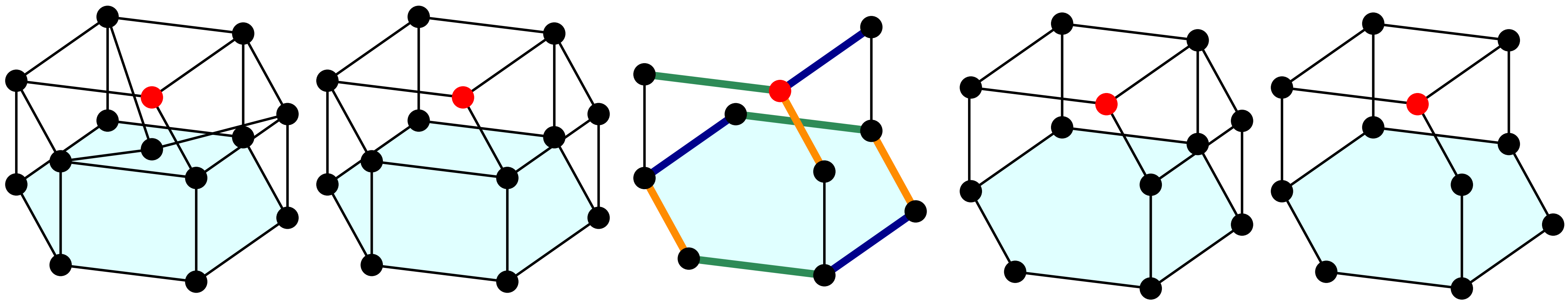


all these are minor-minimally non AG   but more generally:

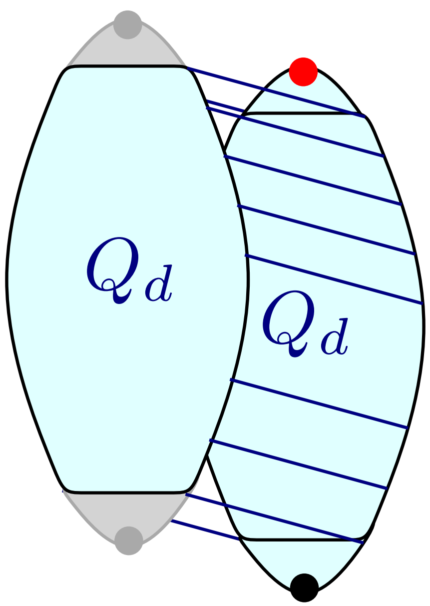


Antipodal gated partial cubes and $\mathcal{Q}^-$

AG= $\{G$ partial cube$|$ all antipodal subgraphs gated$\}$

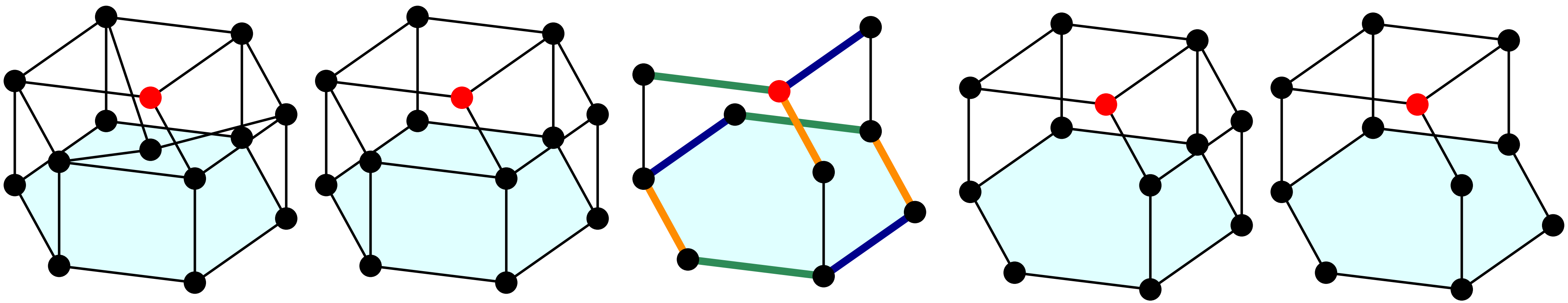


all these are minor-minimally non AG but more generally:

