Walking in Poisson Delaunay triangulations

Olivier Devillers

Inria

[D. & Hemsley, 2016]

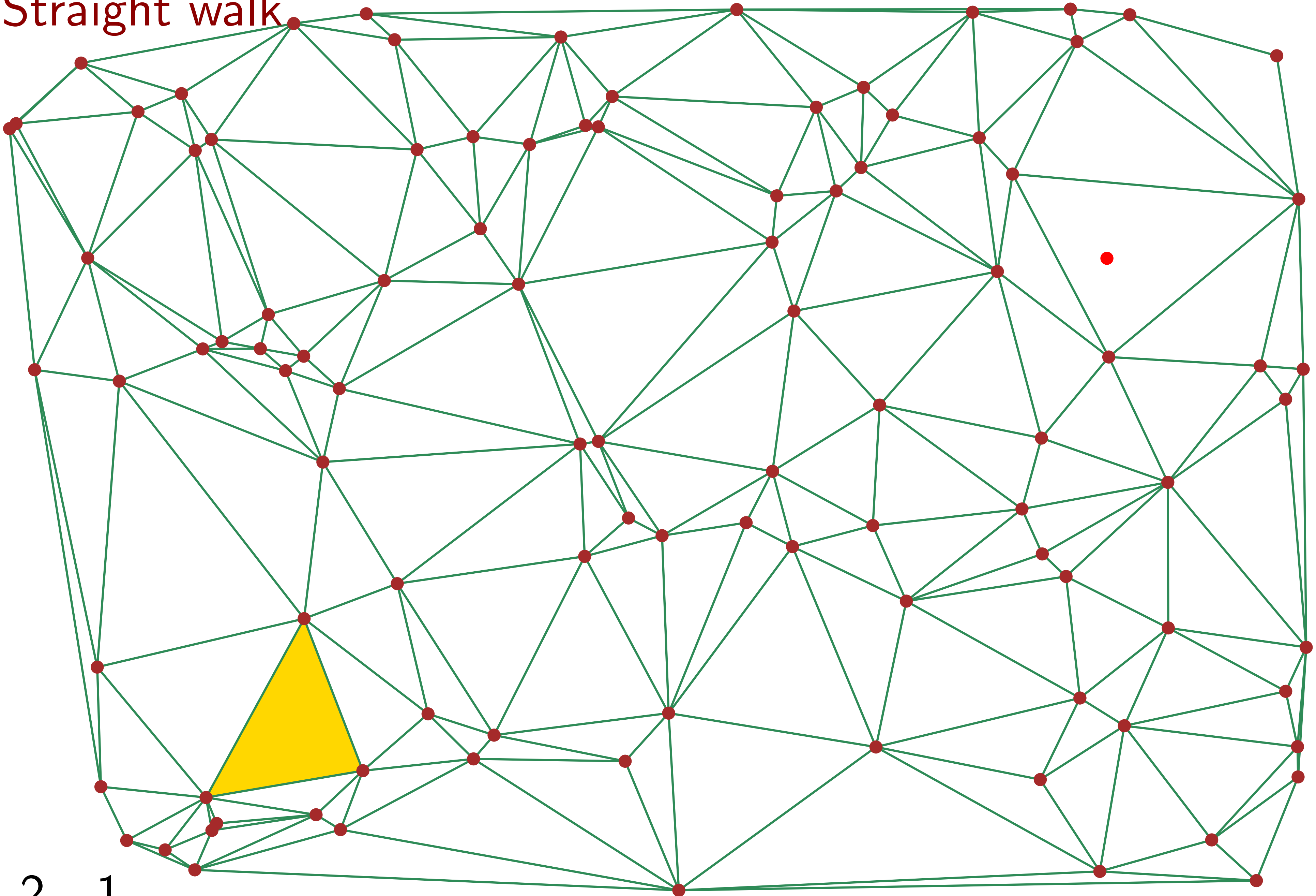
[Chenavier & D.,2018]

[de Castro & D., 2018]

[D. & Noizet, 2018]

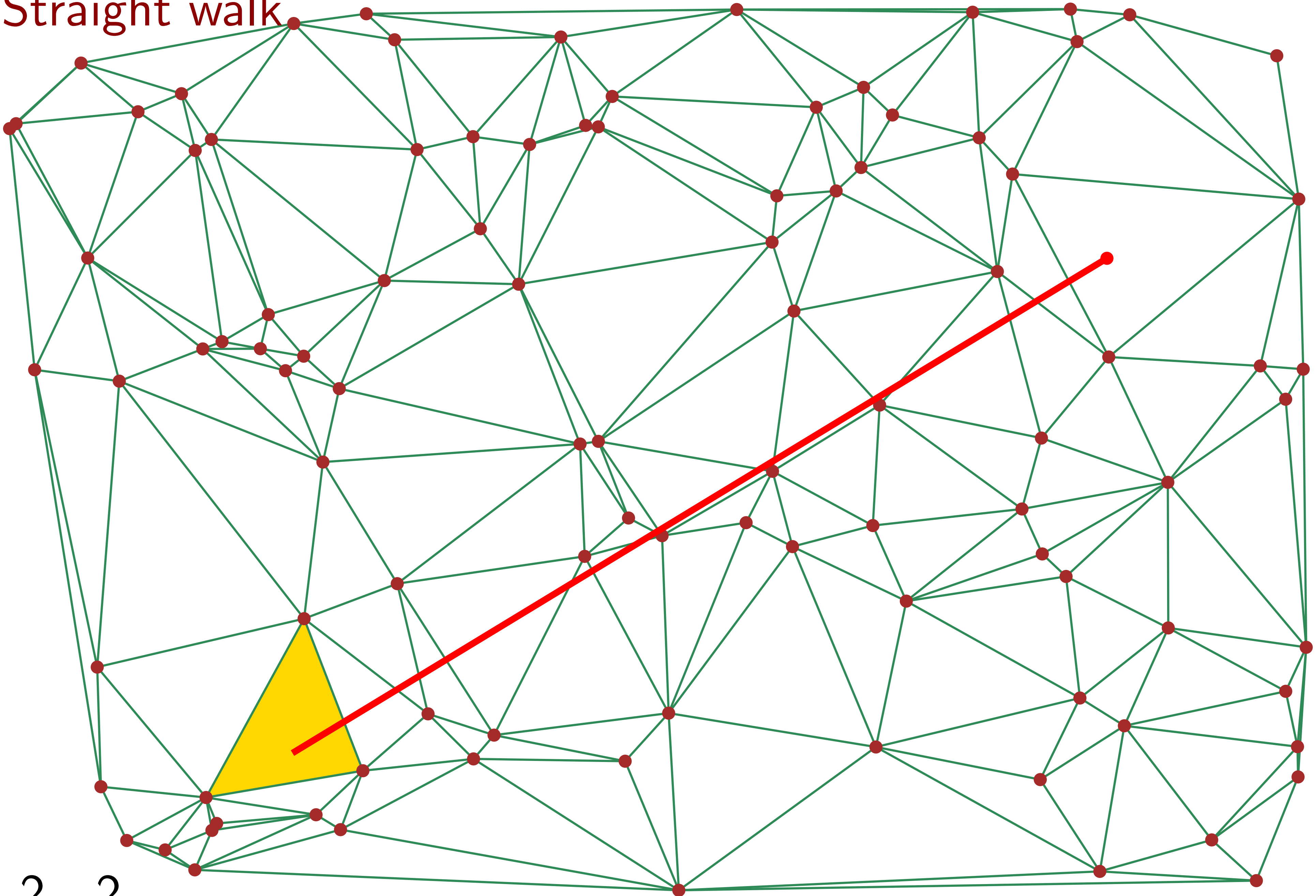
Walking in Delaunay triangulations

Straight walk



Walking in Delaunay triangulations

Straight walk



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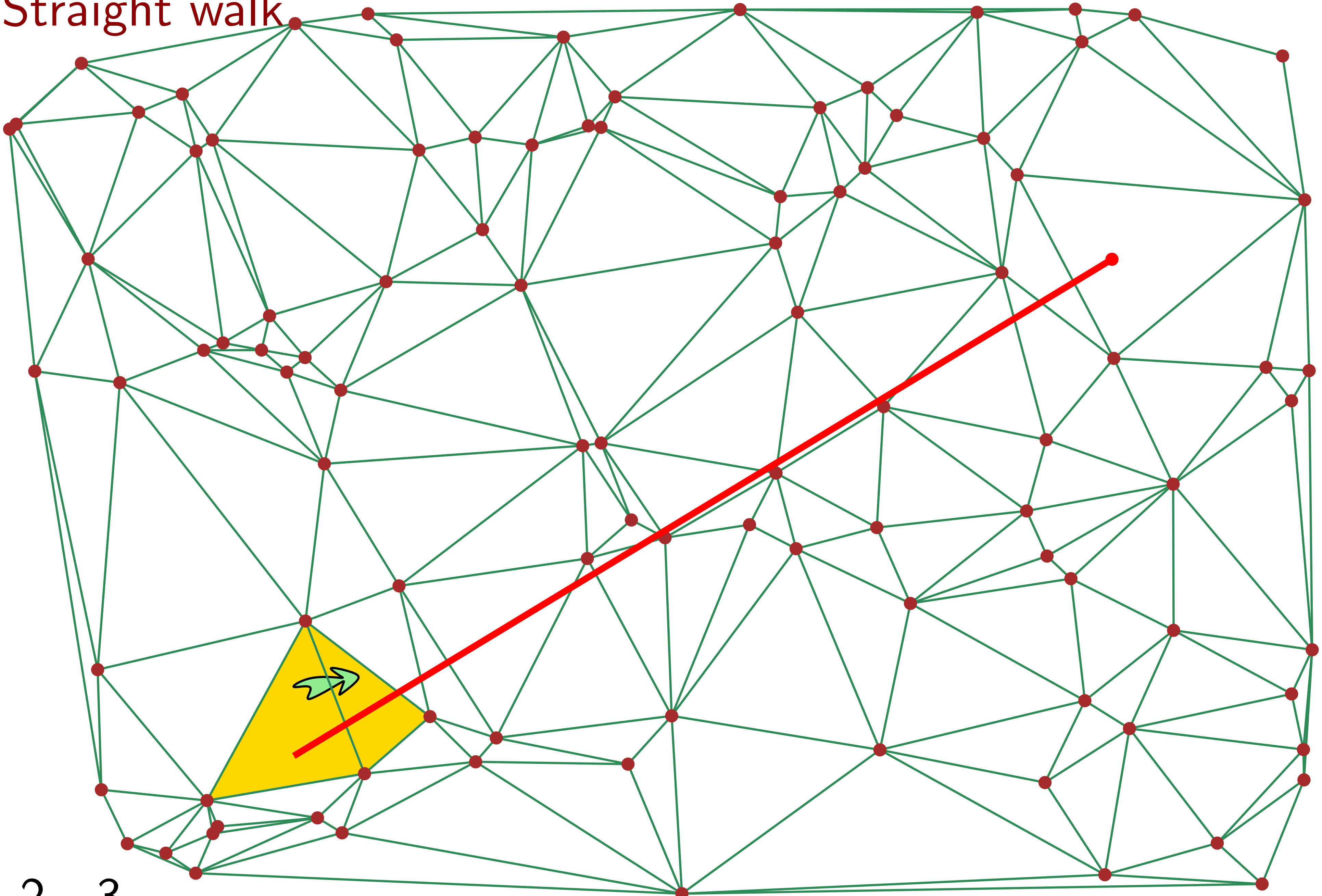

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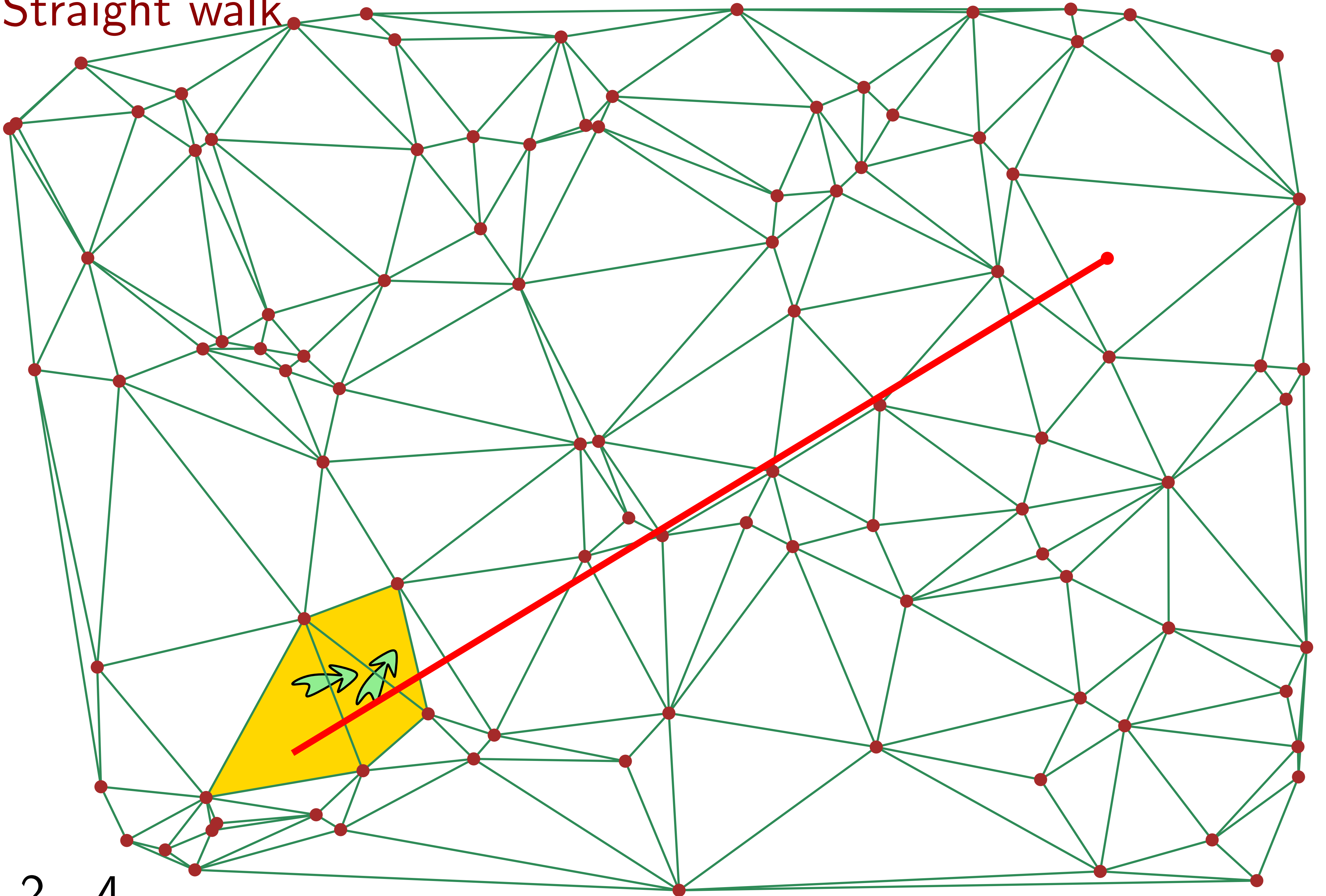
Straight walk



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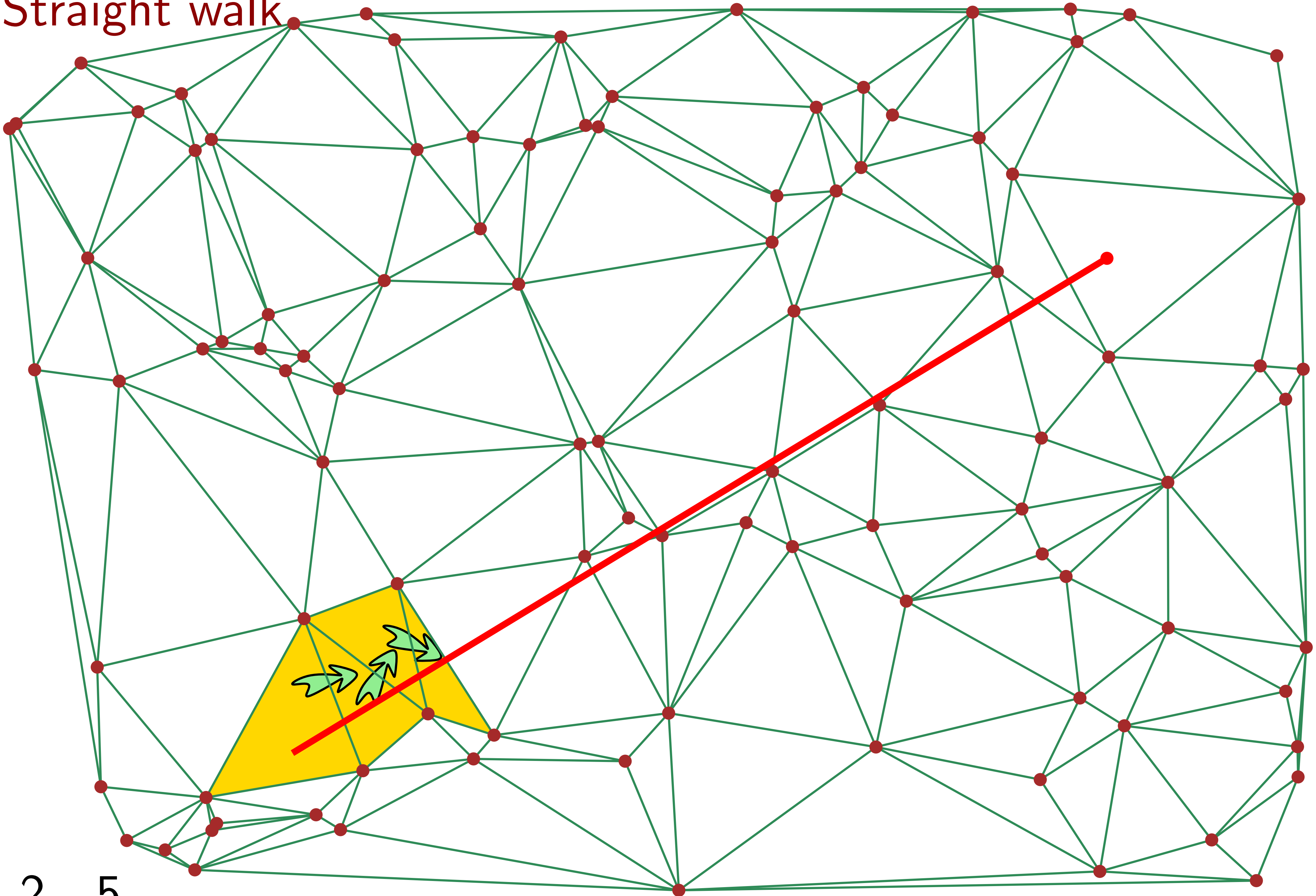
Straight walk



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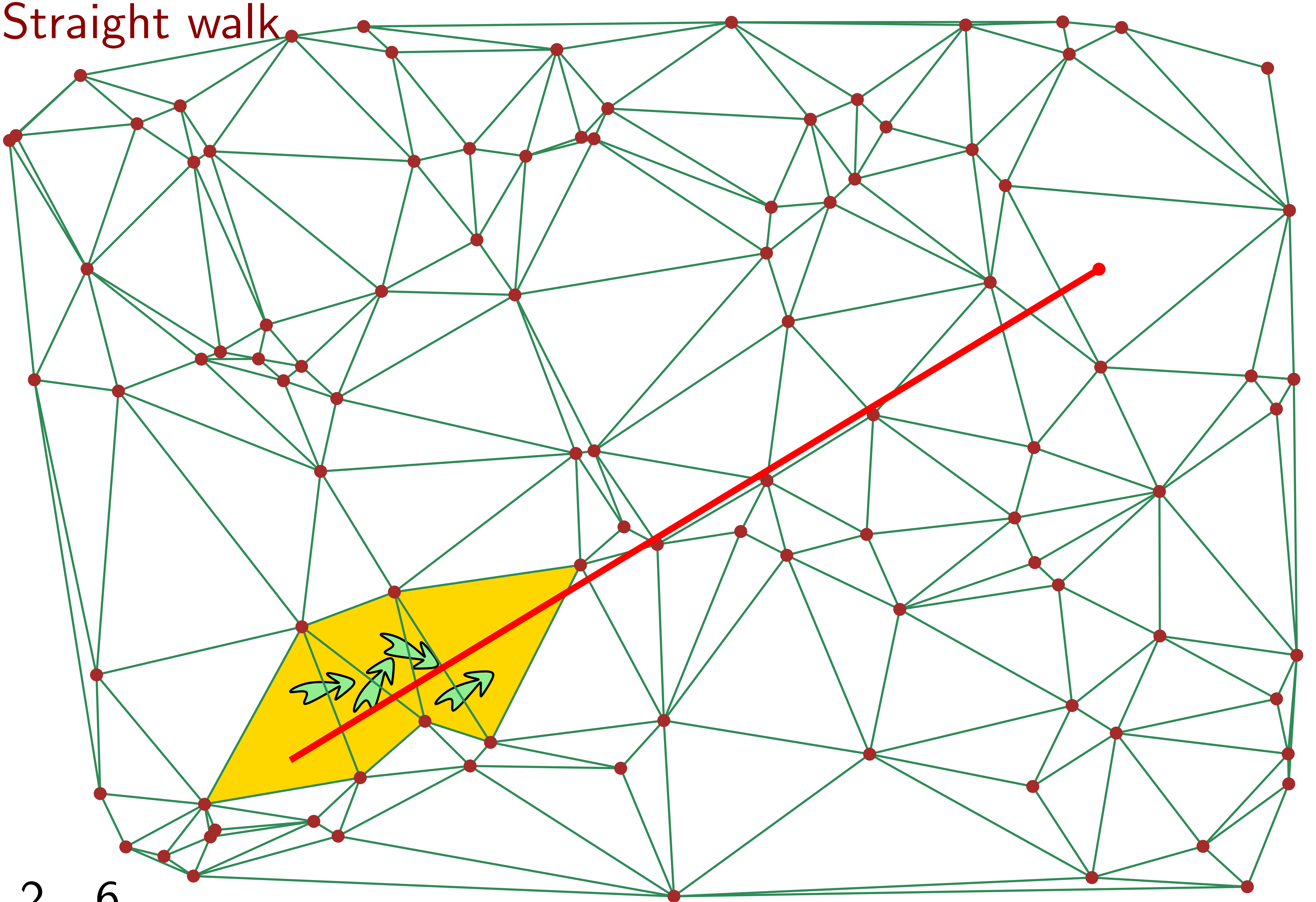