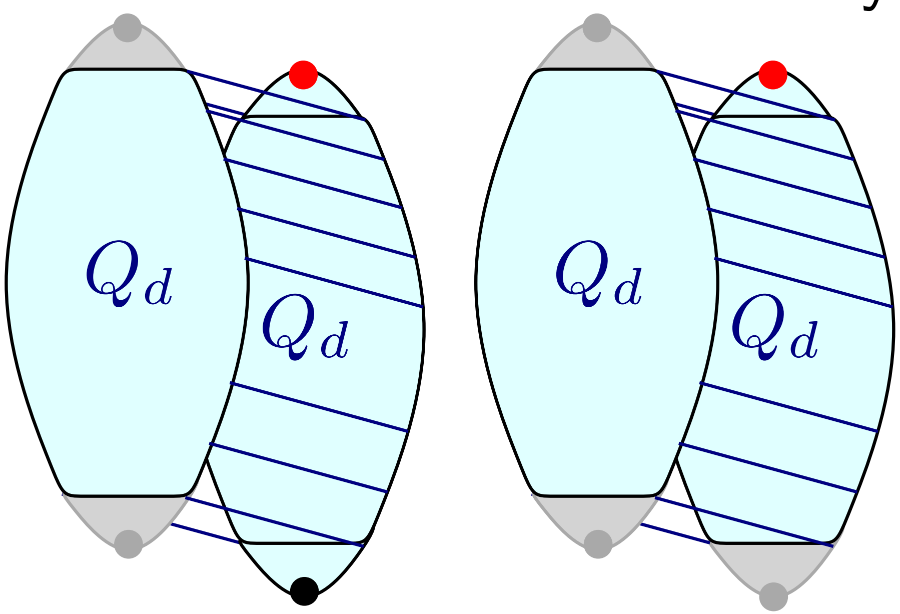


Antipodal gated partial cubes and $\mathcal{Q}^-$

AG$=\{G$ partial cube$|$ all antipodal subgraphs gated$\}$

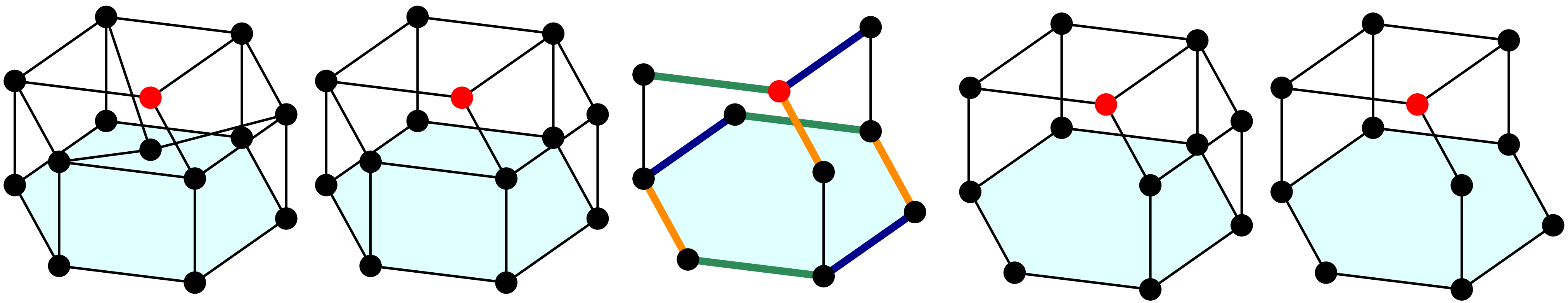


all these are minor-minimally non AG but more generally:

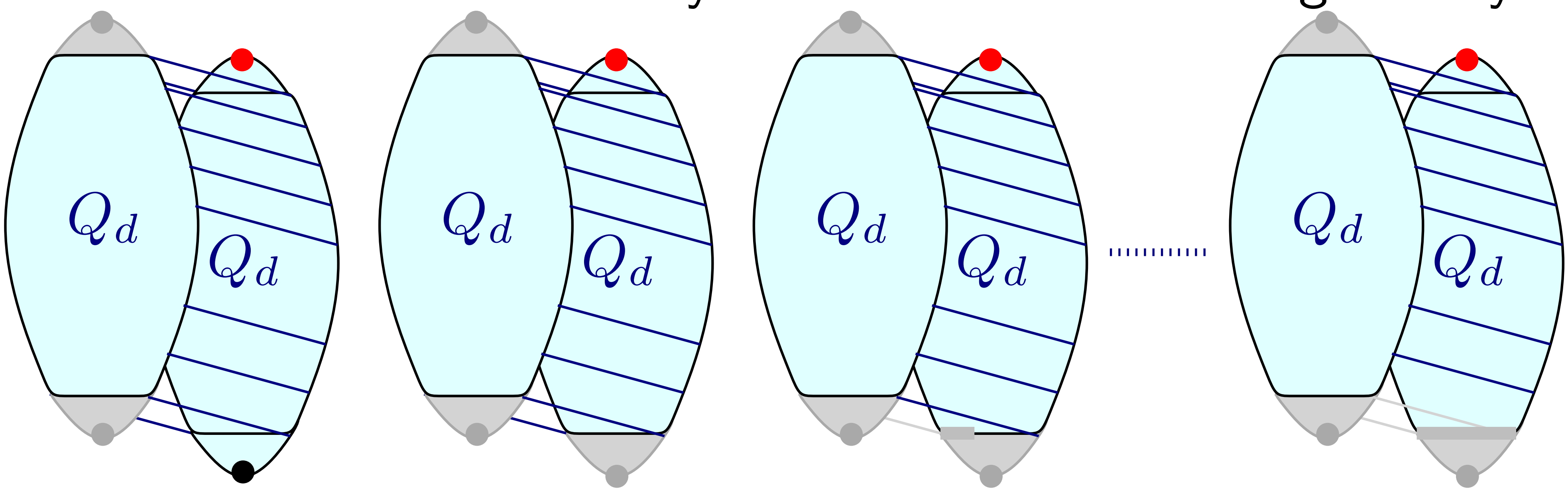


Antipodal gated partial cubes and $\mathcal{Q}^-$

AG= $\{G$ partial cube$|$ all antipodal subgraphs gated$\}$

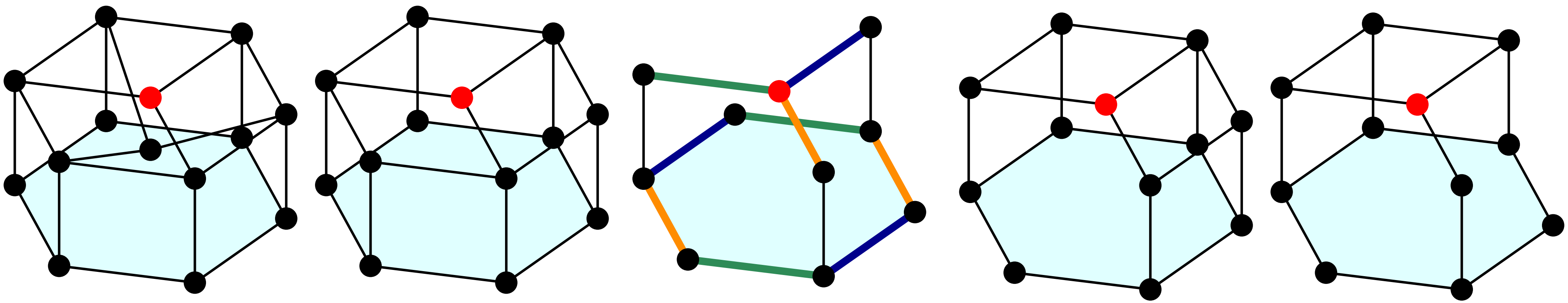


all these are minor-minimally non AG but more generally:

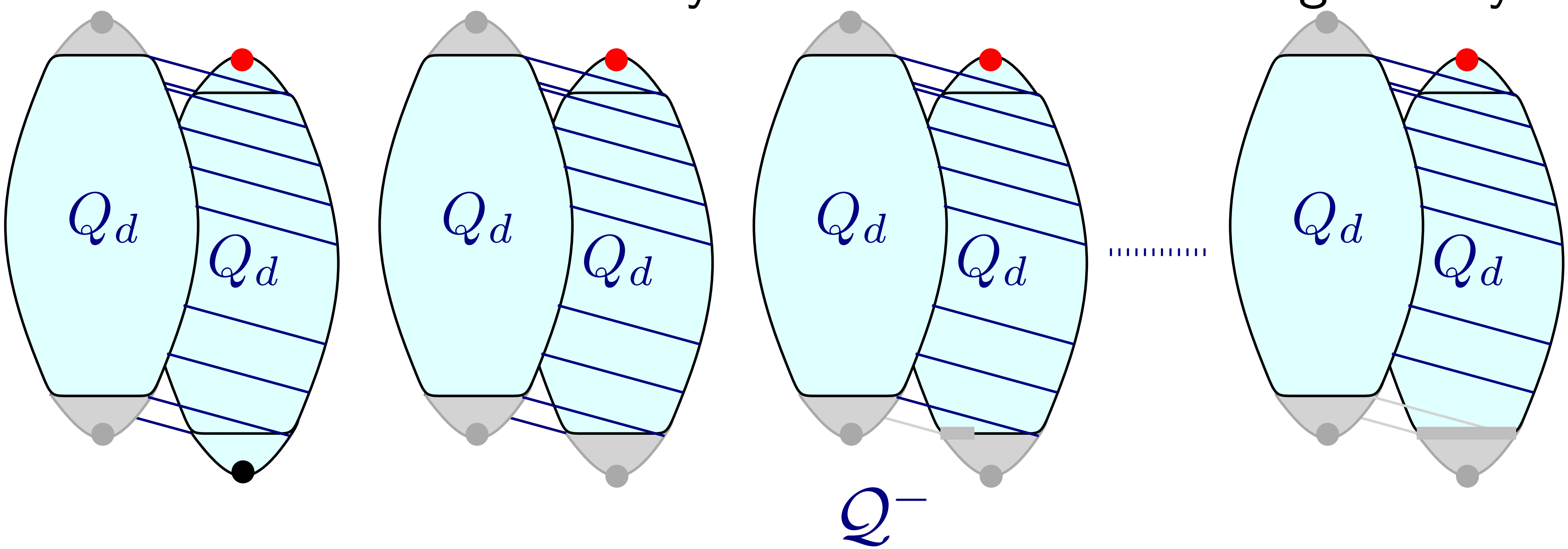


Antipodal gated partial cubes and $\mathcal{Q}^-$

AG= $\{G$ partial cube$|$ all antipodal subgraphs gated$\}$

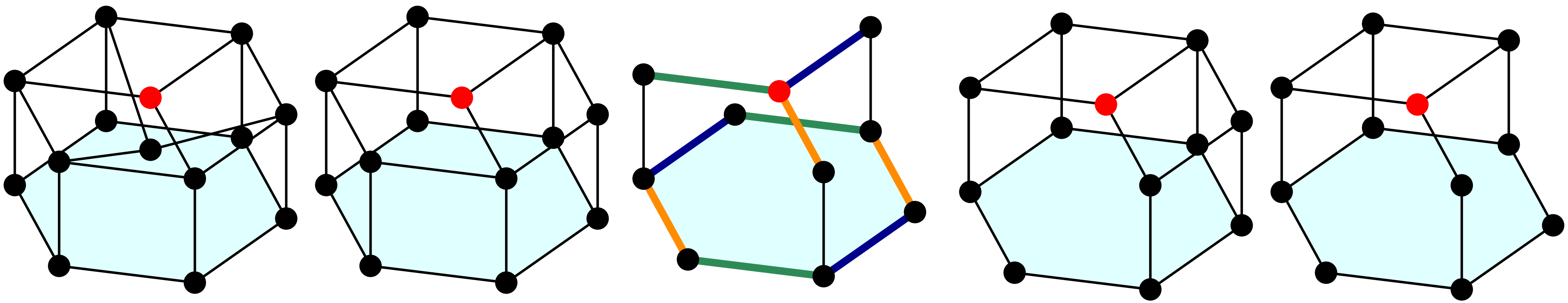


all these are minor-minimally non AG but more generally:

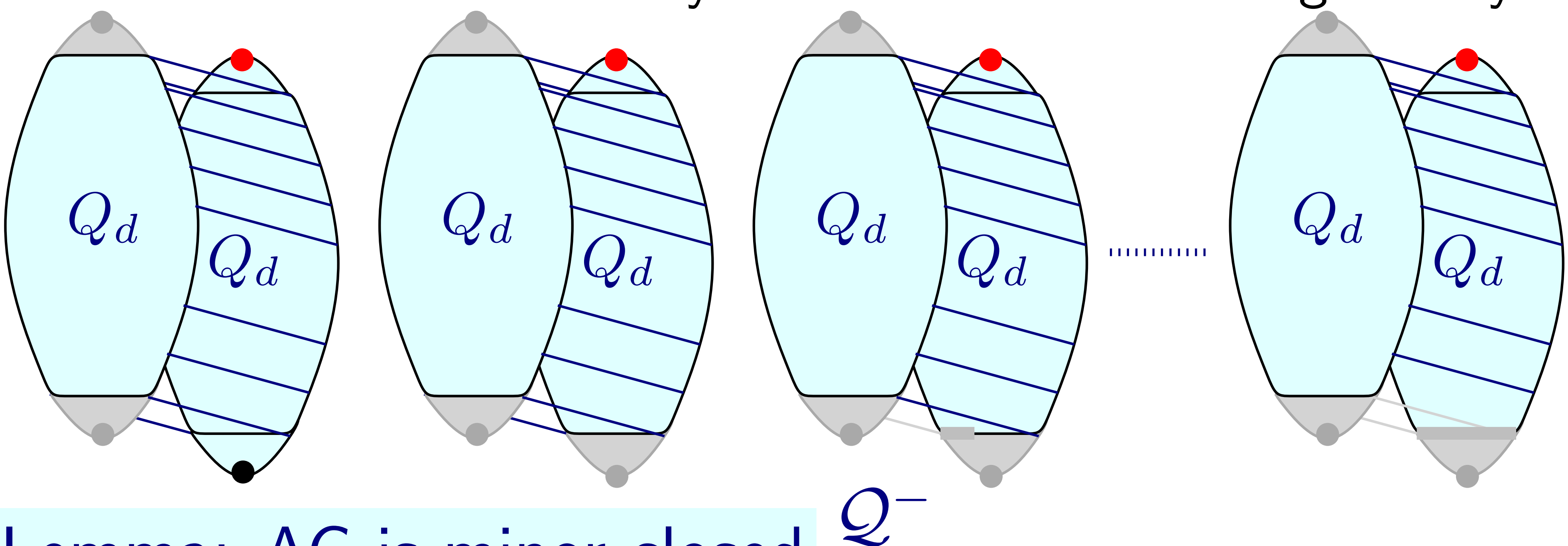


Antipodal gated partial cubes and $\mathcal{Q}^-$

AG= $\{G$ partial cube$|$ all antipodal subgraphs gated$\}$



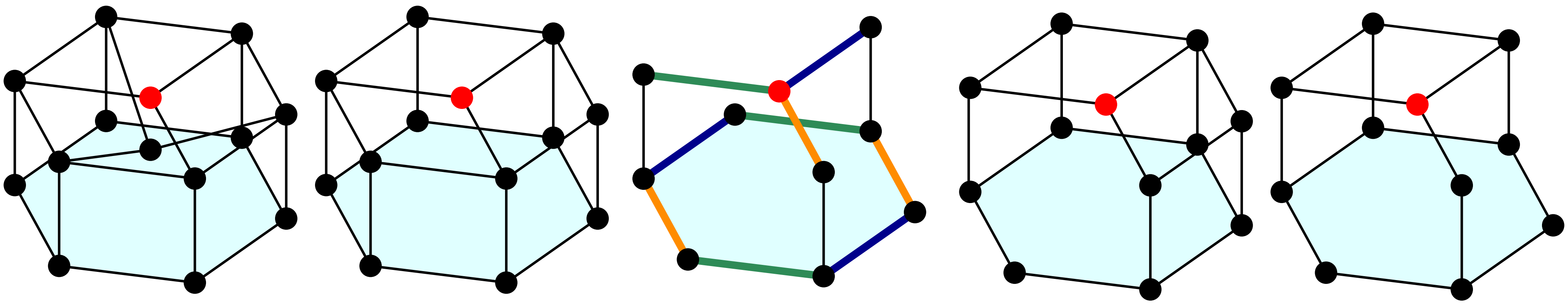
all these are minor-minimally non AG   but more generally:



Lemma: AG is minor-closed

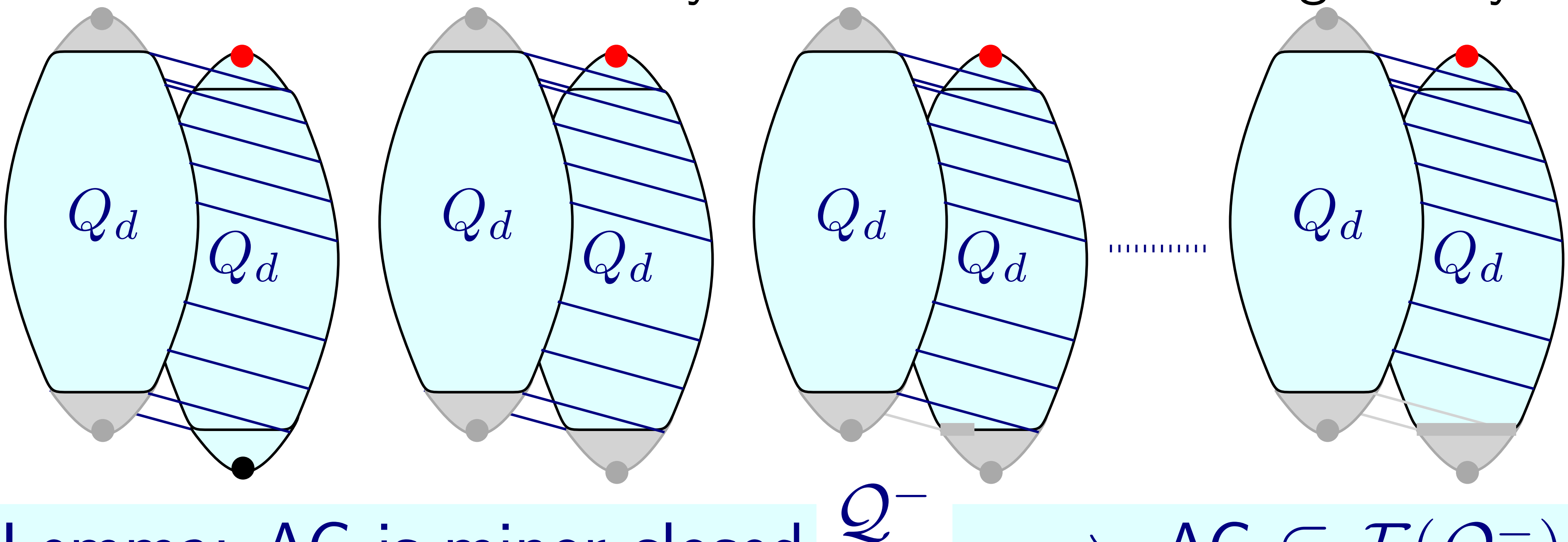
Antipodal gated partial cubes and $\mathcal{Q}^-$

AG$= \{G$ partial cube$|$ all antipodal subgraphs gated$\}$



all these are minor-minimally non AG

but more generally:



Lemma: AG is minor-closed $\implies$ AG $\subseteq \mathcal{F}(\mathcal{Q}^-)$

Characterization

THM[K, Marc '17]:
for a partial cube G the following are equivalent:
- G is tope graph of a COM
- all antipodal subgraphs of G are gated
- G has no partial cube minor from $\mathcal{Q}^-$

Corollaries:
- characterization, recognition for oriented matroids and affine oriented matroids
- polytime recognition

A common generalization

THM[K, Marc 17]:
G tope graph of COM iff G partial cube such that all antipodal subgraphs gated.

COR:
G tope graph of OM iff G *antipodal* partial cube such that all antipodal subgraphs gated.

COR:
G tope graph of AOM iff G *affine* partial cube such that all antipodal and *conformal* subgraphs gated.

Recognition

THM[K, Marc 17]:
G tope graph of COM iff G partial cube such that all antipodal subgraphs gated.