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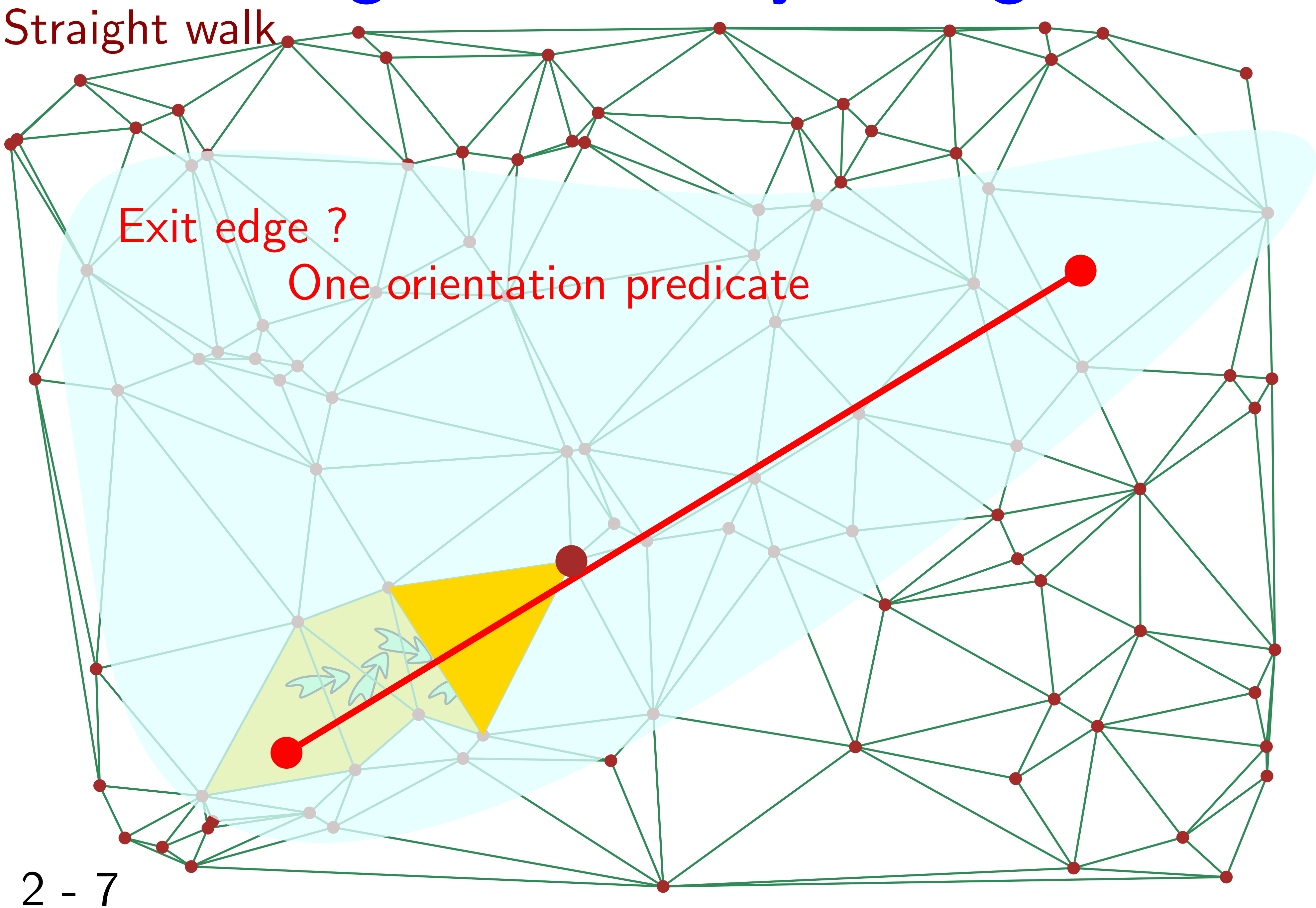

Walking in Delaunay triangulations

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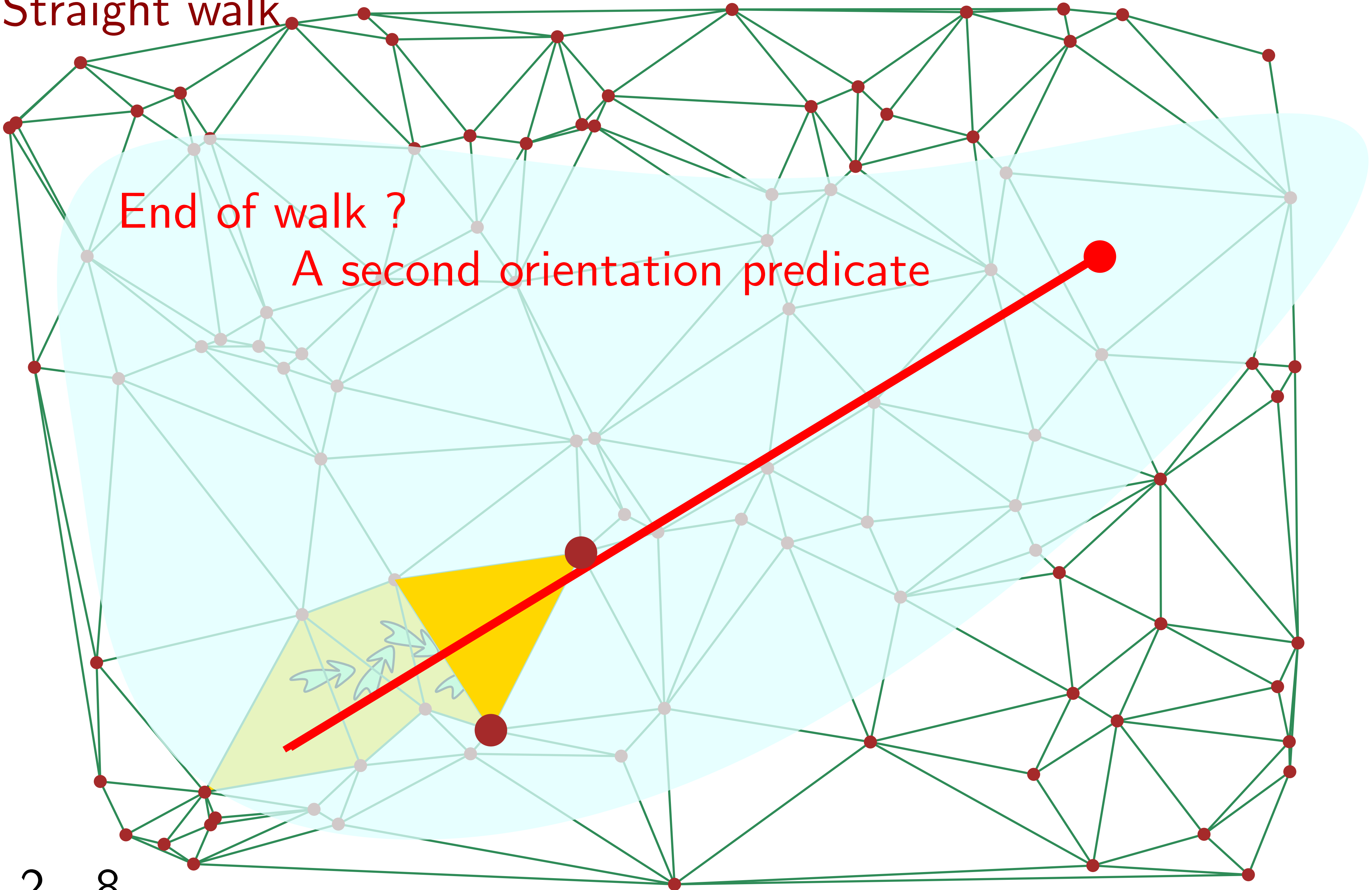

Walking in Delaunay triangulations

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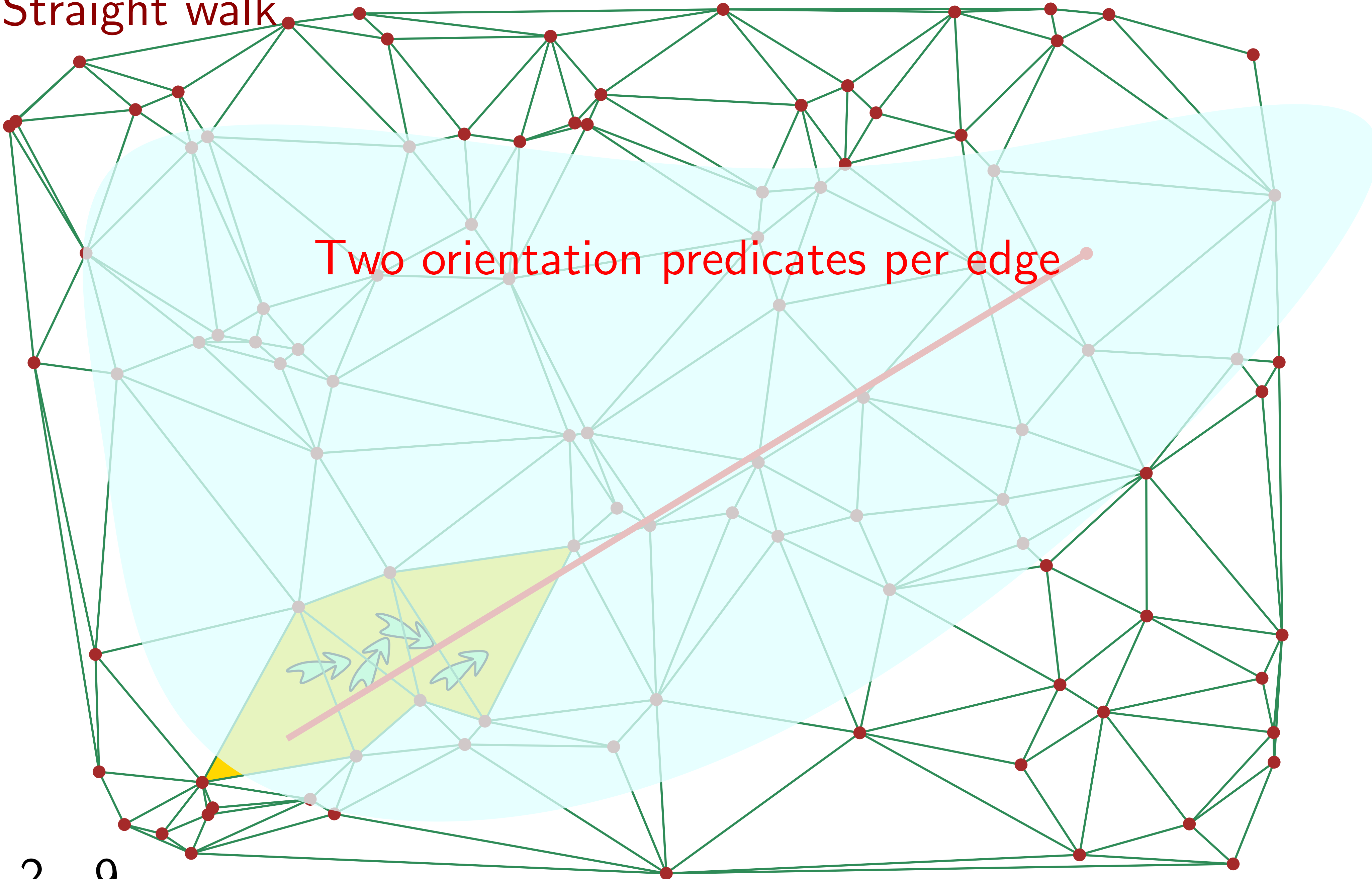
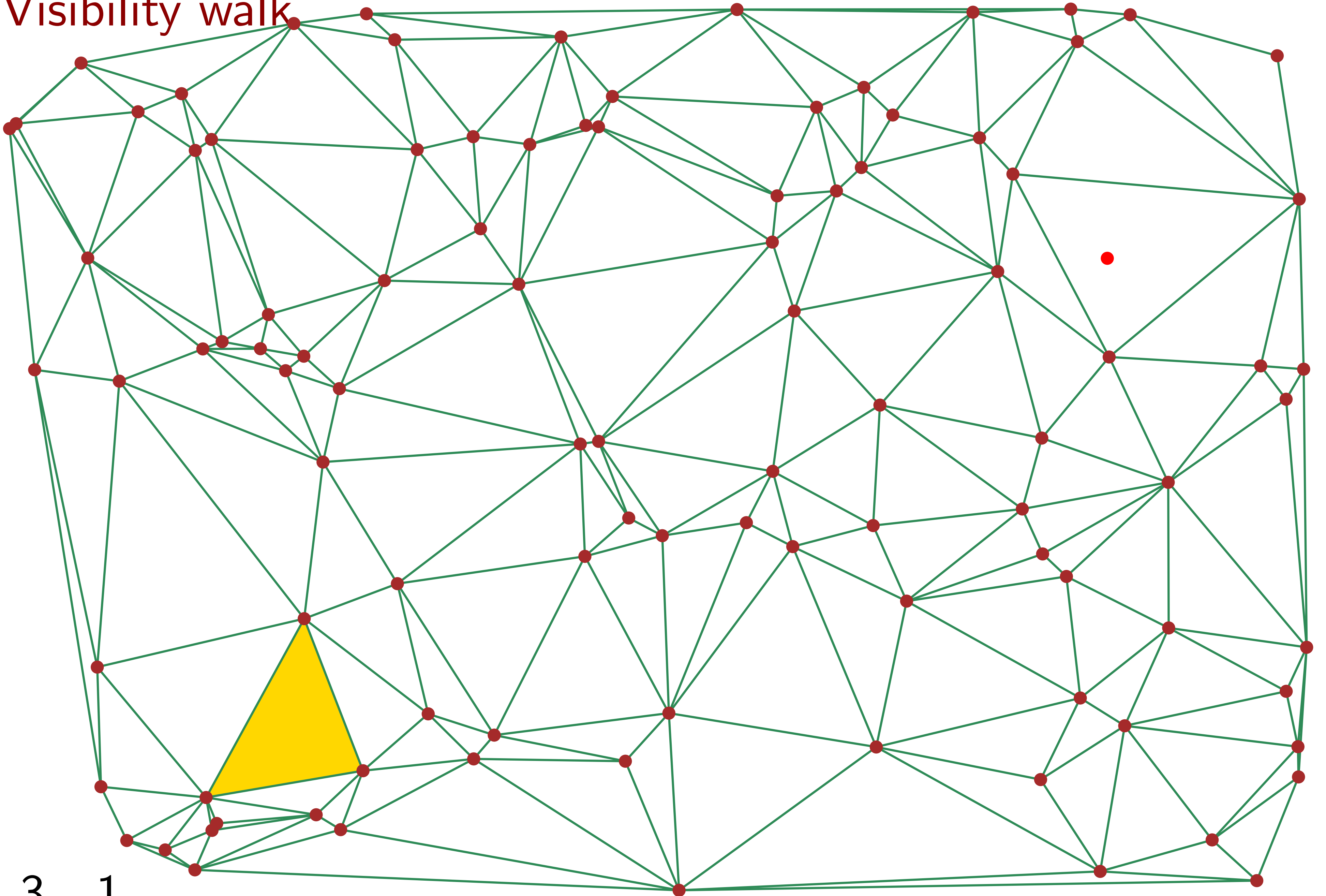