naive polytime alogrithm

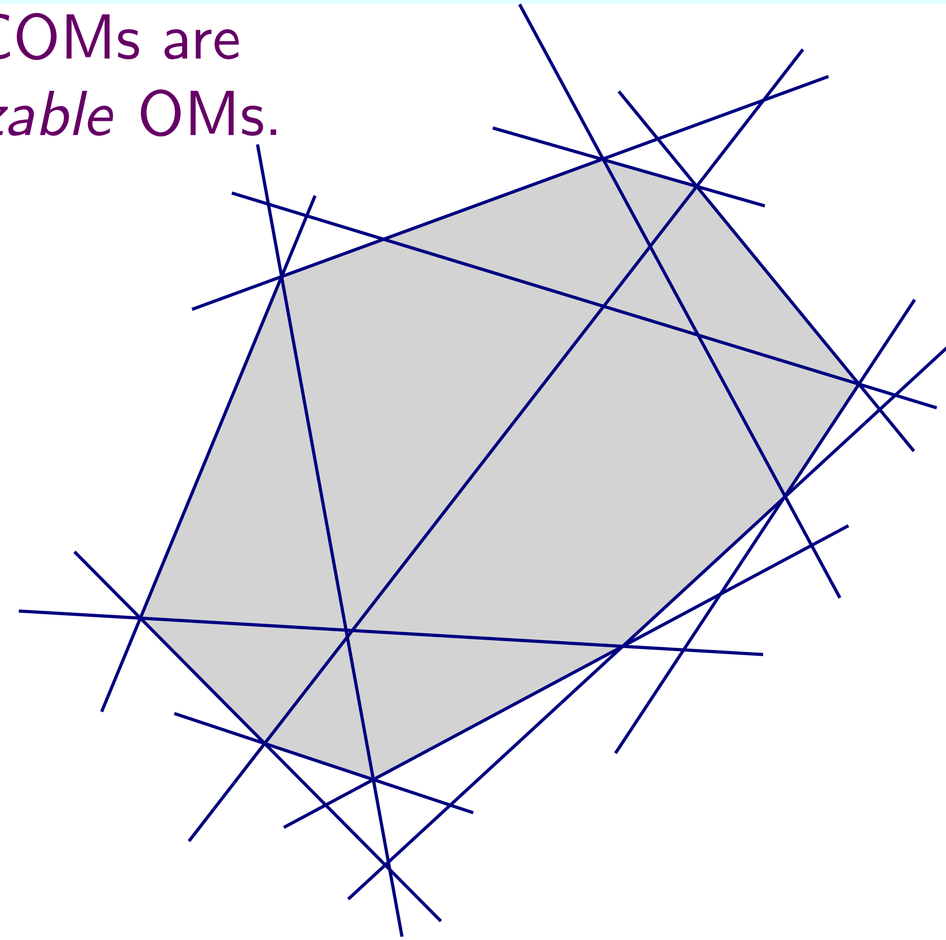
- check if partial cube $O(n^2)$
- find antipodal subgraphs $O(n^2)$ shortest path intervals
  - check if antipodal
- for each check if gated do some distances

Further questions

Observation: tope graphs of *realizable* COMs are convex subgraphs of tope graphs of *realizable* OMs.

Further questions

Observation: tope graphs of *realizable* COMs are convex subgraphs of tope graphs of *realizable* OMs.



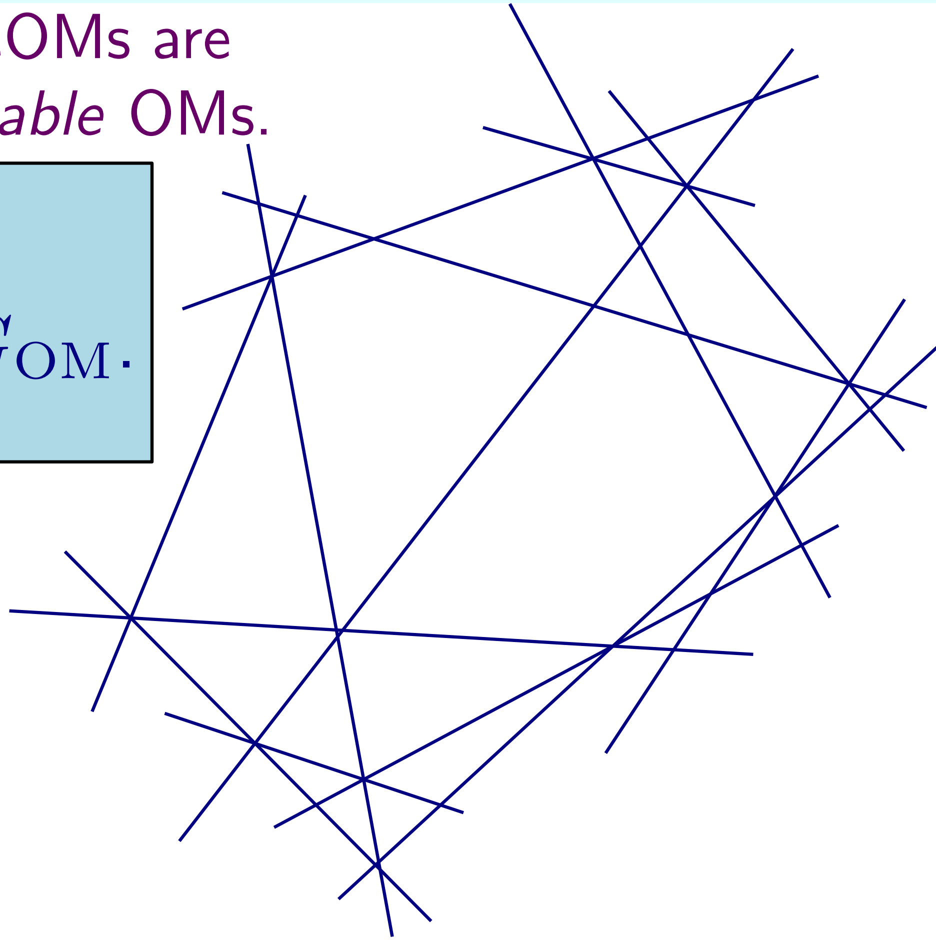
Further questions

Observation: tope graphs of *realizable* COMs are convex subgraphs of tope graphs of *realizable* OMs.

Further questions

Observation: tope graphs of *realizable* COMs are convex subgraphs of tope graphs of *realizable* OMs.

Conjecture [Bandelt, Chepoi, K '15]:
every G_{COM} is convex subgraph of G_{OM}.

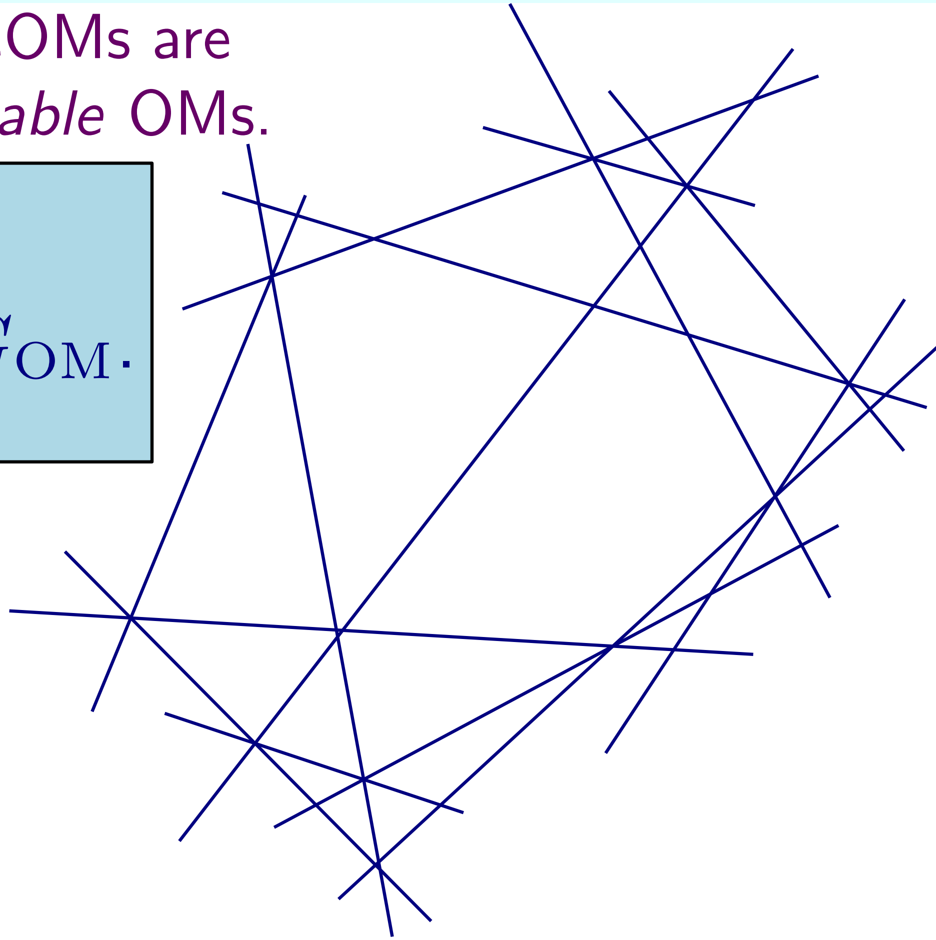


Further questions

Observation: tope graphs of *realizable* COMs are convex subgraphs of tope graphs of *realizable* OMs.

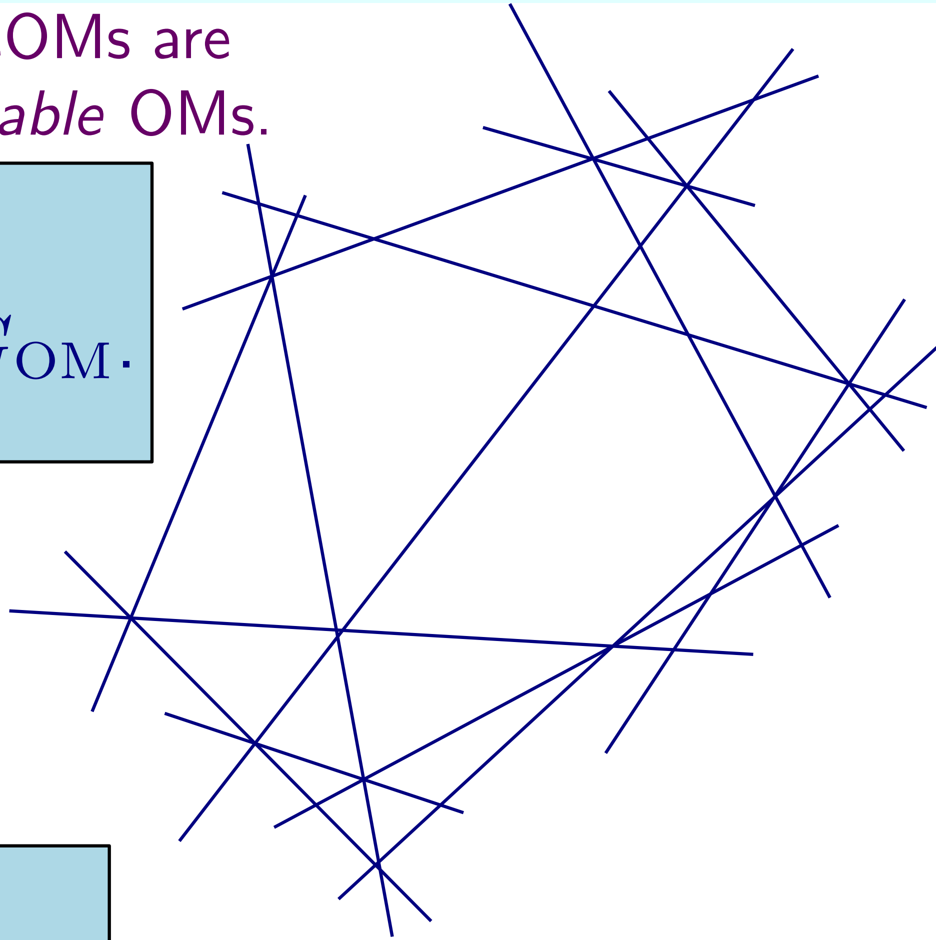
Conjecture [Bandelt, Chepoi, K '15]: every G_{COM} is convex subgraph of G_{OM}.