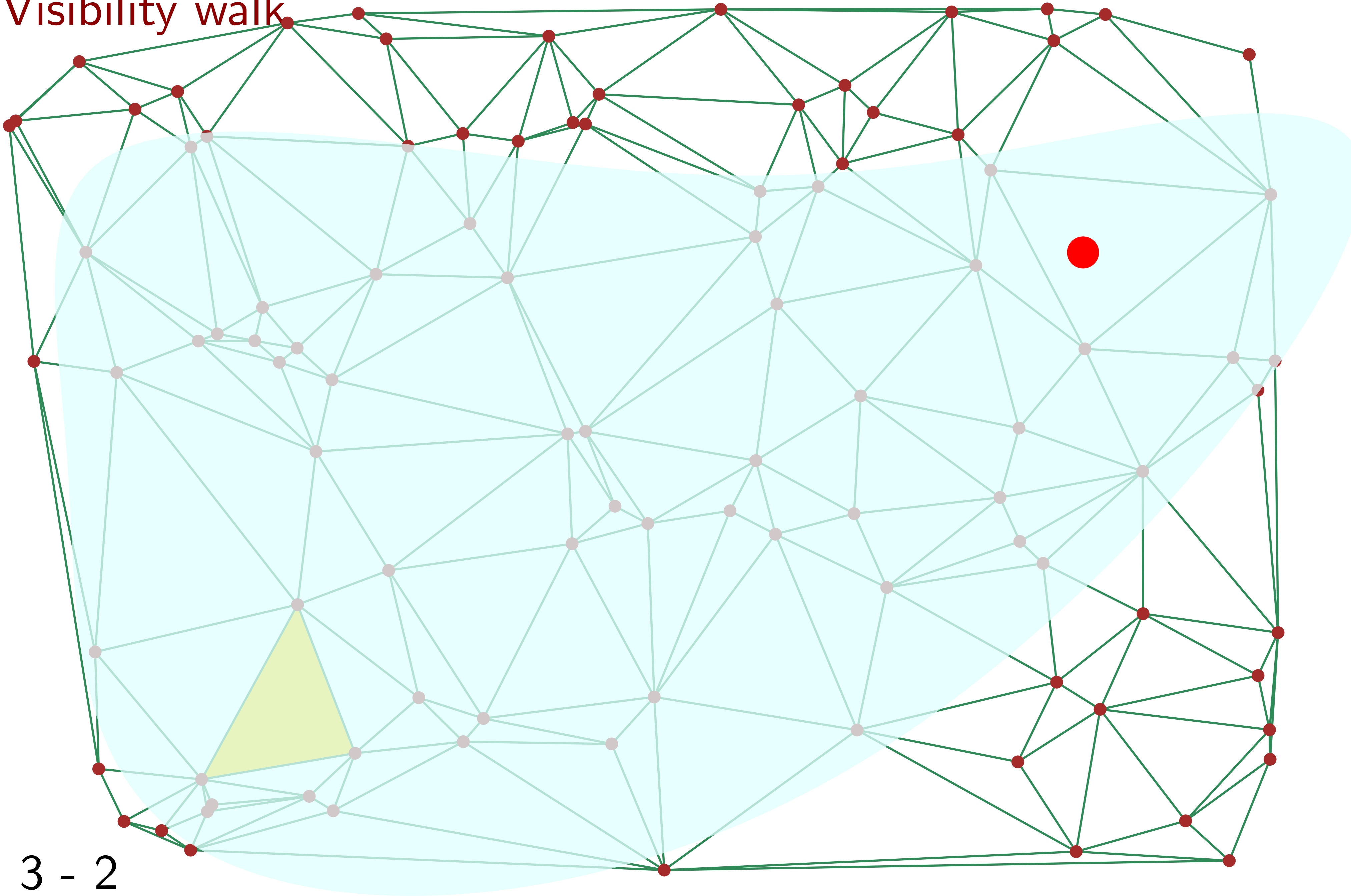
Walking in Delaunay triangulations

Visibility walk



Walking in Delaunay triangulations

Visibility walk



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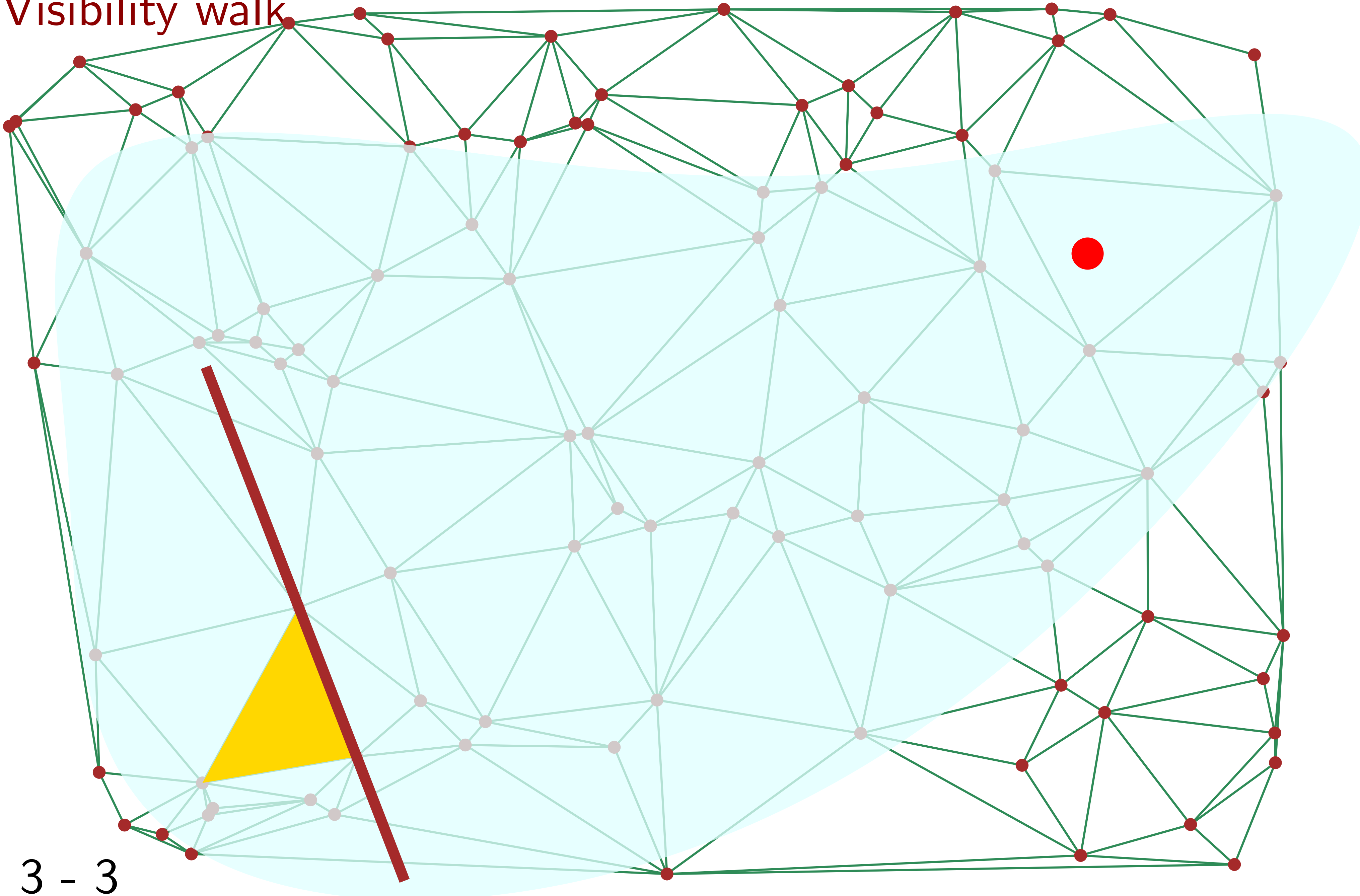
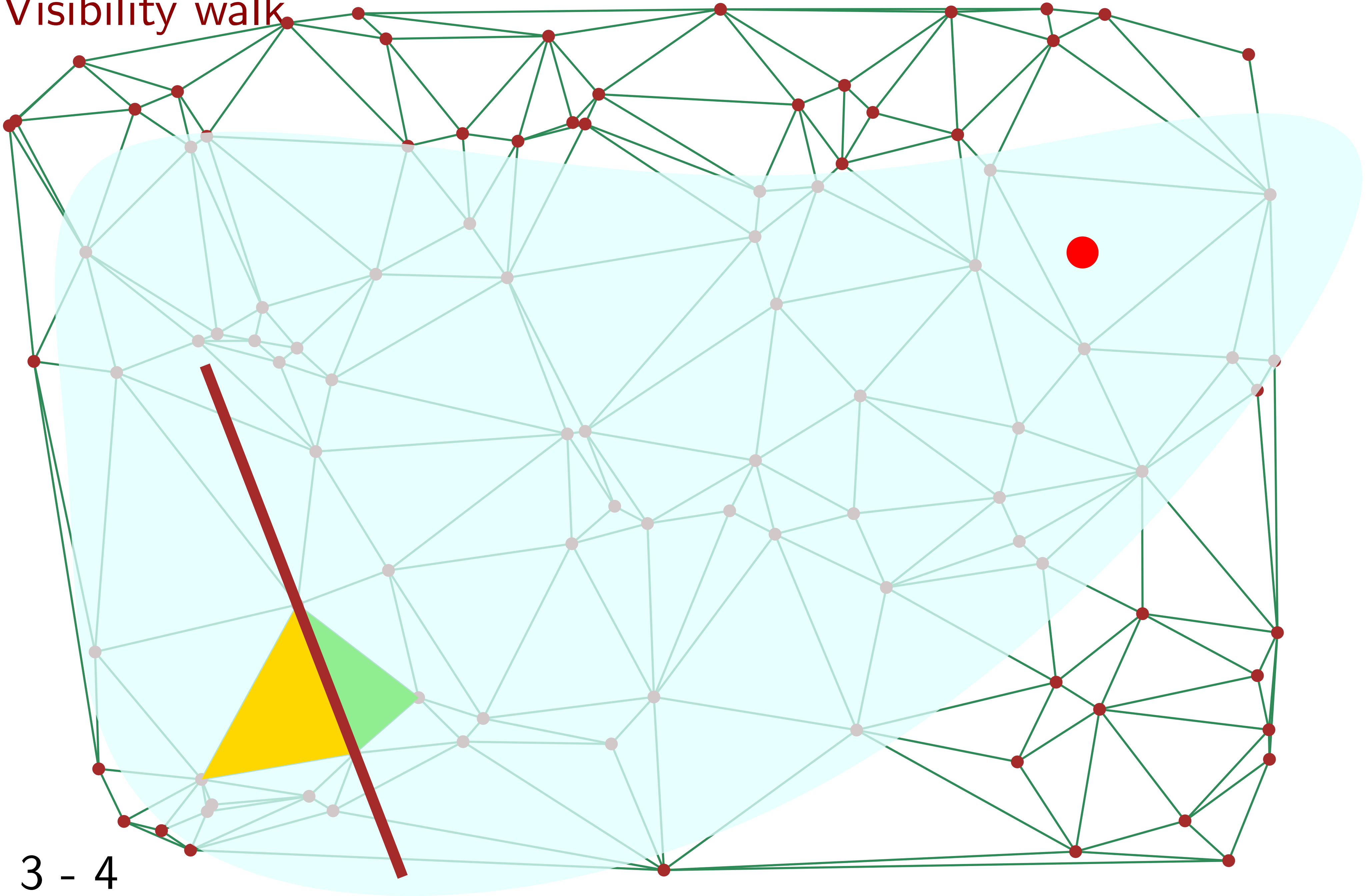
Visibility walk



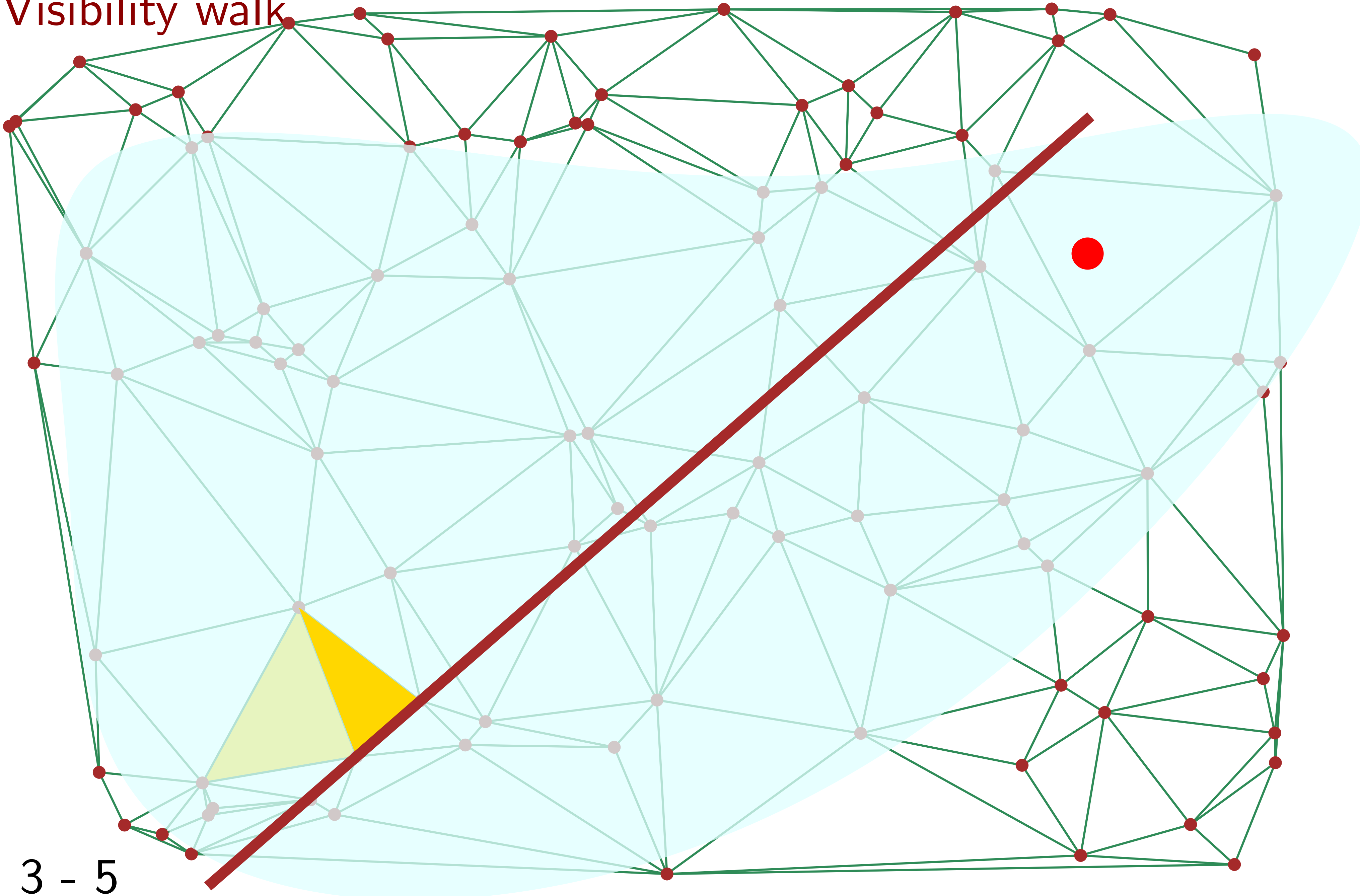
Walking in Delaunay triangulations

Visibility walk



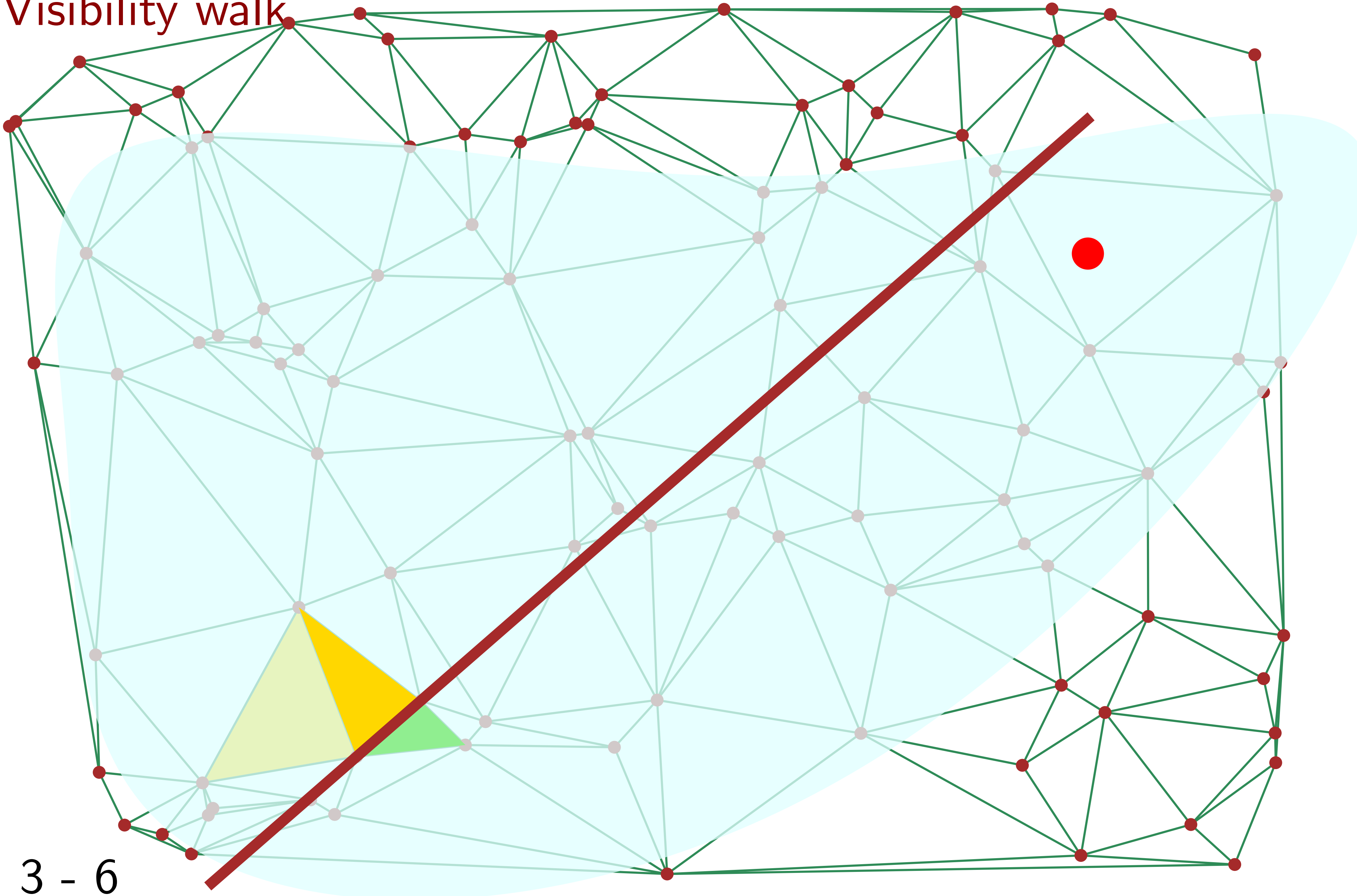
Walking in Delaunay triangulations

Visibility walk



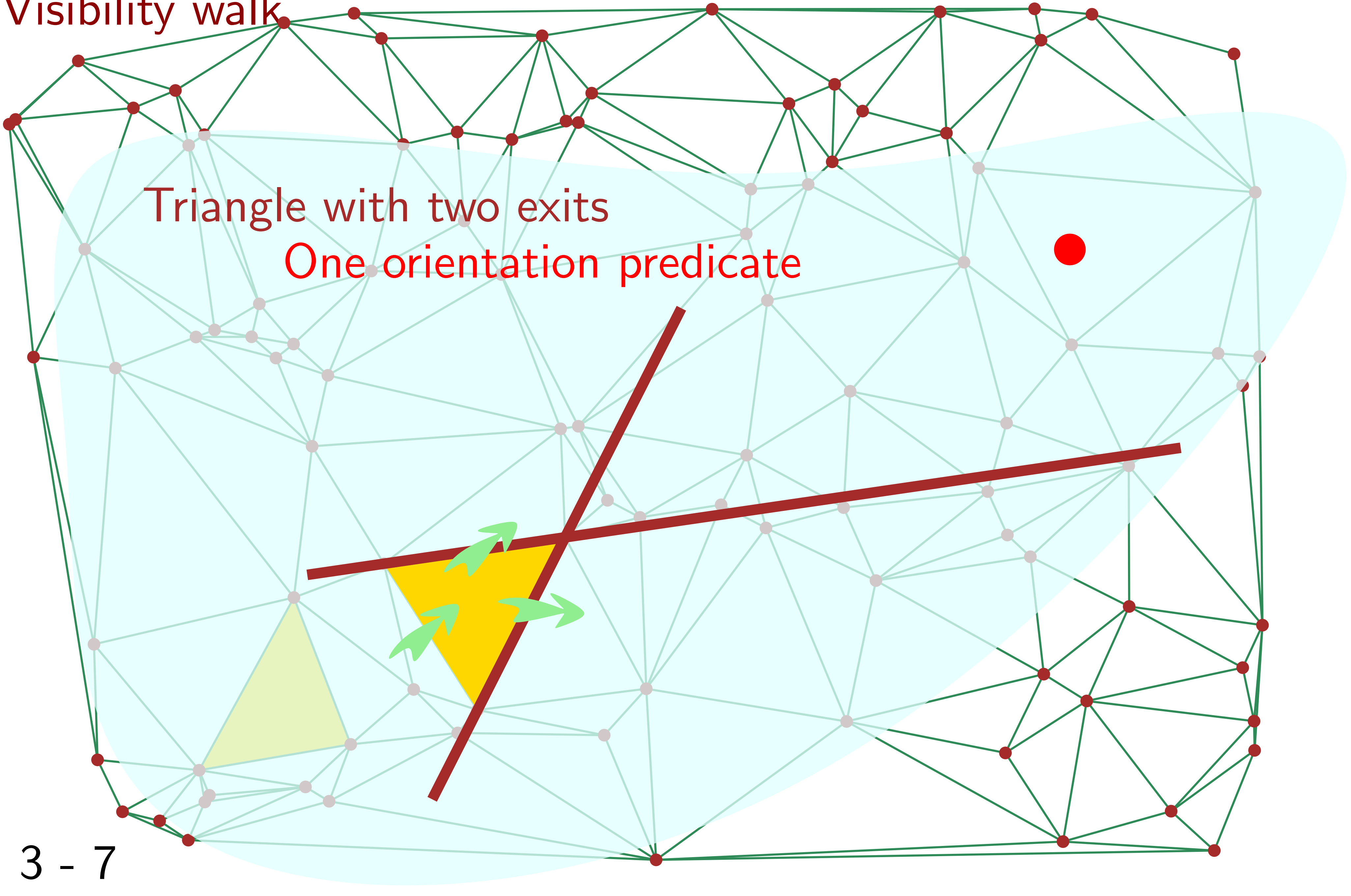
Walking in Delaunay triangulations

Visibility walk



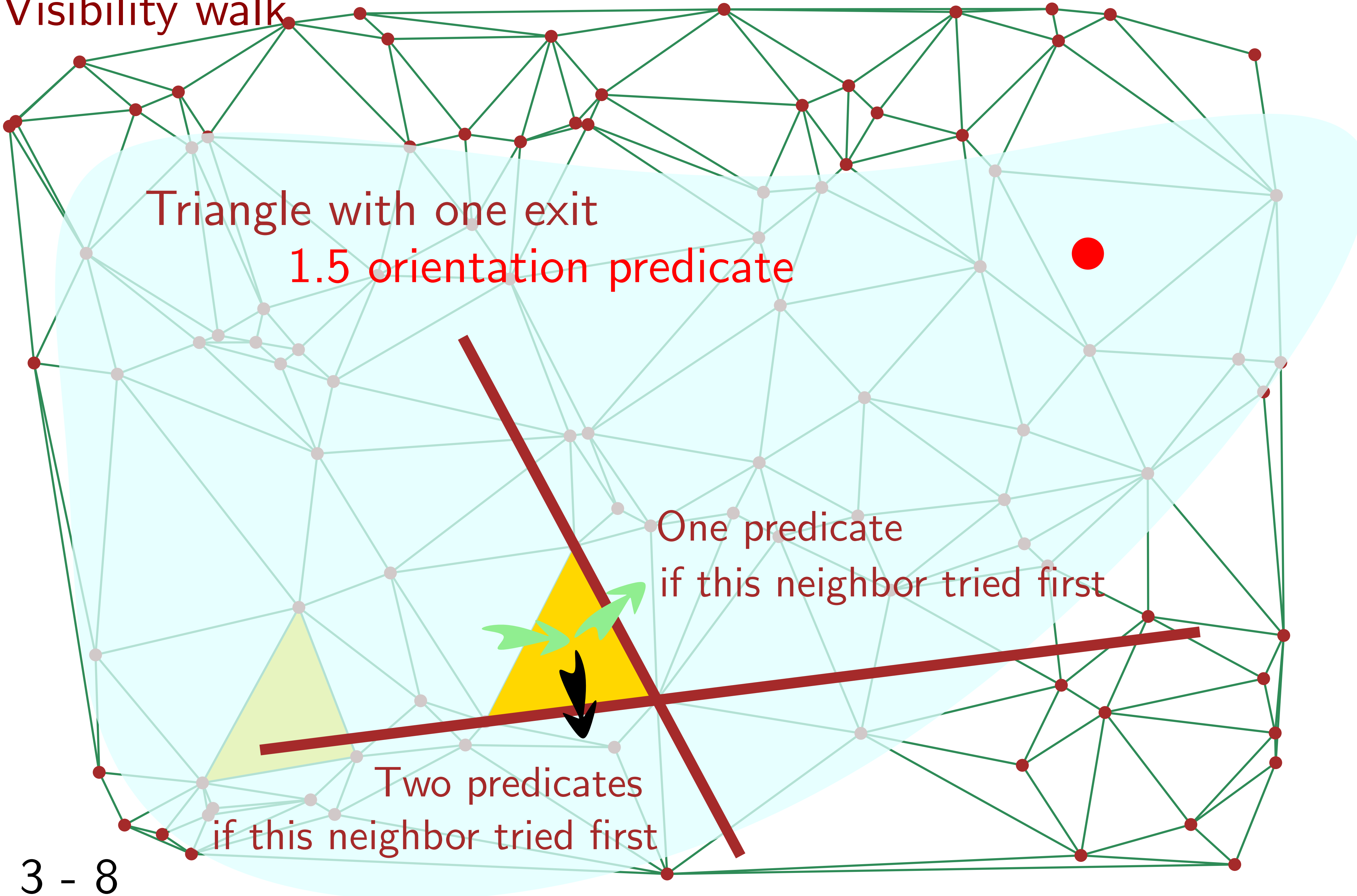
Walking in Delaunay triangulations

Visibility walk



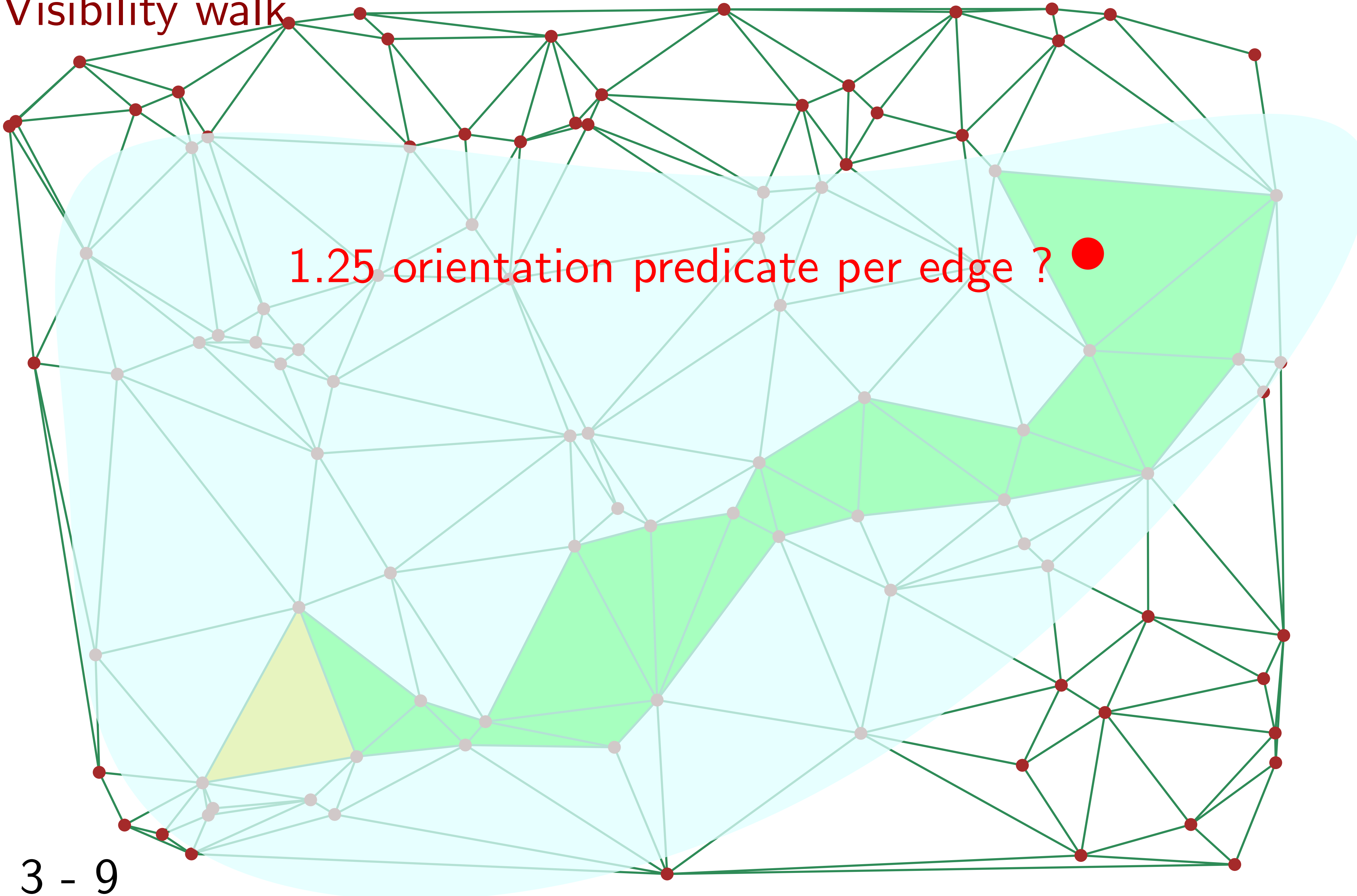
Walking in Delaunay triangulations

Visibility walk



Walking in Delaunay triangulations

Visibility walk



Walking in Delaunay triangulations

How many edges crossed ?

Straight walk

Visibility walk

Walking in Delaunay triangulations

How many edges crossed ?

Straight walk

$2n$

Visibility walk

Worst case in a triangulation (non Delaunay)

Walking in Delaunay triangulations

How many edges crossed ?

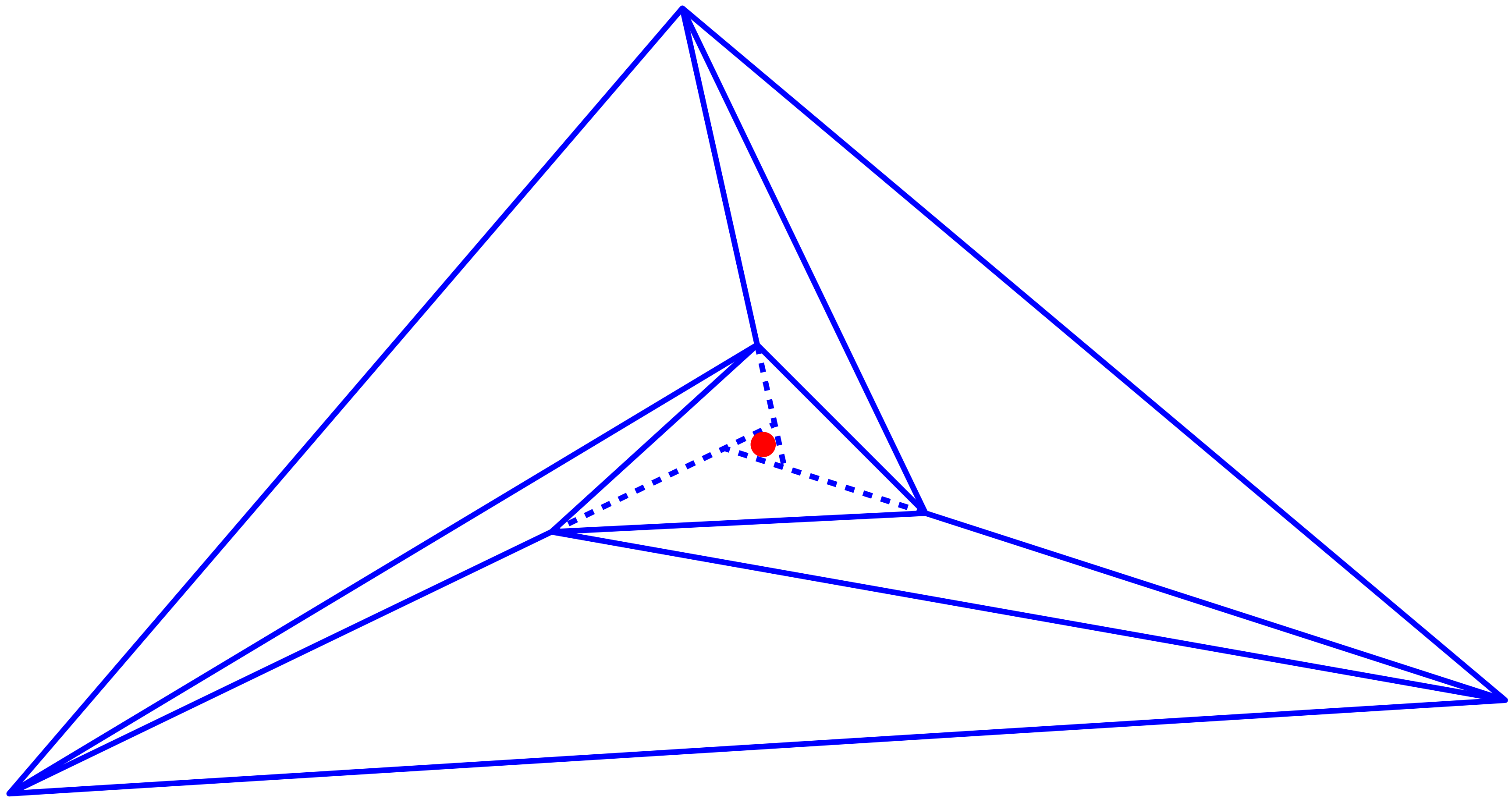
Straight walk

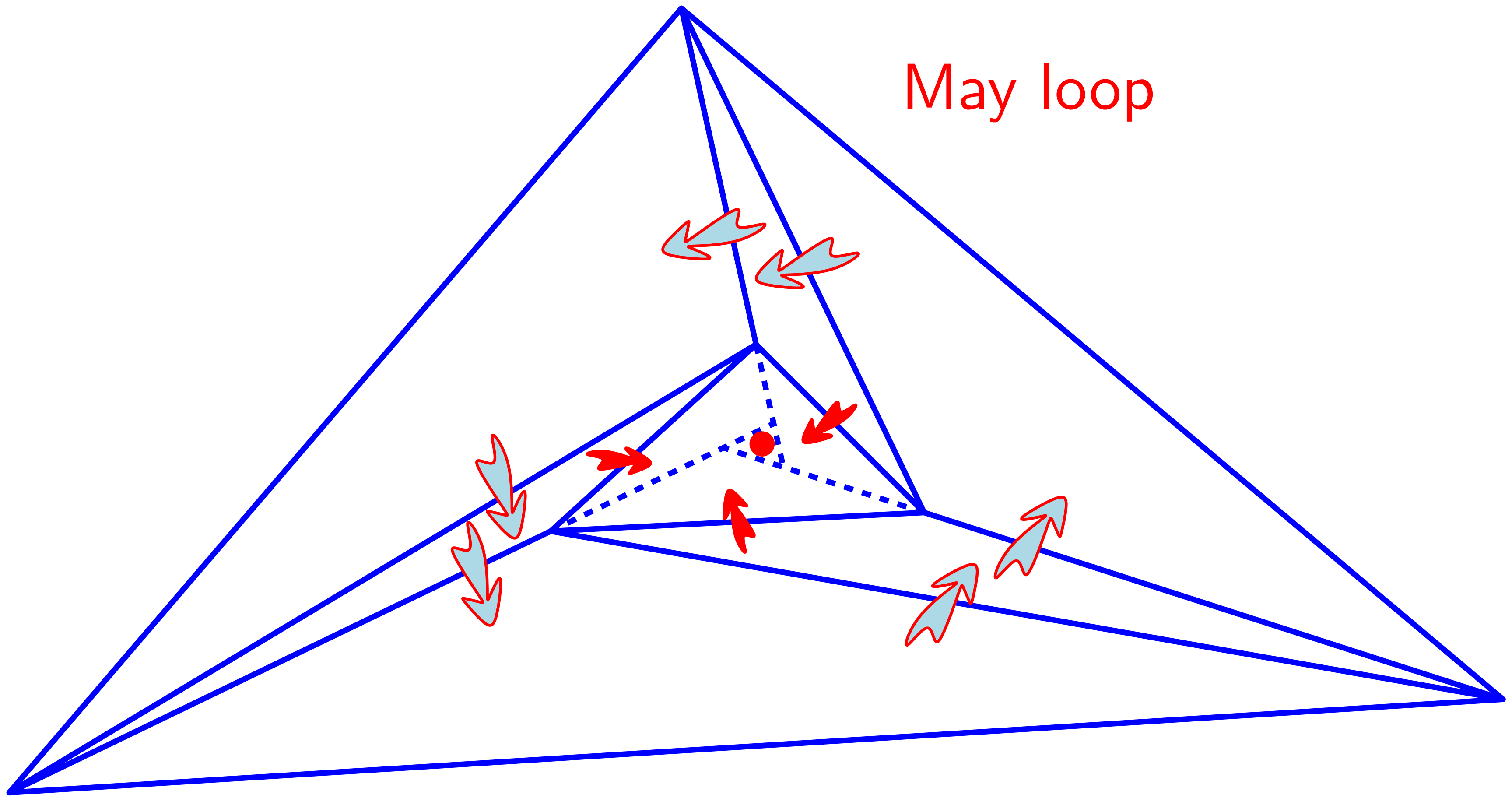
$2n$

Visibility walk

∞

Worst case in a triangulation (non Delaunay)




May loop

Walking in Delaunay triangulations

How many edges crossed ?

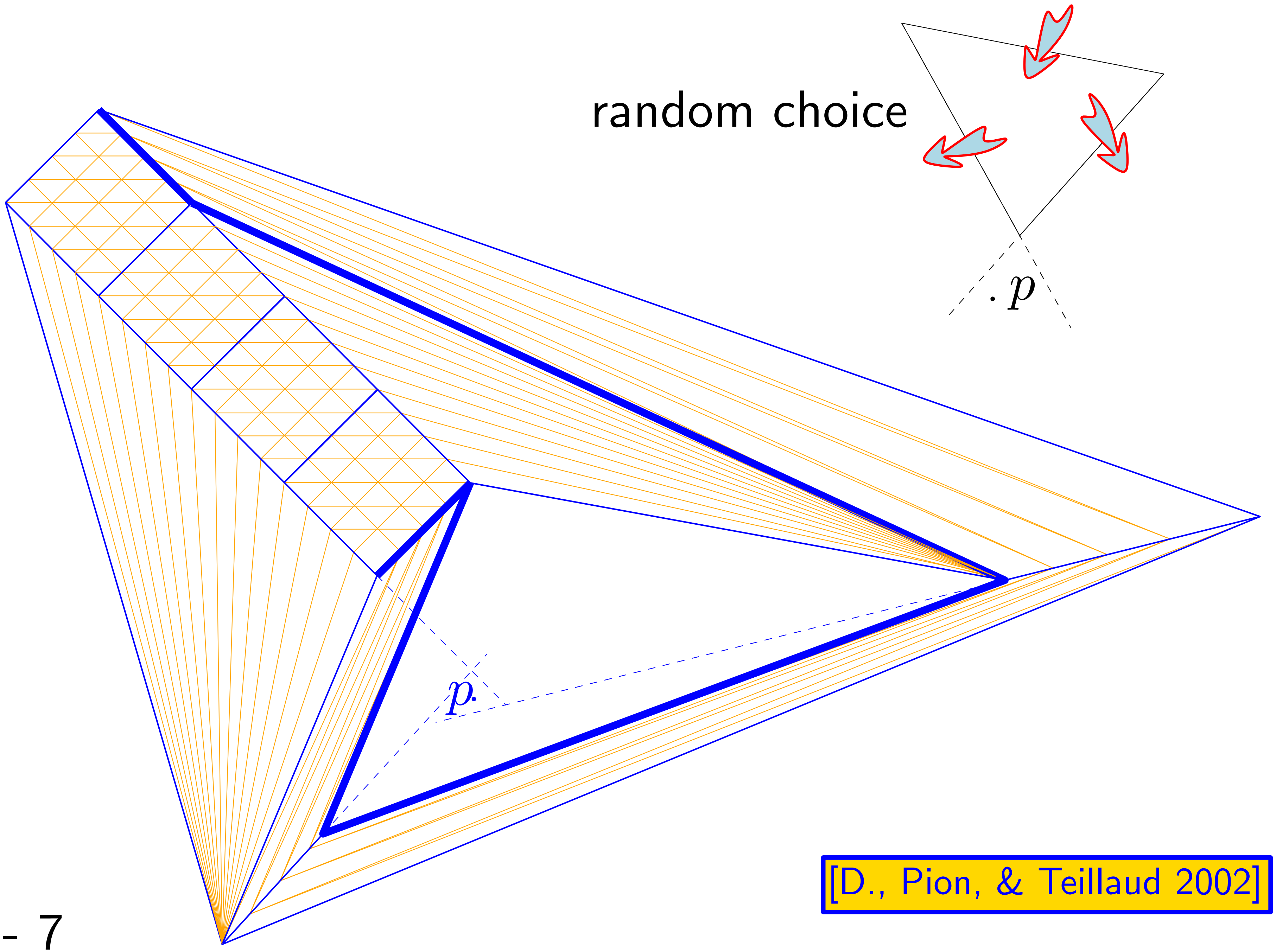
Straight walk

$2n$

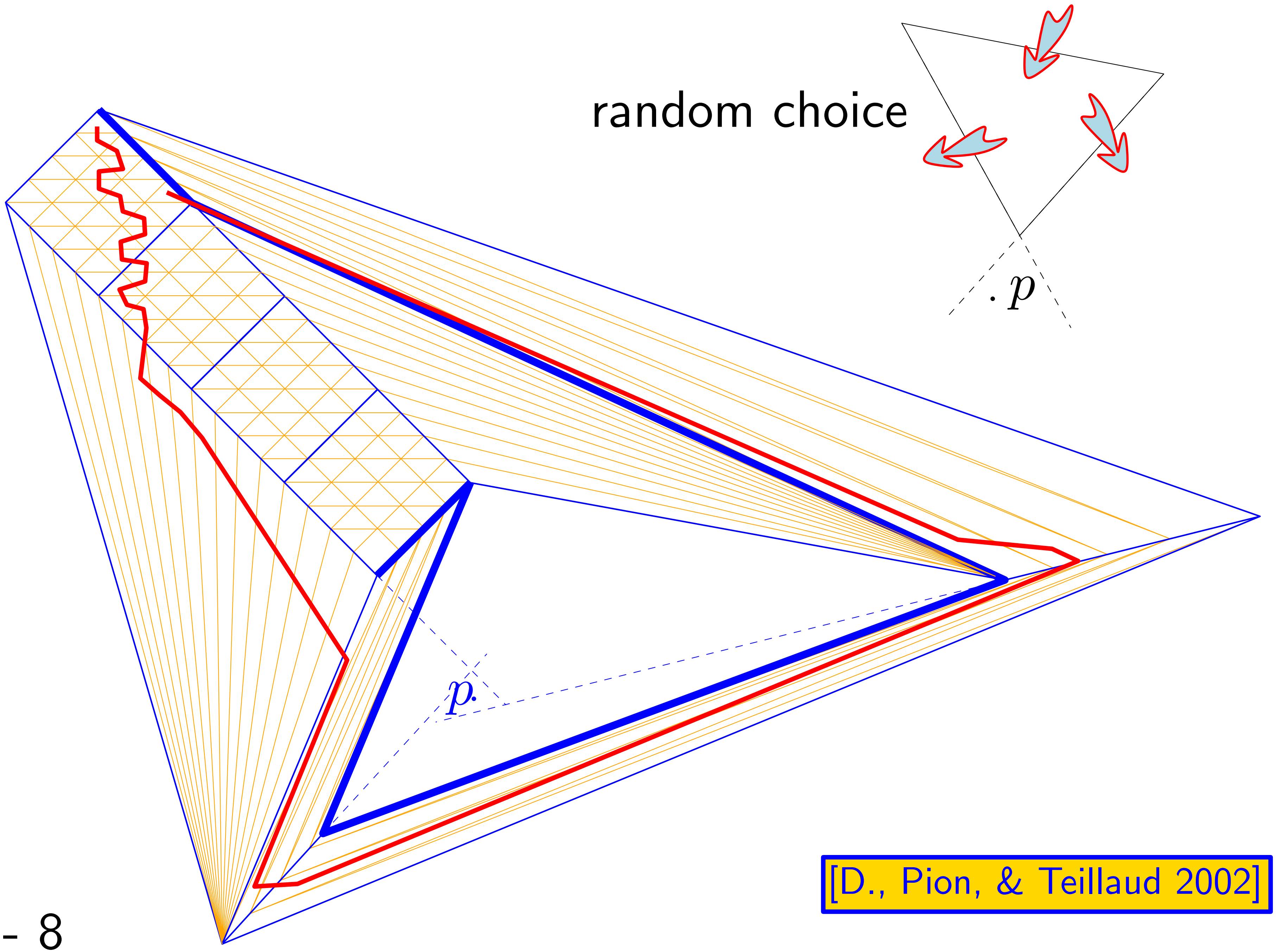
Visibility walk

∞ $\geq 2^{\sqrt[3]{n}}$ randomized

Worst case in a triangulation (non Delaunay)



[D., Pion, & Teillaud 2002]



[D., Pion, & Teillaud 2002]

Walking in Delaunay triangulations

How many edges crossed ?

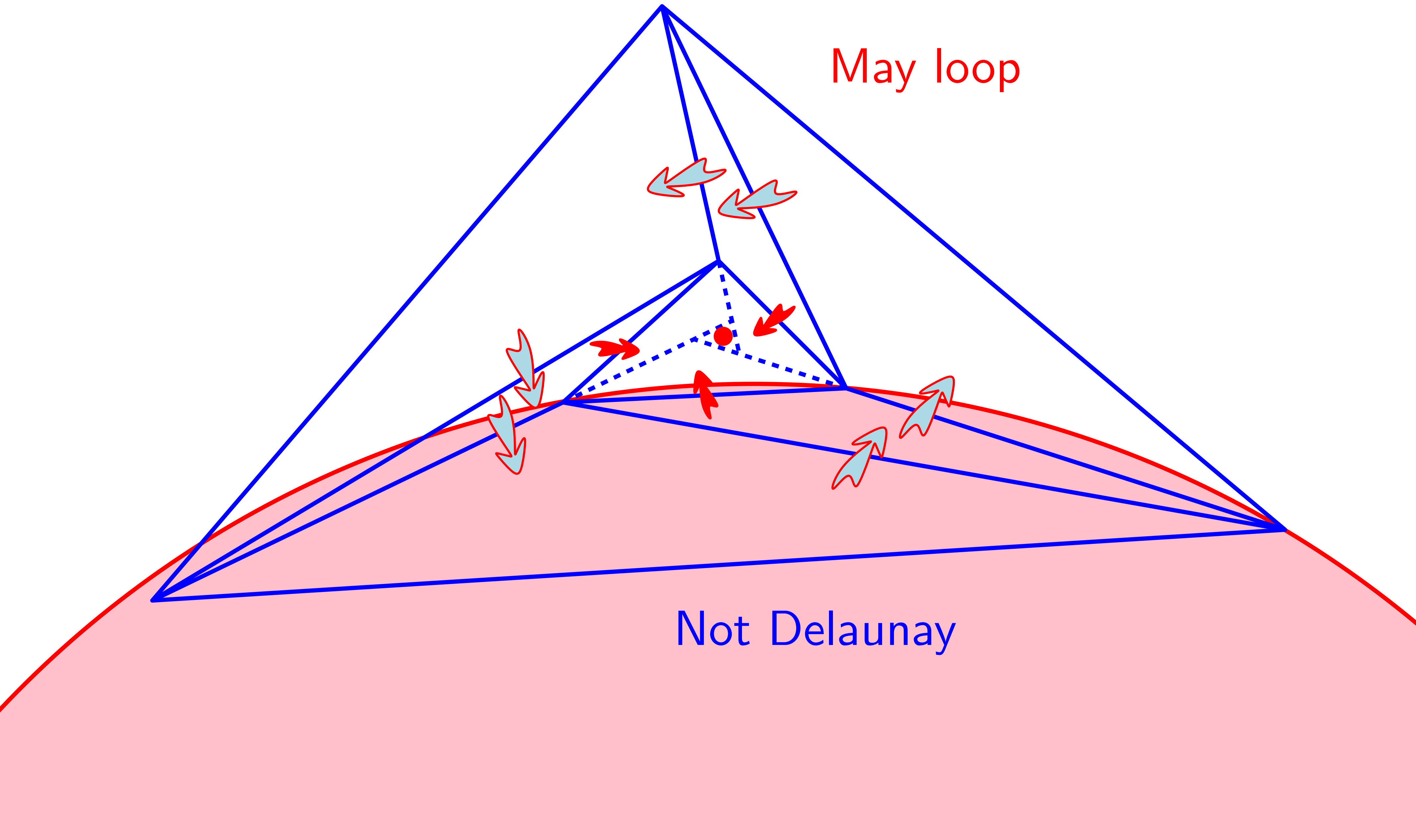
Straight walk

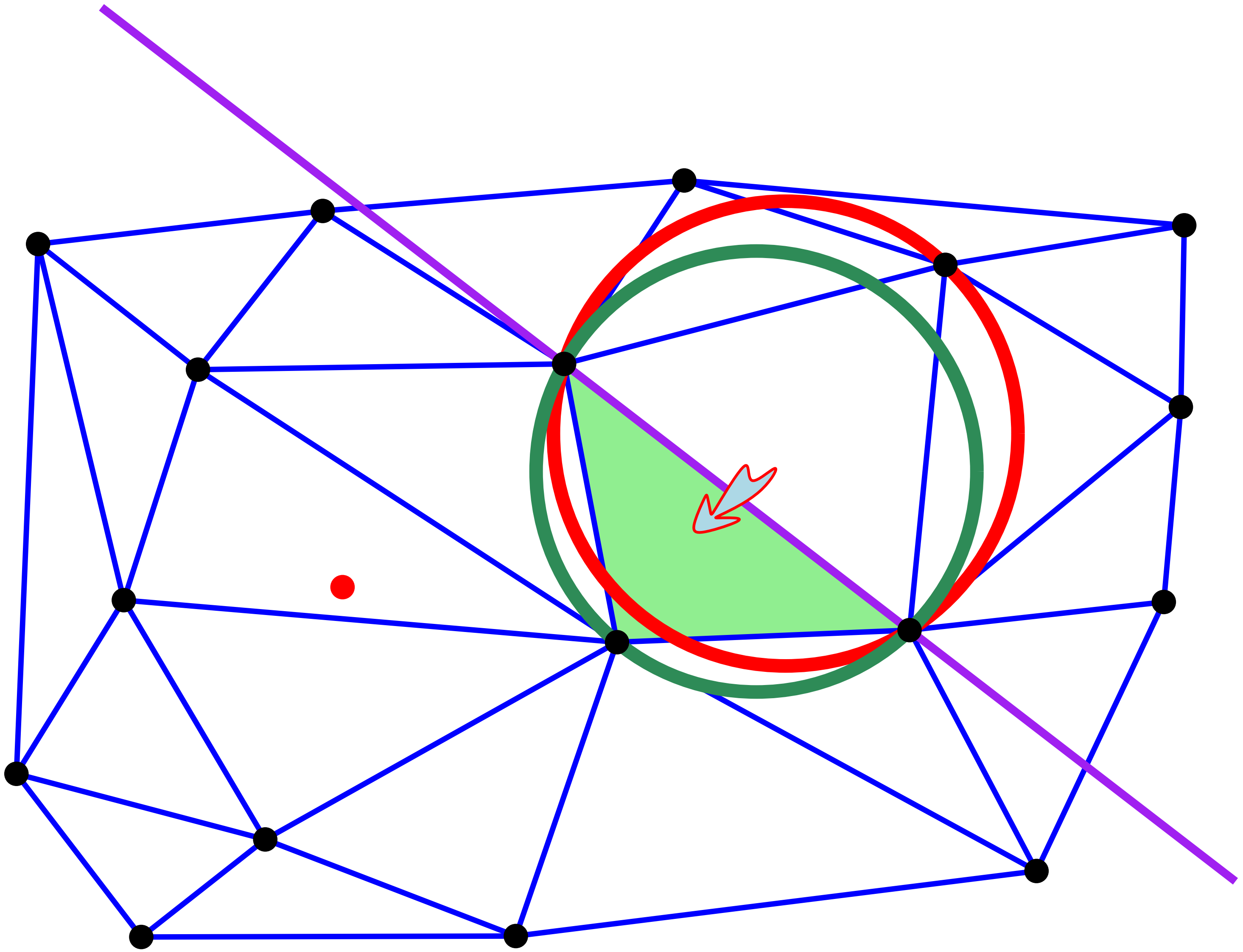
$2n$

Visibility walk

$2n$

Worst case in a Delaunay triangulation


May loop
Not Delaunay



Green power < Red power

Power decreases

Walking in Delaunay triangulations

How many edges crossed ?