would yield a Topological Representation Theorem with pseudohyperplanes and pseudohalfspaces for COMs



Further questions

Observation: tope graphs of *realizable* COMs are convex subgraphs of tope graphs of *realizable* OMs.



Conjecture [Bandelt, Chepoi, K '15]: every G_{COM} is convex subgraph of G_{OM}.

would yield a Topological Representation Theorem with pseudohyperplanes and pseudohalfspaces for COMs

Find excluded minors for:

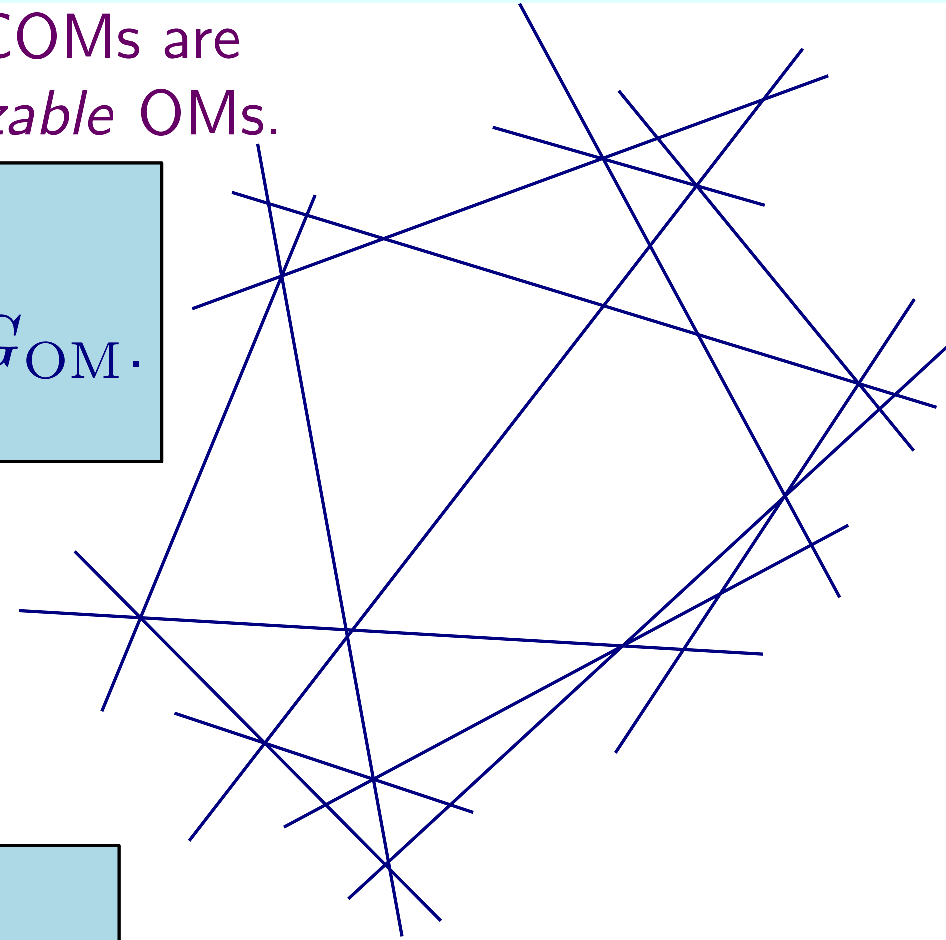
- planar partial cubes
- realizable COMs
- flip graphs of acyclic orientations of mixed graphs

Further questions

Observation: tope graphs of *realizable* COMs are convex subgraphs of tope graphs of *realizable* OMs.

Conjecture [Bandelt, Chepoi, K '15]: every G_{COM} is convex subgraph of G_{OM}.

would yield a Topological Representation Theorem with pseudohyperplanes and pseudohalfspaces for COMs



Find excluded minors for:

- planar partial cubes
- realizable COMs
- flip graphs of acyclic orientations of mixed graphs

Merci!