Straight walk

$2n$ $O(\sqrt{n})$ [Devroye, Lemaire, & Moreau, 2004]

uniform in domain

$$\frac{64}{3\pi^2}\sqrt{n} + O\left(\frac{1}{\sqrt{n}}\right) \simeq 2.16\sqrt{n}$$

Stretch in infinite Poisson distribution

Visibility walk

[D. & Hemsley, 2016]

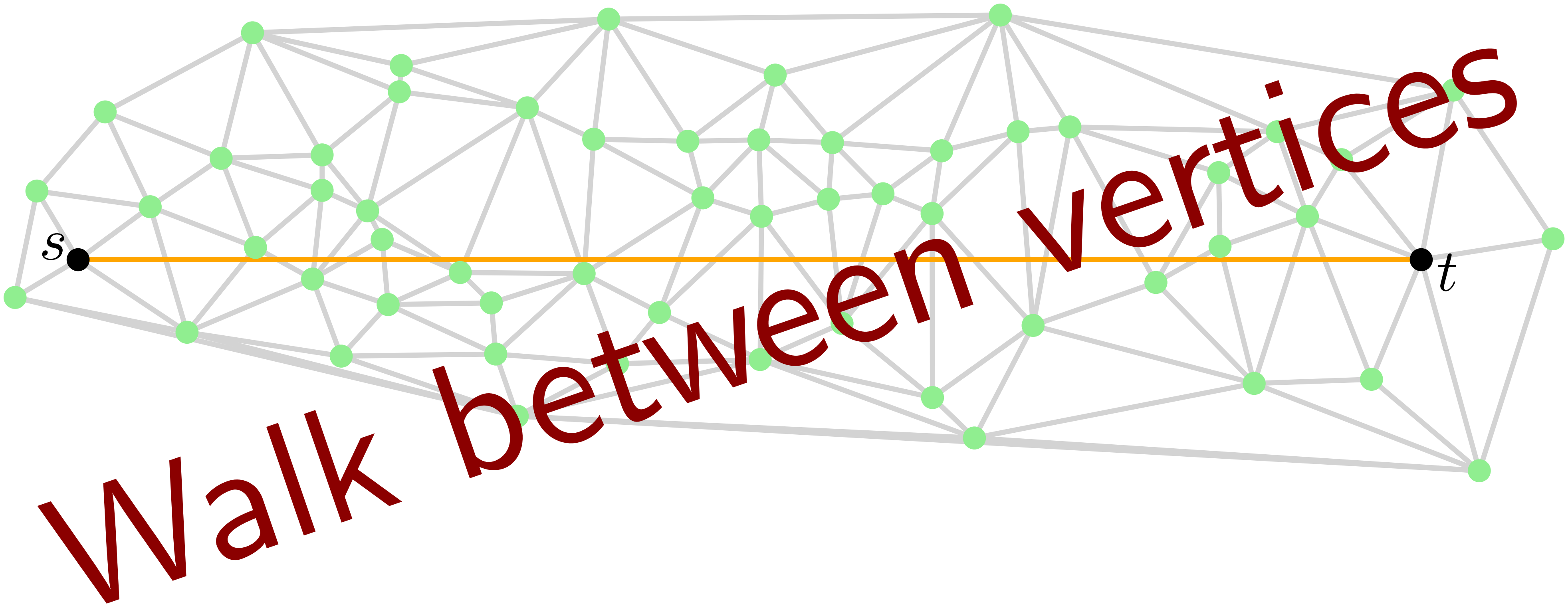
$2n$ $O(\sqrt{n})$

random

Stretch in infinite Poisson distribution

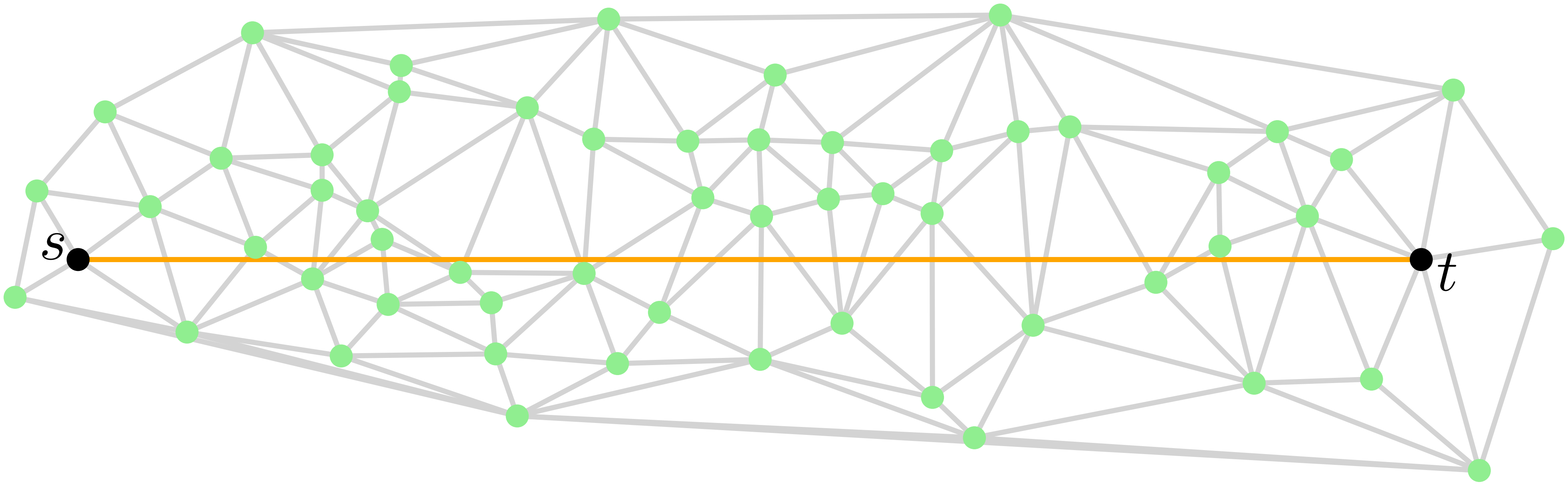
Worst case in a Delaunay triangulation

Walking in Delaunay triangulations



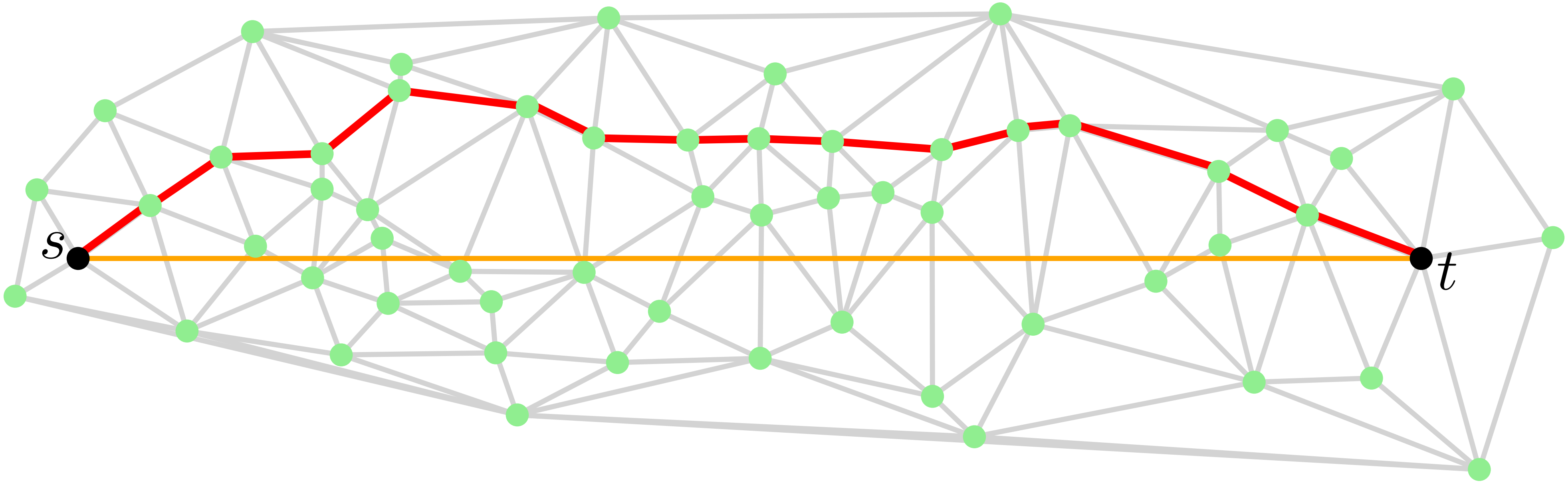
Walking in Delaunay triangulations

Walk between vertices



Walking in Delaunay triangulations

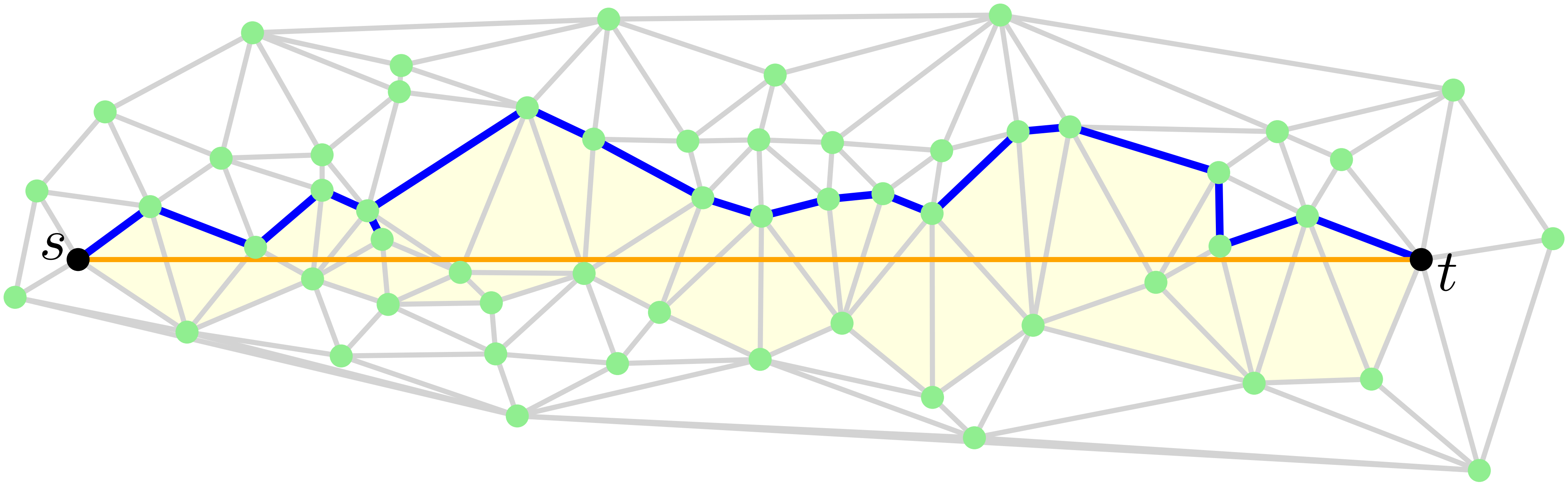
Walk between vertices



Shortest path

Walking in Delaunay triangulations

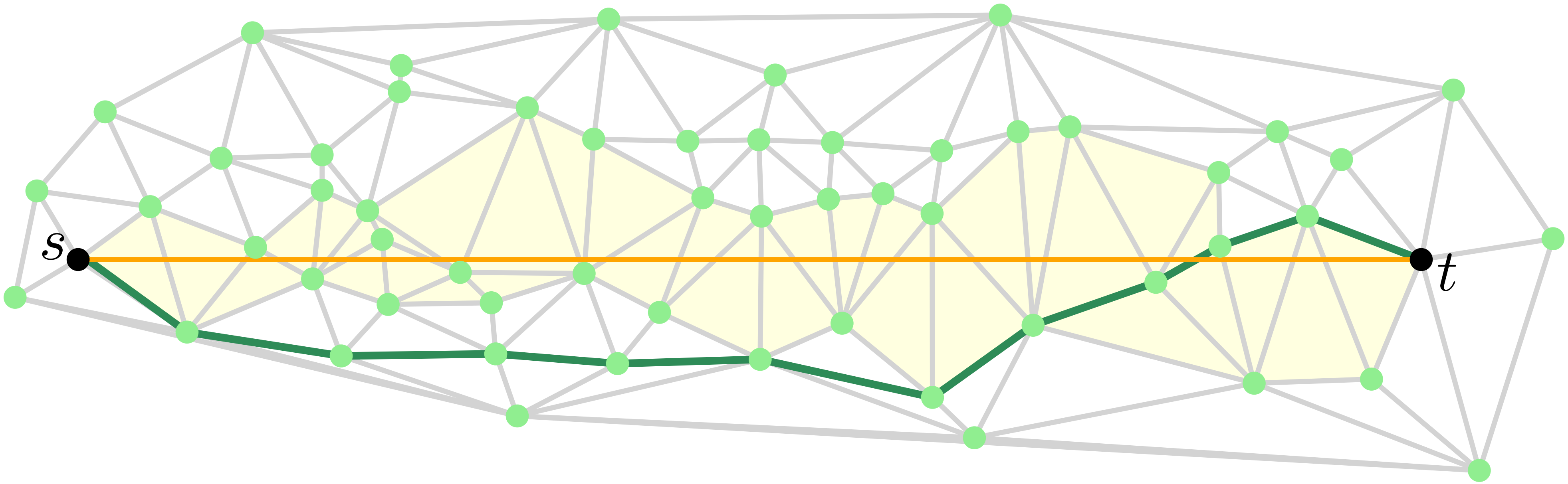
Walk between vertices



Upper path

Walking in Delaunay triangulations

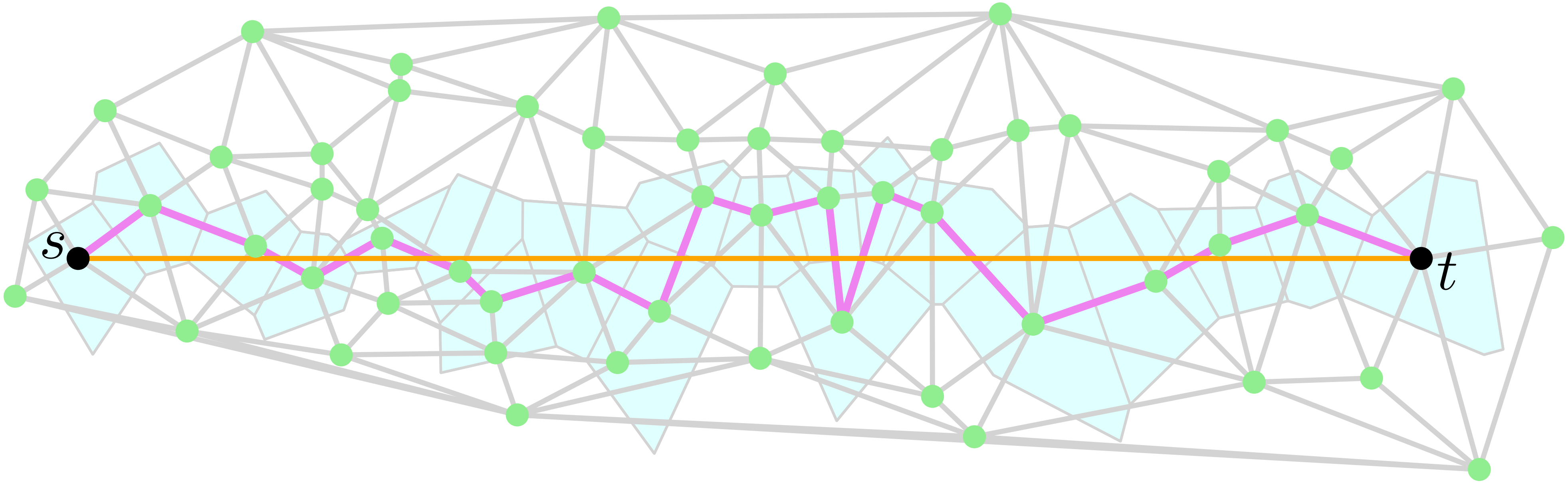
Walk between vertices



Compass walk

Walking in Delaunay triangulations

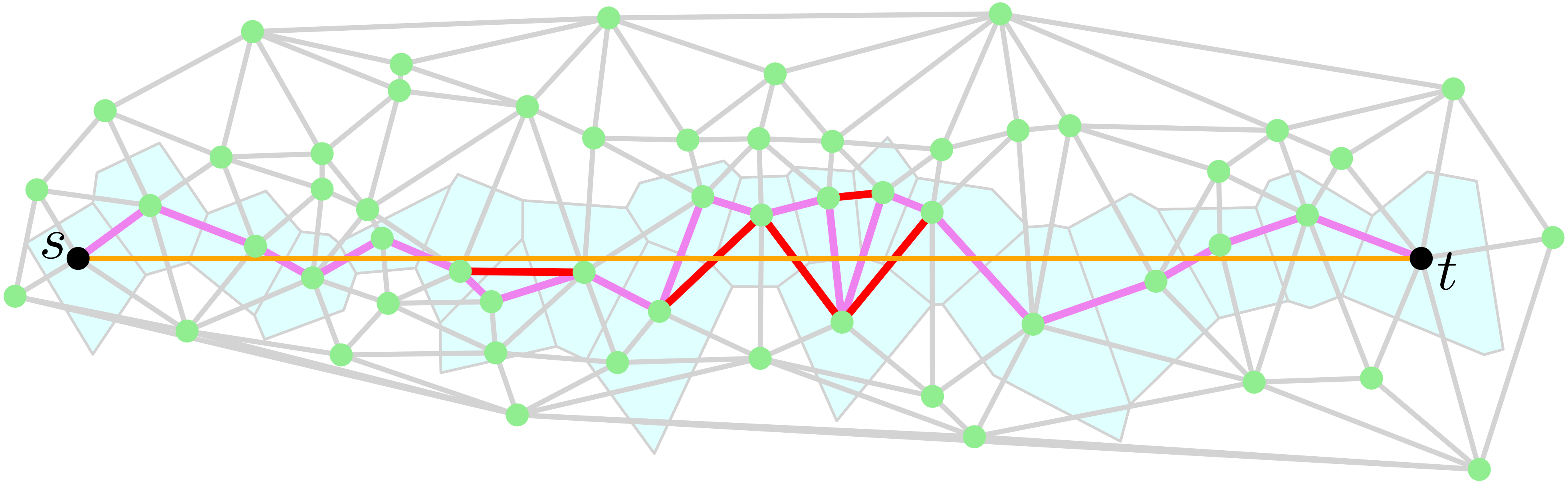
Walk between vertices



Voronoi path

Walking in Delaunay triangulations

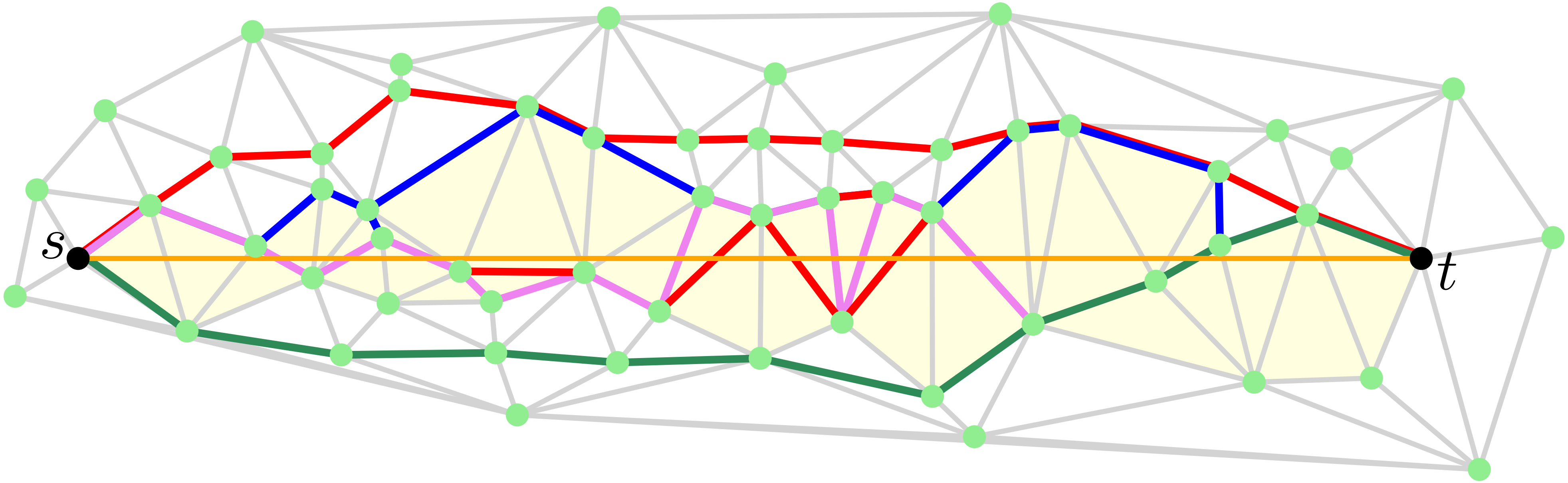
Walk between vertices



Voronoi path with shortcuts

Walking in Delaunay triangulations

Walk between vertices

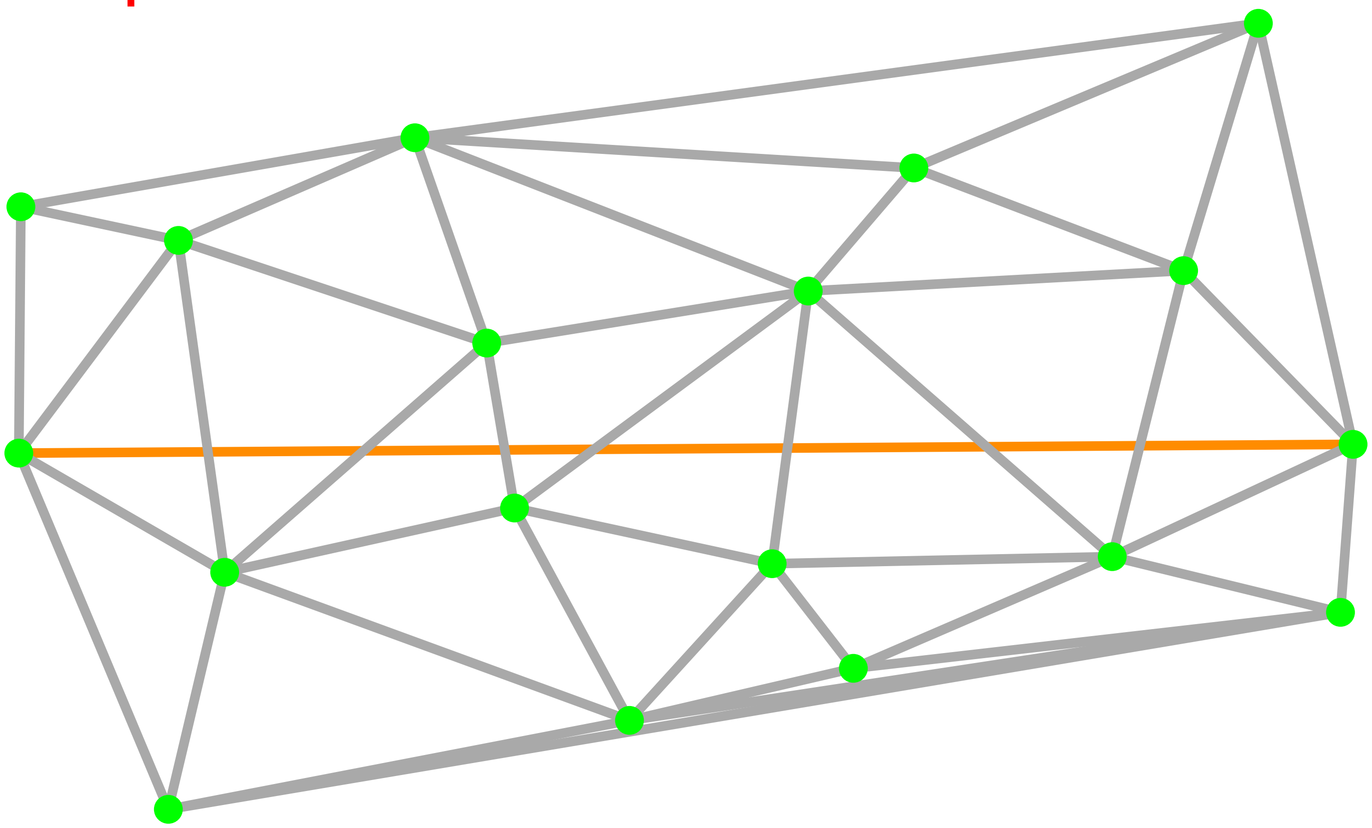


Shortest path
Upper path
Compass walk
Voronoi path with shortcuts

Walking in Delaunay triangulations

Walk between vertices, worst case

Shortest path

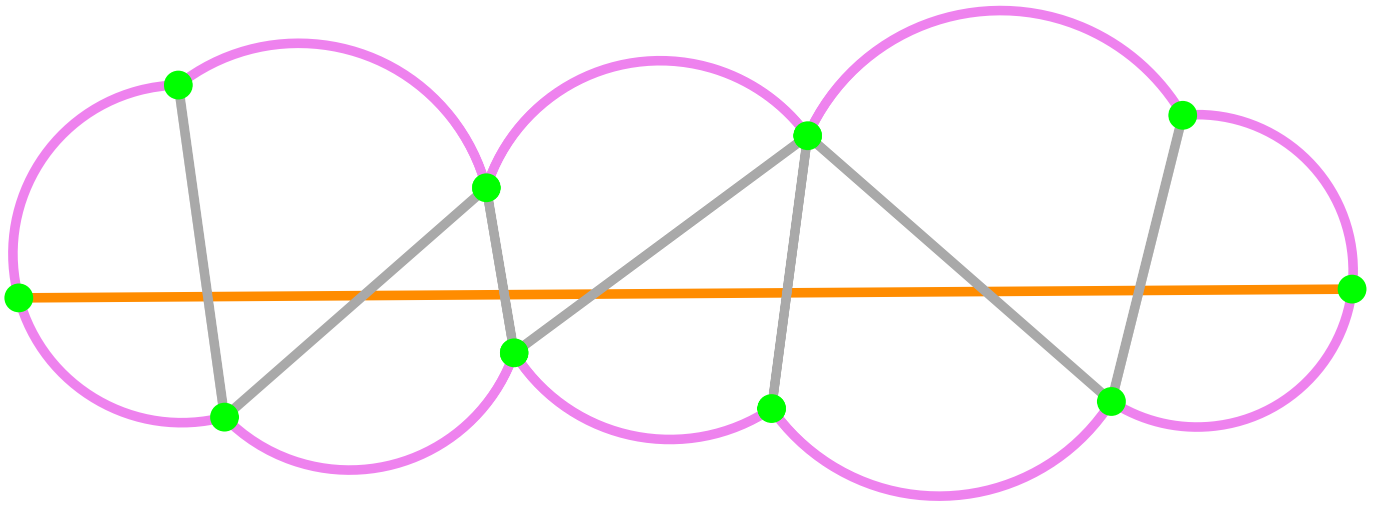


Walking in Delaunay triangulations

Walk between vertices, worst case

Shortest path

Search this "subgraph"

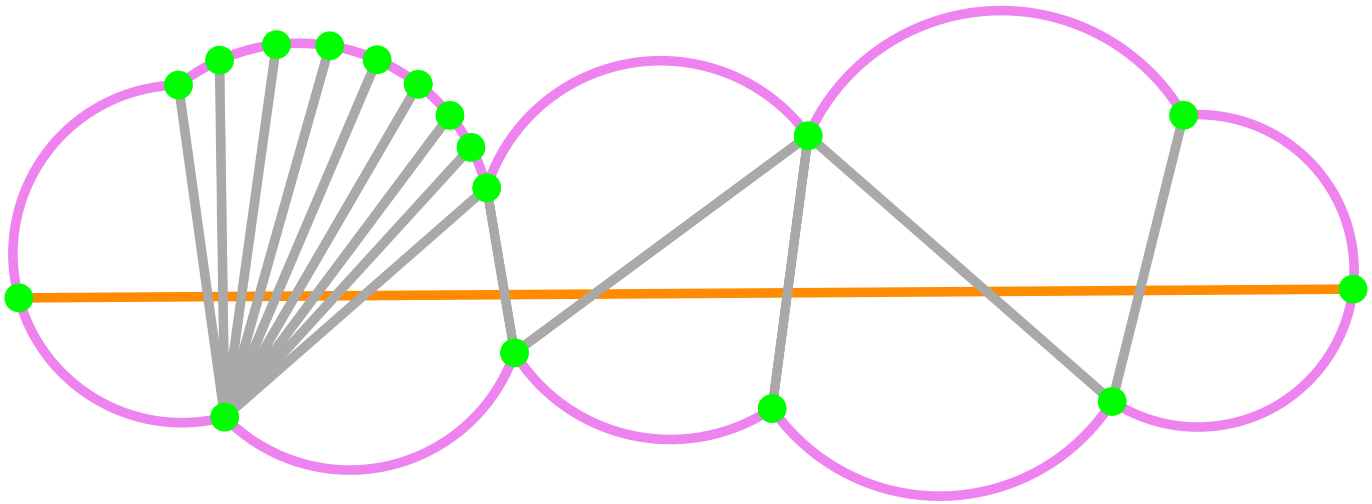


Walking in Delaunay triangulations

Walk between vertices, worst case

Shortest path

Search this "subgraph"

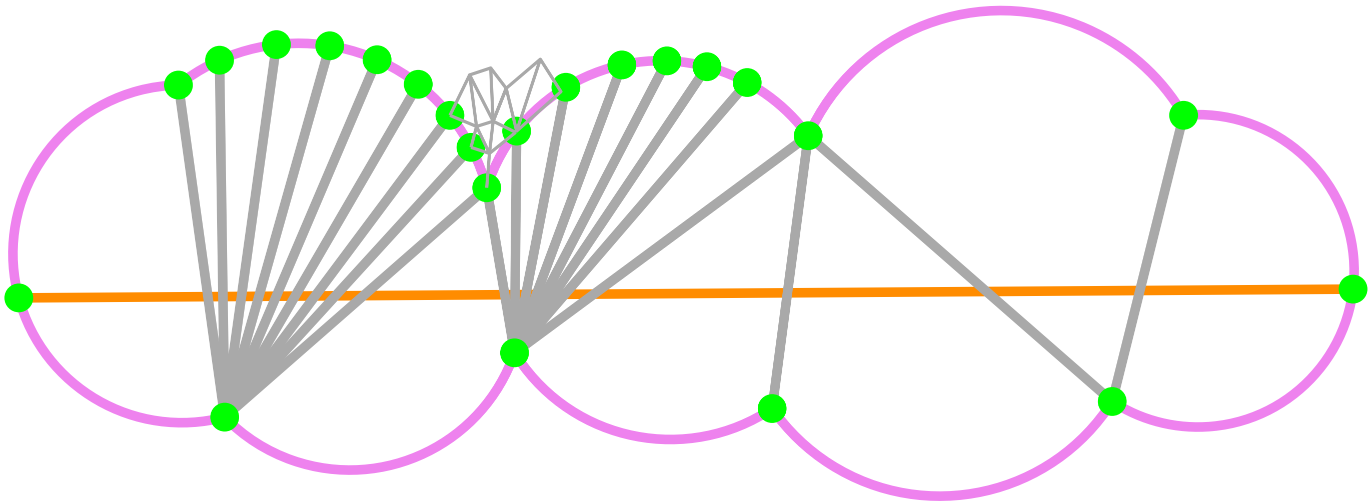


Walking in Delaunay triangulations

Walk between vertices, worst case

Shortest path

Search this "subgraph"

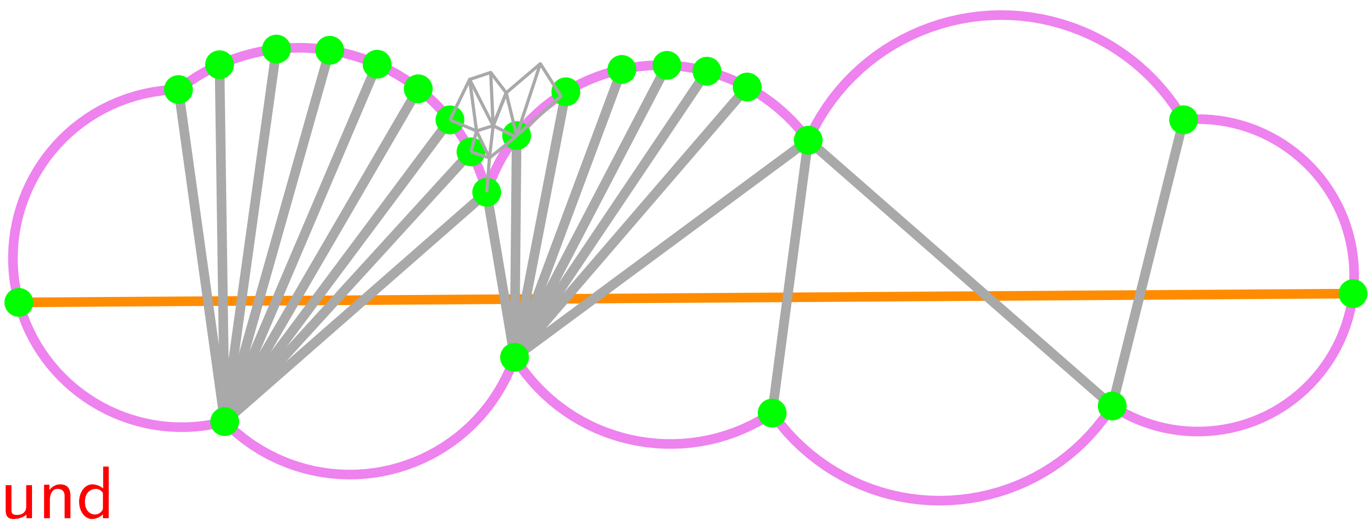


Walking in Delaunay triangulations

Walk between vertices, worst case

Shortest path

Search this "subgraph"



Upper bound

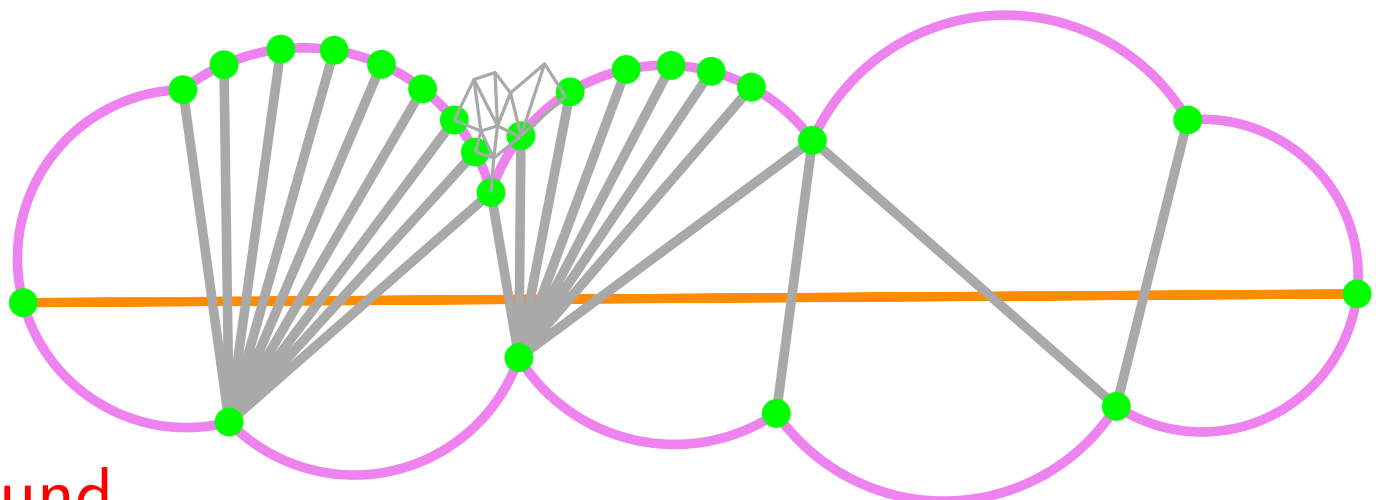
Dobkin, Friedman, and Supowit 1987	5.08	$\frac{1+\sqrt{5}}{2}\pi$
Keil and Gutwin 1989	2.42	$\frac{2\pi}{3\cos(\pi/6)}$
Xia 2011	1.998	

Walking in Delaunay triangulations

Walk between vertices, worst case

Shortest path

Search this "subgraph"



Lower bound

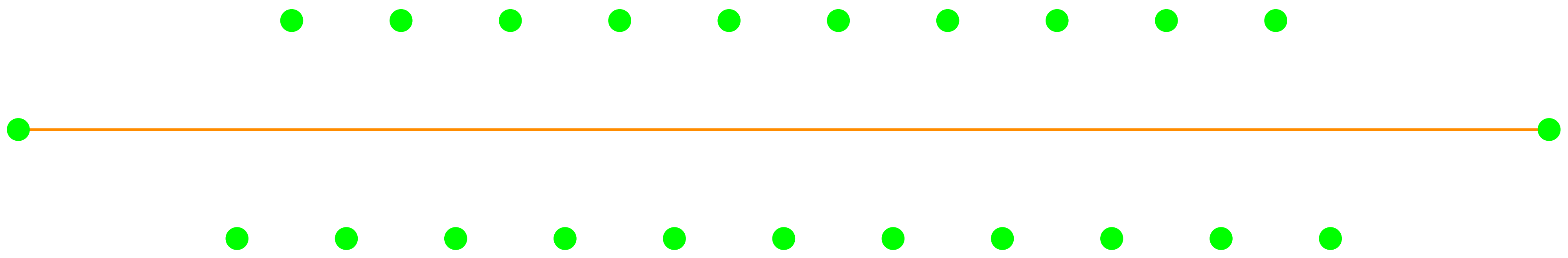
Chew 1989	1.5708	$\frac{\pi}{2}$
Bose, Devroye, Löffler, Snoeyink, & Verma 2011	1.5846	
Xia & Zhang 2011	1.5932	

Walking in Delaunay triangulations

Walk between vertices, worst case

Voronoi path

Unbounded

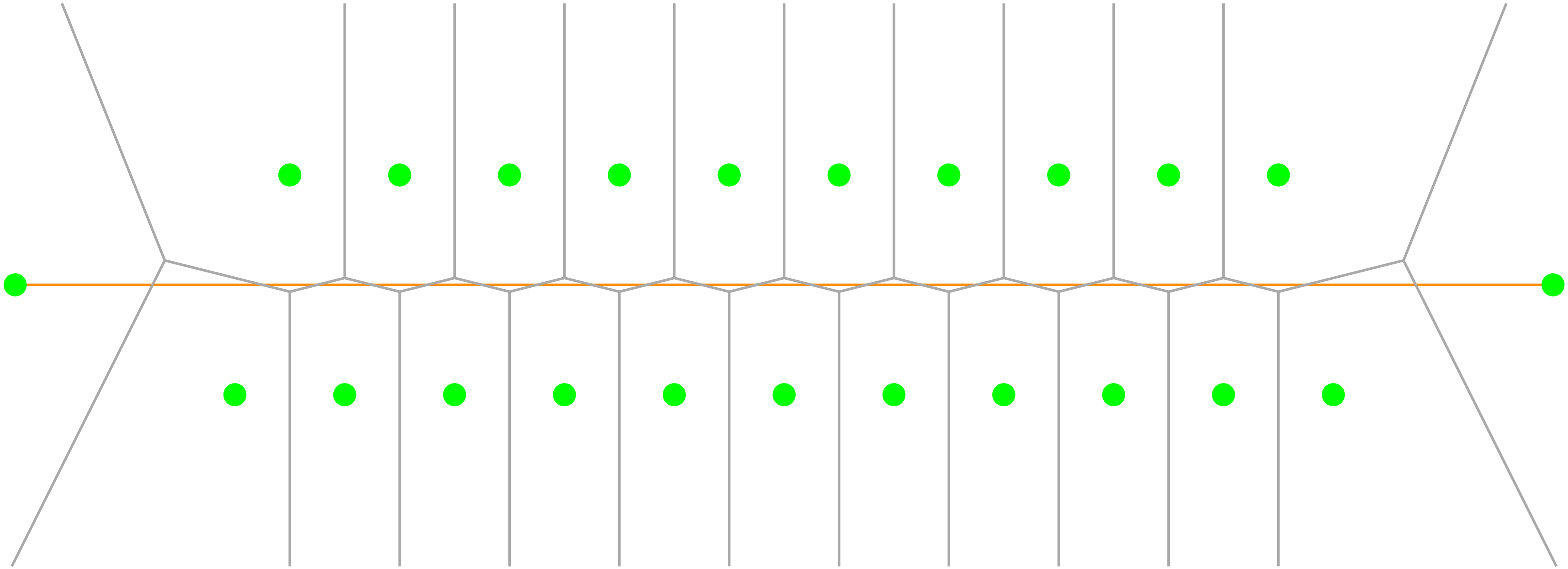


Walking in Delaunay triangulations

Walk between vertices, worst case

Voronoi path

Unbounded

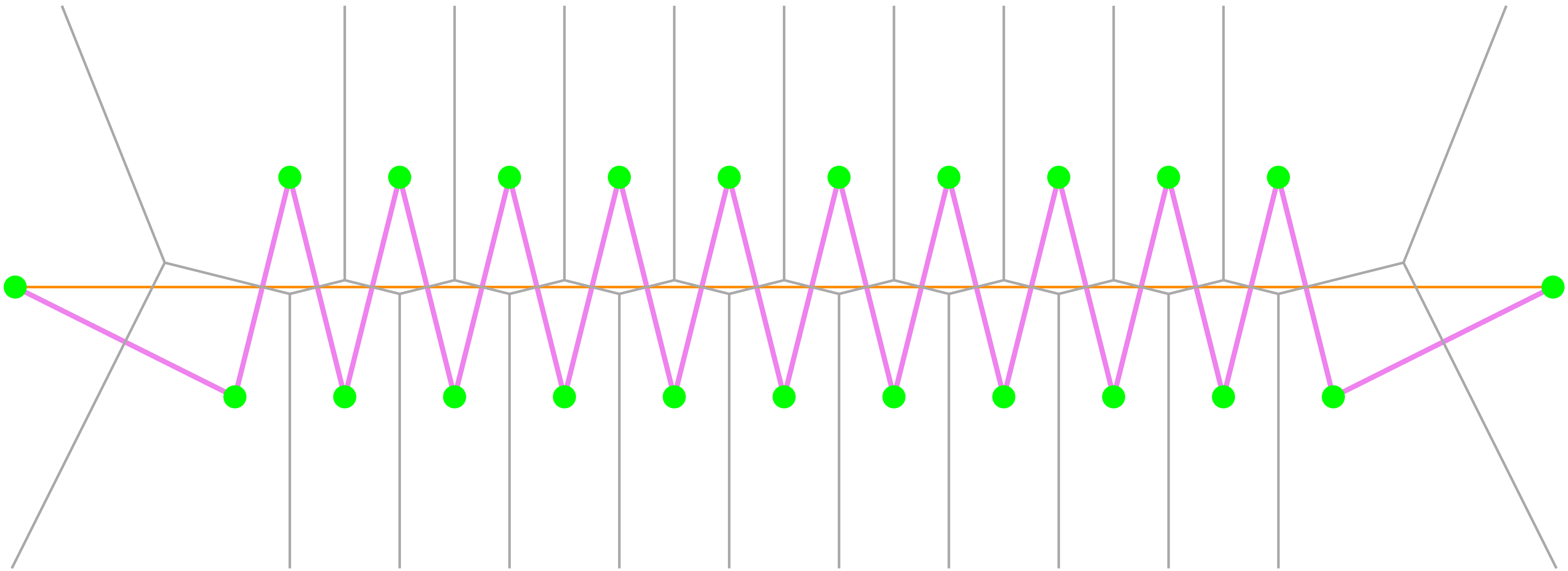


Walking in Delaunay triangulations

Walk between vertices, worst case

Voronoi path

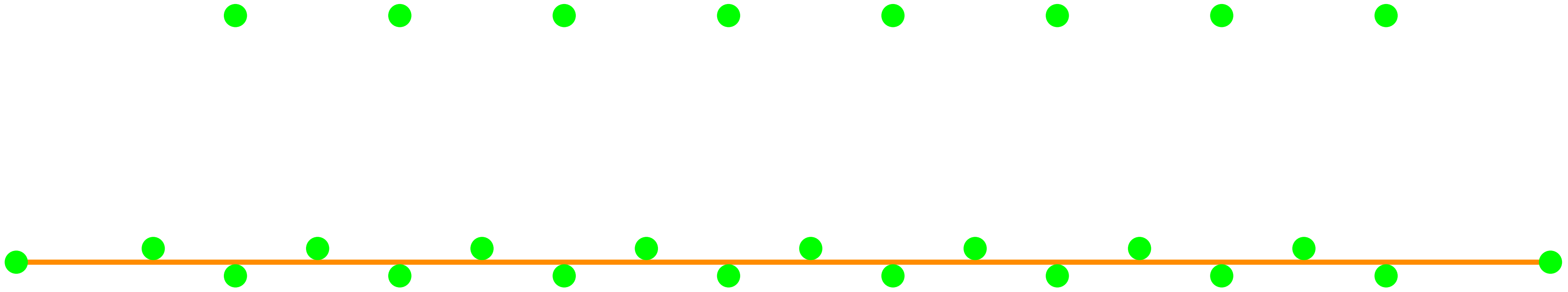
Unbounded



Walking in Delaunay triangulations

Walk between vertices, worst case

Upper path Unbounded

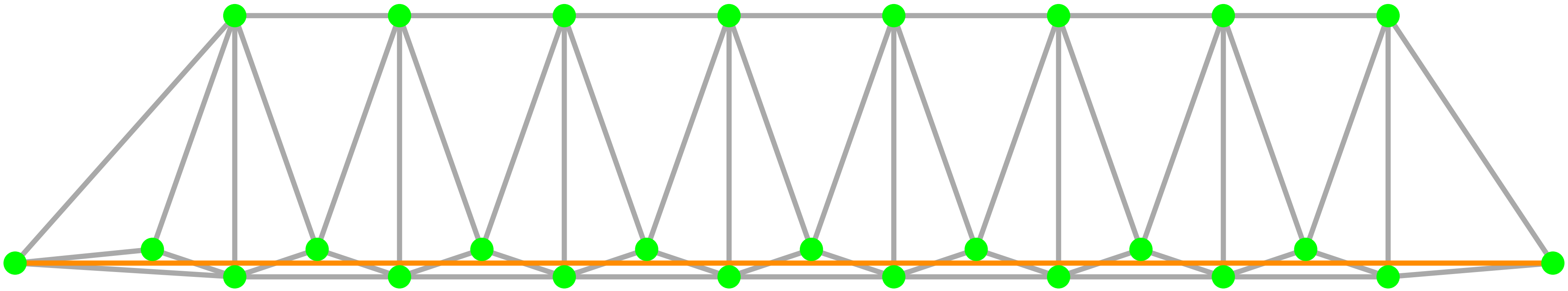


Walking in Delaunay triangulations

Walk between vertices, worst case

Upper path

Unbounded

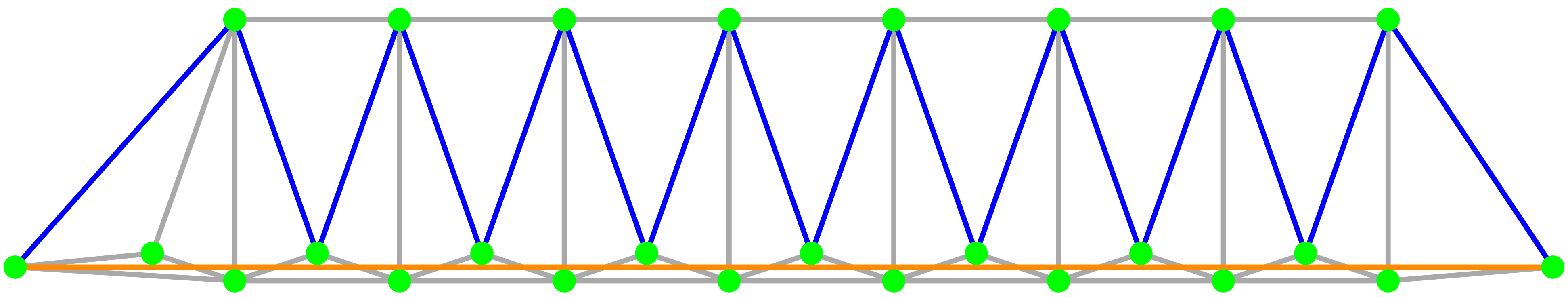


Walking in Delaunay triangulations

Walk between vertices, worst case

Upper path

Unbounded

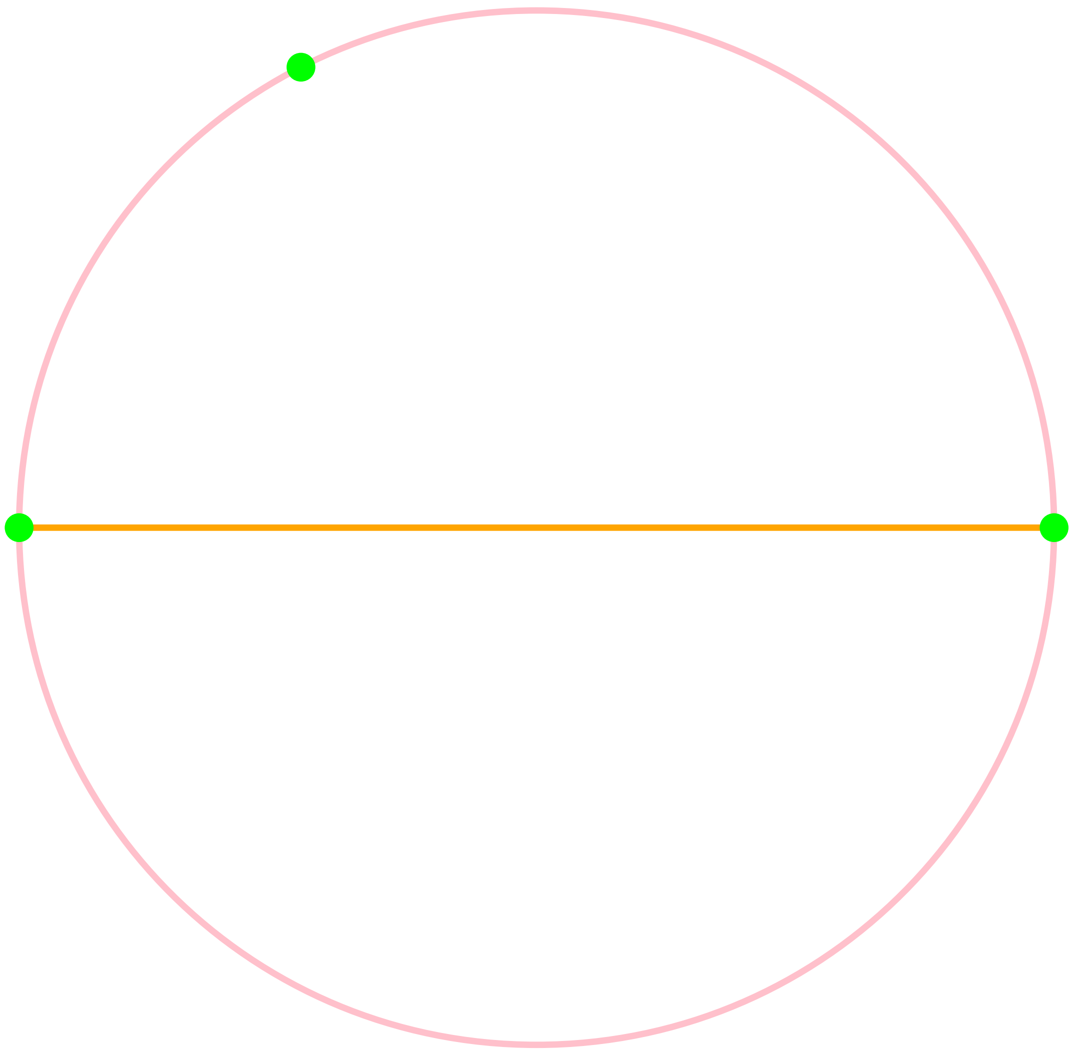


Walking in Delaunay triangulations

Walk between vertices, worst case

Compass walk Unbounded

[Bose & Morin 2004]

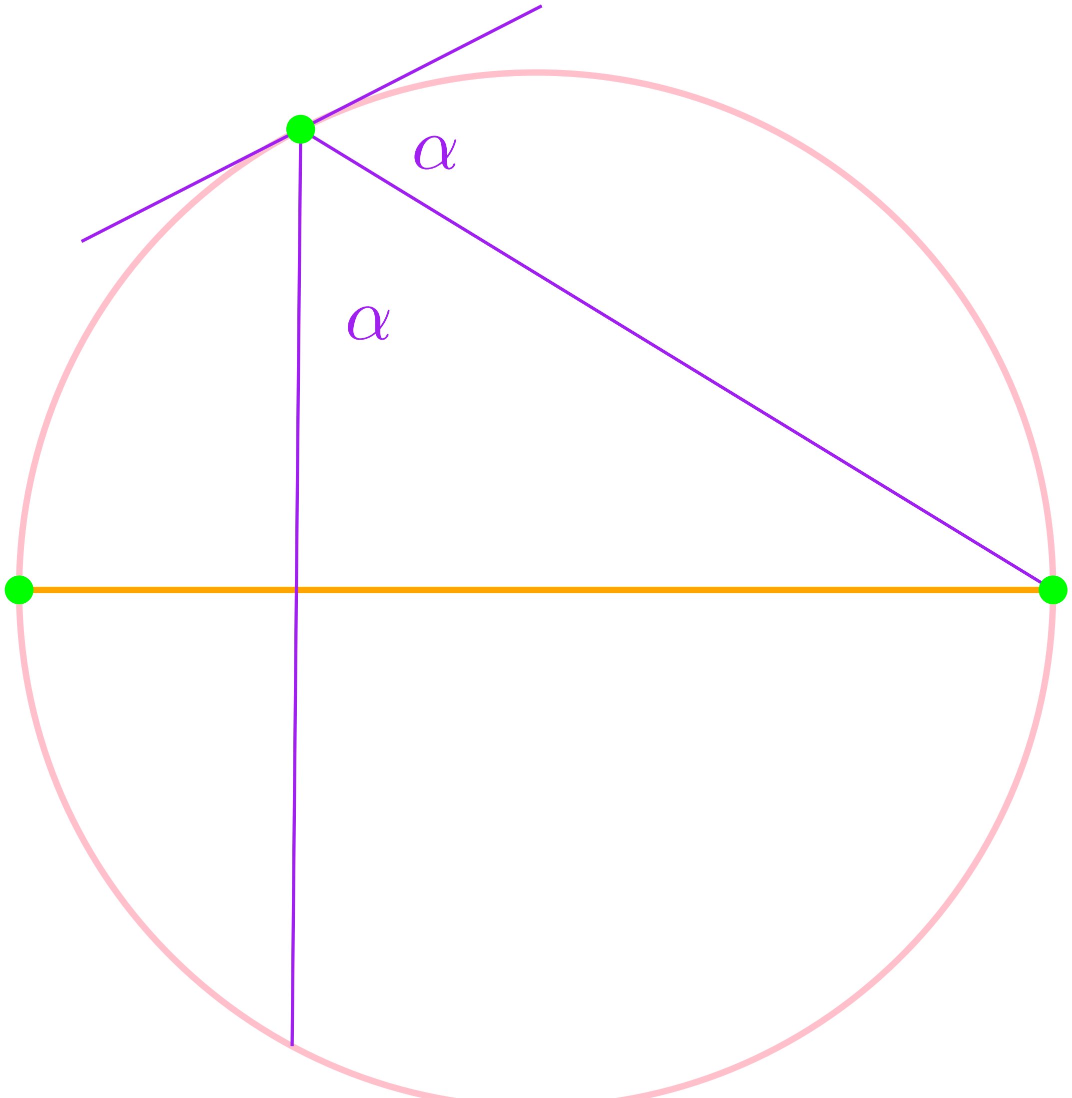


Walking in Delaunay triangulations

Walk between vertices, worst case

Compass walk   Unbounded

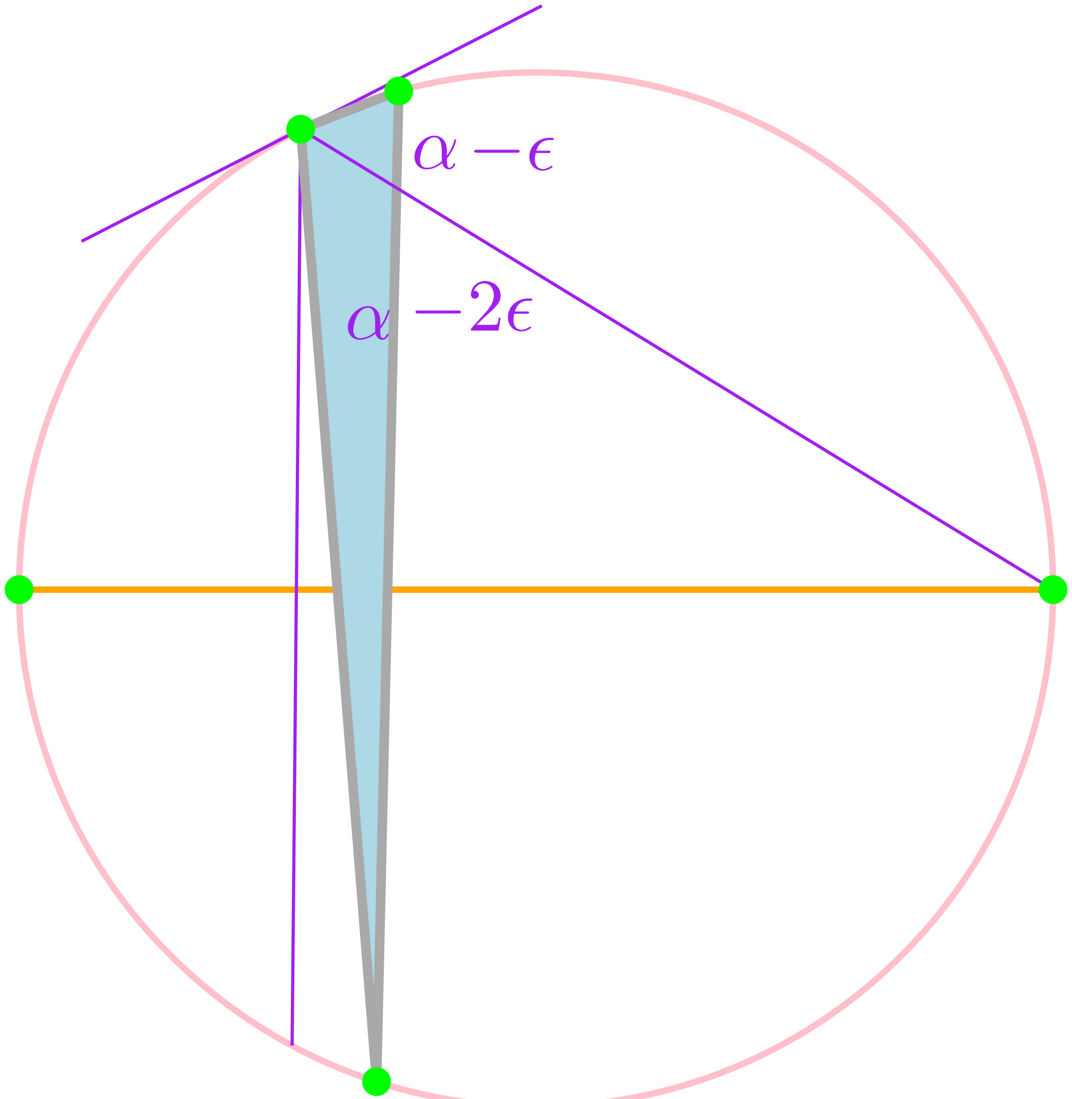
[Bose & Morin 2004]



Walking in Delaunay triangulations

Walk between vertices, worst case

Compass walk Unbounded



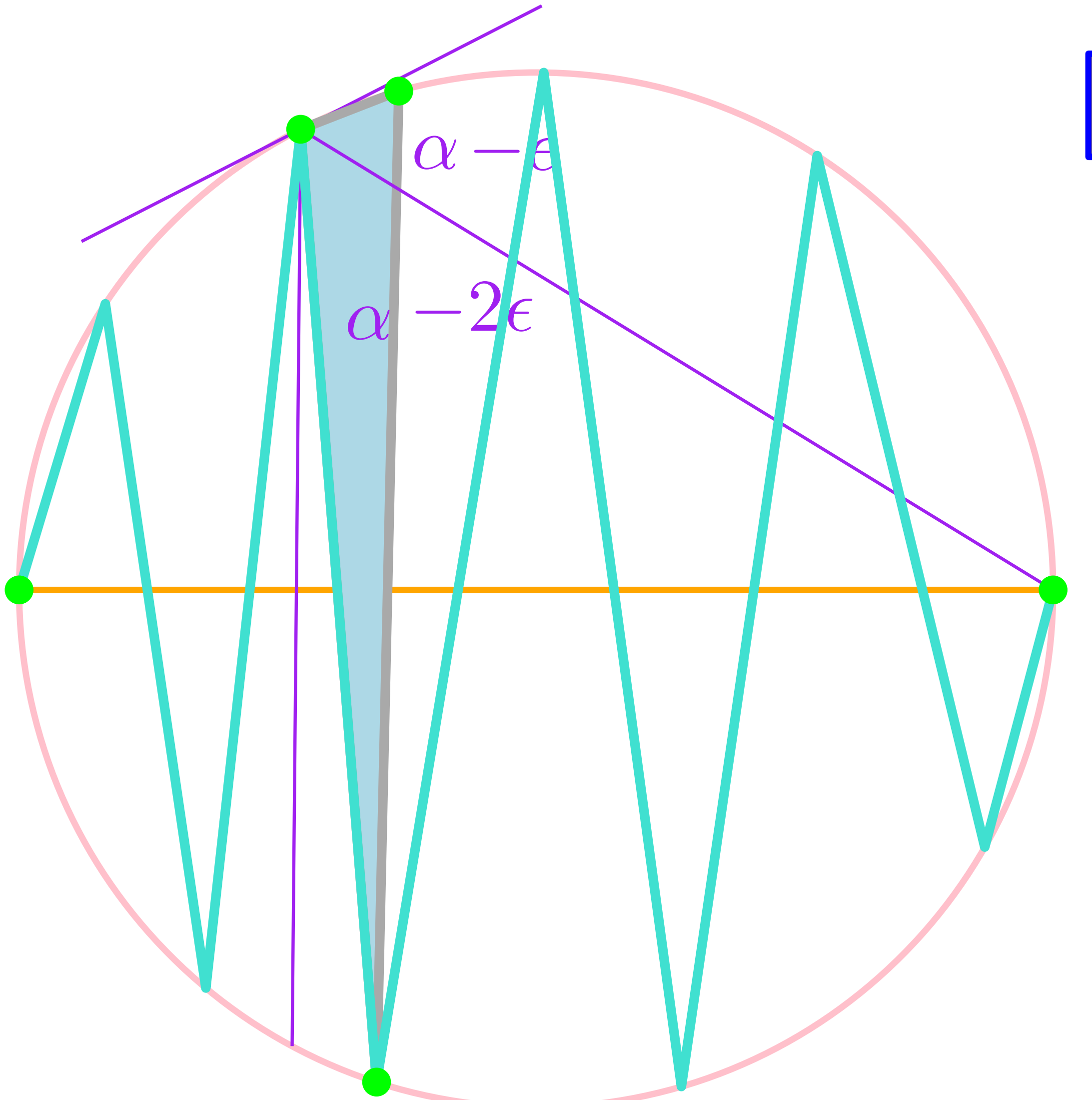
[Bose & Morin 2004]

Walking in Delaunay triangulations

Walk between vertices, worst case

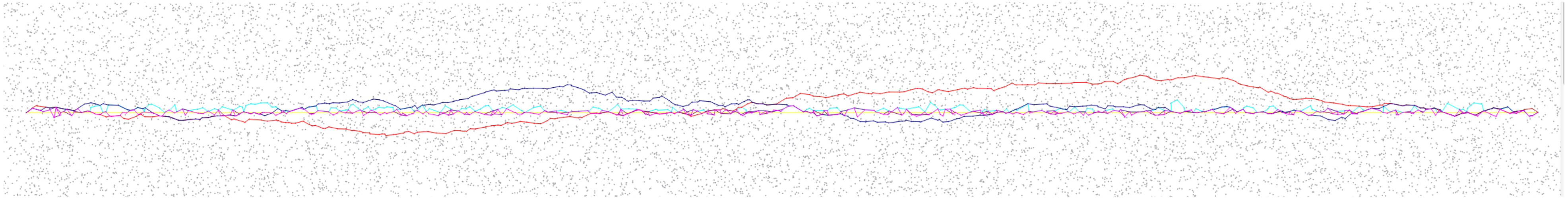
Compass walk　　　　Unbounded

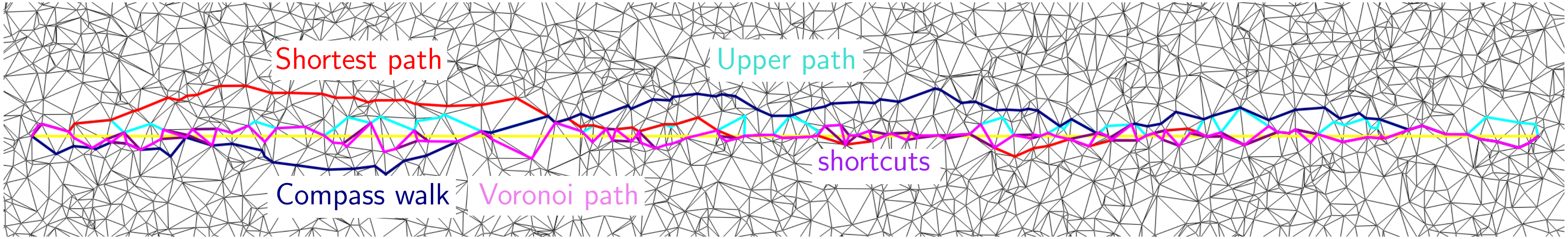
[Bose & Morin 2004]



Walking in Delaunay triangulations

Walk between vertices





Walking in Delaunay triangulations

Expected length (experiments)

Euclidean length	1
Shortest path	1.04
Compass walk	1.07
Shortened V. path	1.16
Upper path	1.18
Voronoi path	1.27

Walking in Delaunay triangulations

Expected length (experiments)		theory
Euclidean length	1	
Shortest path	1.04	$\geq 1+10^{-11}$ [Chenavier & D.,2018]
Compass walk	1.07	
Shortened V. path	1.16	1.16 [D. & Noizet, 2018] numerical integration
Upper path	1.18	$\frac{35}{3\pi^2} \simeq 1.18$ [Chenavier & D.,2018]
Voronoi path	1.27	$\frac{4}{\pi} \simeq 1.27$ [Baccelli et al., 2000]

Walking in Delaunay triangulations

Expected length (experiments)		theory
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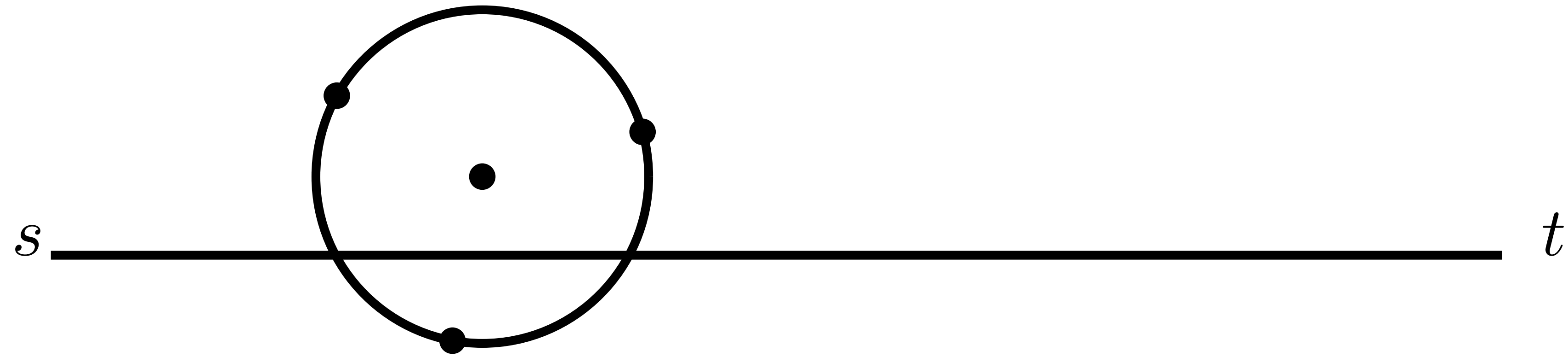
Expected length of upper path

Poisson Delaunay triangulation, rate n

$$\mathbb{E}\left[length\right] = \mathbb{E}\left[\sum_{triangle \in X_n^3} \mathbb{1}_{[triangle\text{ is Delaunay}]}\mathbb{1}_{[\text{first edge above }st]}\text{length(first edge)}\right]$$



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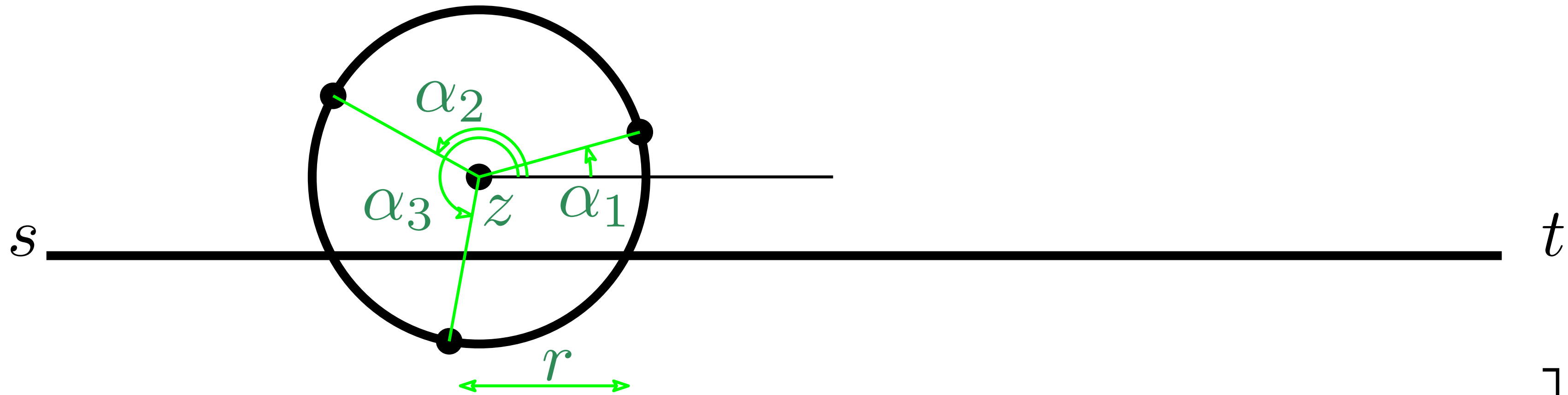
$$= n^3 \int_{(\mathbb{R}^2)^3} \mathbb{P}\left[triangle \text{ is Delaunay}\right] \mathbb{1}_{[\text{first edge above } st]} \text{length(first edge)} \mathrm{d}triangle$$

Slivnyak-Mecke



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$$= n^3 \int_{(\mathbb{R}^2)^3} \mathbb{P}[triangle \text{ is Delaunay}]\, \mathbb{1}_{[\text{first edge above } st]} \text{length(first edge)} \mathrm{d} triangle$$

$$= n^3 \int_{r=0}^{\infty} \int_{x_z=0}^{1} \int_{y_z=-r}^{r} \int_{[0,2\pi]^3} e^{-n\pi r^2} \mathbb{1}_{[\text{first edge above } st]} 2r \sin\frac{\alpha_1 - \alpha_2}{2} r^3 2\mathcal{A}(triangle) \mathrm{d}\alpha_{1:3} \mathrm{d}y_z \mathrm{d}x_z \mathrm{d}r$$

Blaschke-Petkantschin

Expected length of upper path

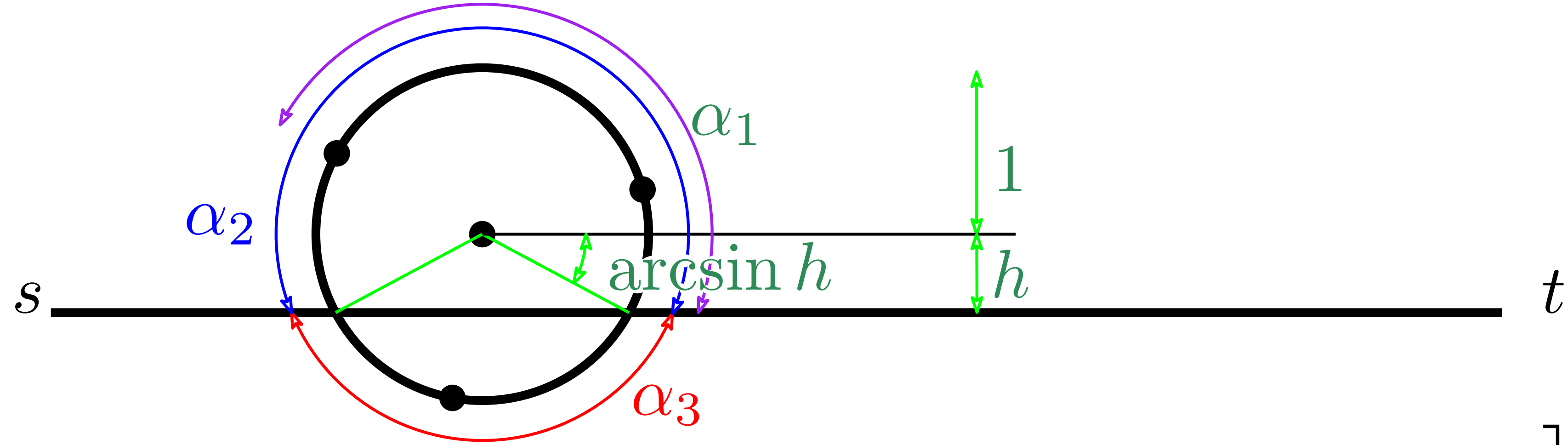
Poisson Delaunay triangulation, rate n

$$\mathbb{E}\left[length\right] = \mathbb{E}\left[\sum_{triangle \in X_n^3} \mathbb{1}_{[triangle \text{ is Delaunay}]} \mathbb{1}_{[\text{first edge above } st]} \text{length(first edge)}\right]$$

$$= n^3 \int_{(\mathbb{R}^2)^3} \mathbb{P}\left[triangle \text{ is Delaunay}\right] \mathbb{1}_{[\text{first edge above } st]} \text{length(first edge)} \mathrm{d}triangle$$

$$= n^3 \int_{r=0}^{\infty} \int_{x_z=0}^{1} \int_{y_z=-r}^{r} \int_{[0,2\pi]^3} e^{-n\pi r^2} \mathbb{1}_{[\text{first edge above } st]} 2r \sin\frac{\alpha_1 - \alpha_2}{2} r^3 2\mathcal{A}(triangle) \mathrm{d}\alpha_{1:3} \mathrm{d}y_z \mathrm{d}x_z \mathrm{d}r$$

$$= 4n^3 \left(\int_{r=0}^{\infty} e^{-n\pi r^2} r^5 \mathrm{d}r\right) \cdot \left(\int_{h=\frac{y_z}{r}=-1}^{1} \int_{[0,2\pi]^3} \mathbb{1}_{[\text{first edge above } st]} \sin\frac{\alpha_1 - \alpha_2}{2} \mathcal{A}(triangle) \mathrm{d}\alpha_{1:3} \mathrm{d}h\right)$$



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$$= 4n^3 \cdot \frac{1}{\pi^3 n^3} \cdot \frac{35\pi}{12} = \frac{35}{3\pi^2}$$

Walking in Delaunay triangulations

Expected length (experiments)		theory
Euclidean length	1	
Shortest path	1.04	$\geq 1 + 10^{-11}$ [Chenavier & D.,2018]
Compass walk	1.07	
Shortened V. path	1.16	1.16 [D. & Noizet, 2018] numerical integration
Upper path	1.18	$\frac{35}{3\pi^2} \simeq 1.18$ [Chenavier & D.,2018]
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Walking in Delaunay triangulations

Shortest path

$$\mathbb{E}\left[length(\text{shortest path})\right]$$

Walking in Delaunay triangulations

Shortest path

$\mathbb{E}\left[length(\text{shortest path})\right]$ $\xrightarrow[\text{density} \to \infty]{}$ limit

Walking in Delaunay triangulations

Shortest path

$\mathbb{E}\left[length(\text{shortest path})\right] \xrightarrow[\text{density} \to \infty]{} \text{limit}$

by subadditivity

remove start and target from point set

look at shortest path between closest neighbor

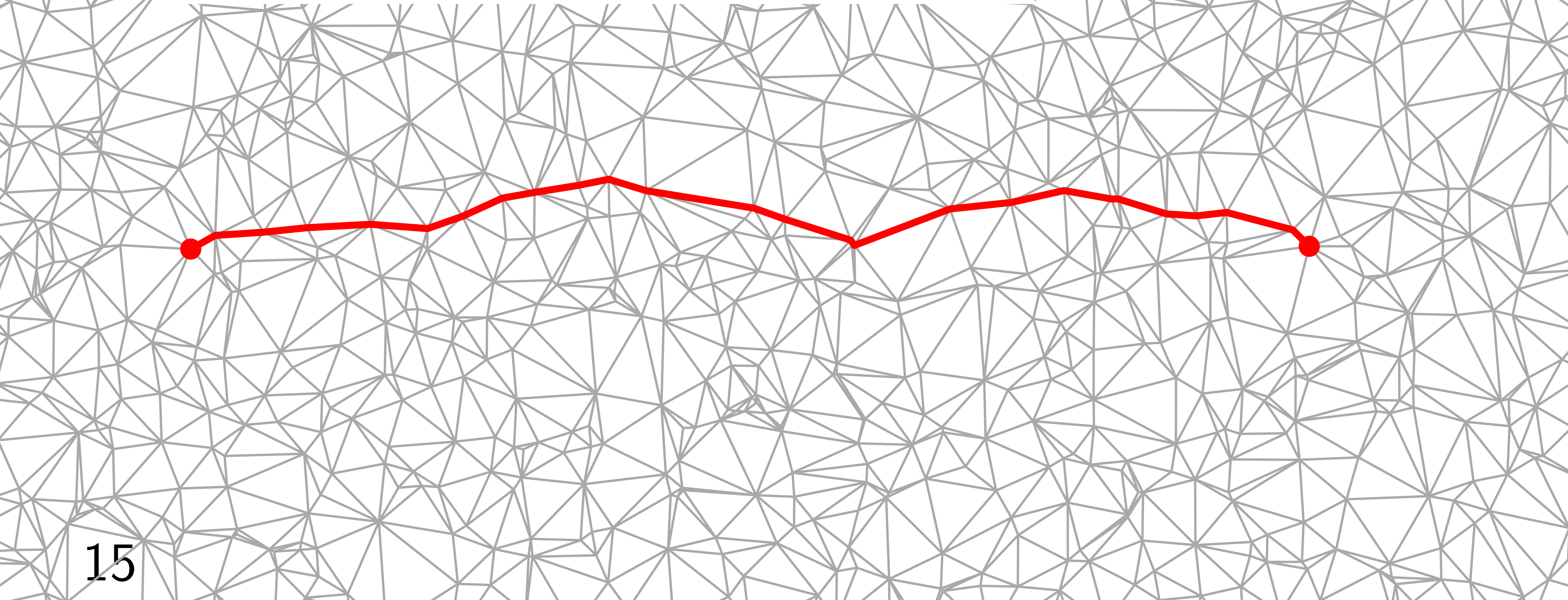
$\neq$ between path from start to target negligible

Bad edge = almost horizontal edge

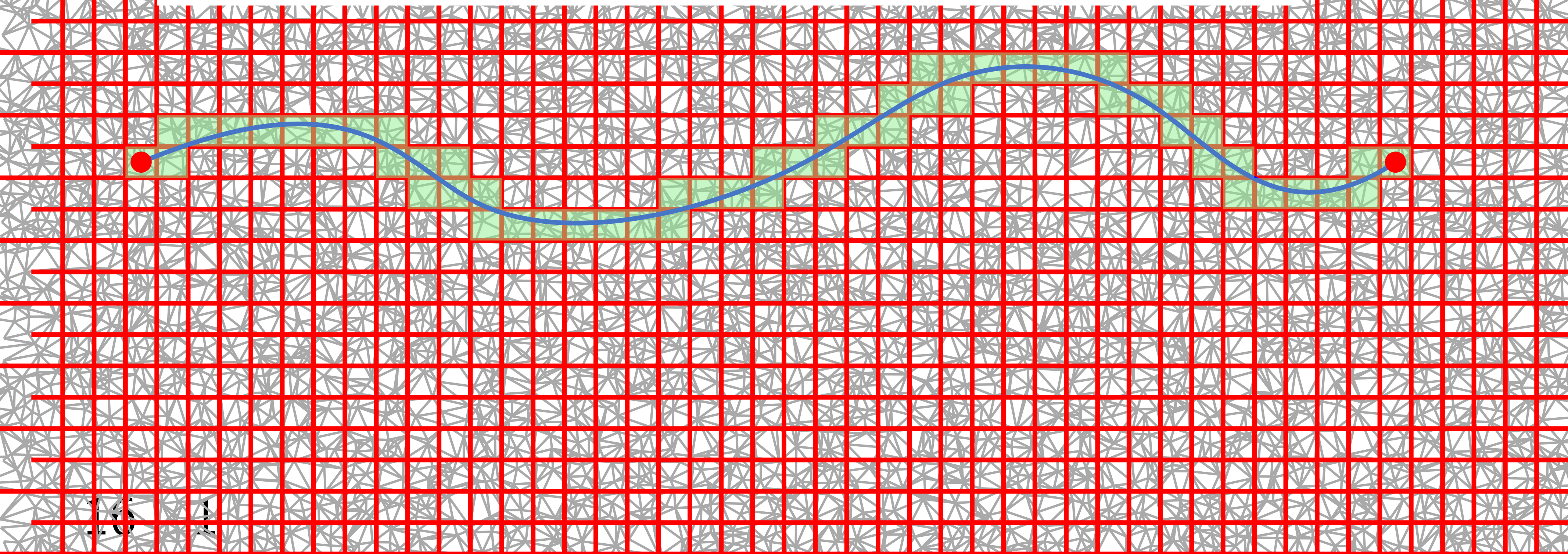
Many bad edges $\Leftarrow$ length close to 1

$\mathbb{P}\left[bad\right]$ = small constant

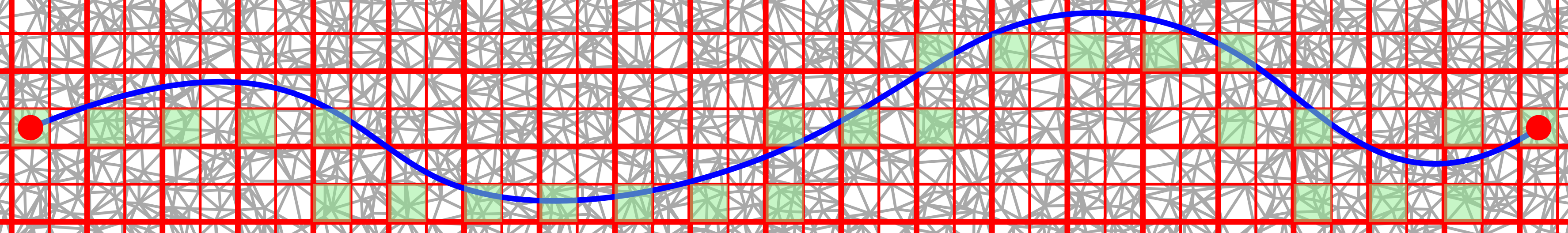
difficult dependencies to handle



Make a grid



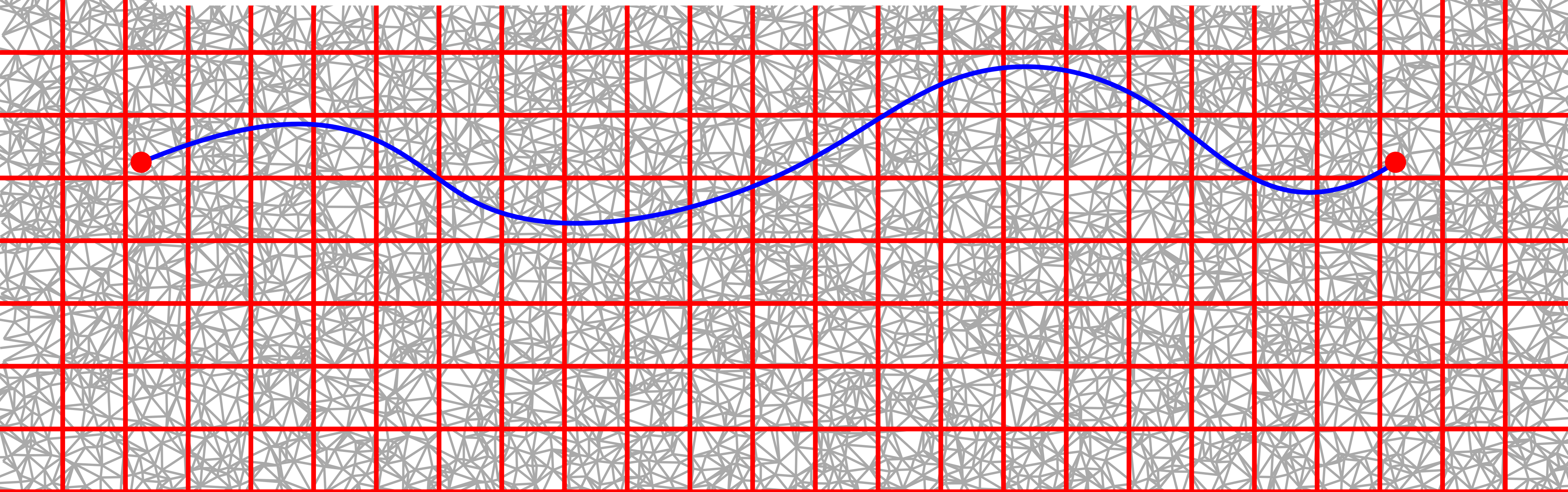
Make a grid



Make a grid

If $\mathbb{E}\left[\sharp \in cell\right] =$ constant

$\sharp$ possible paths $= 4^n$ (too big)

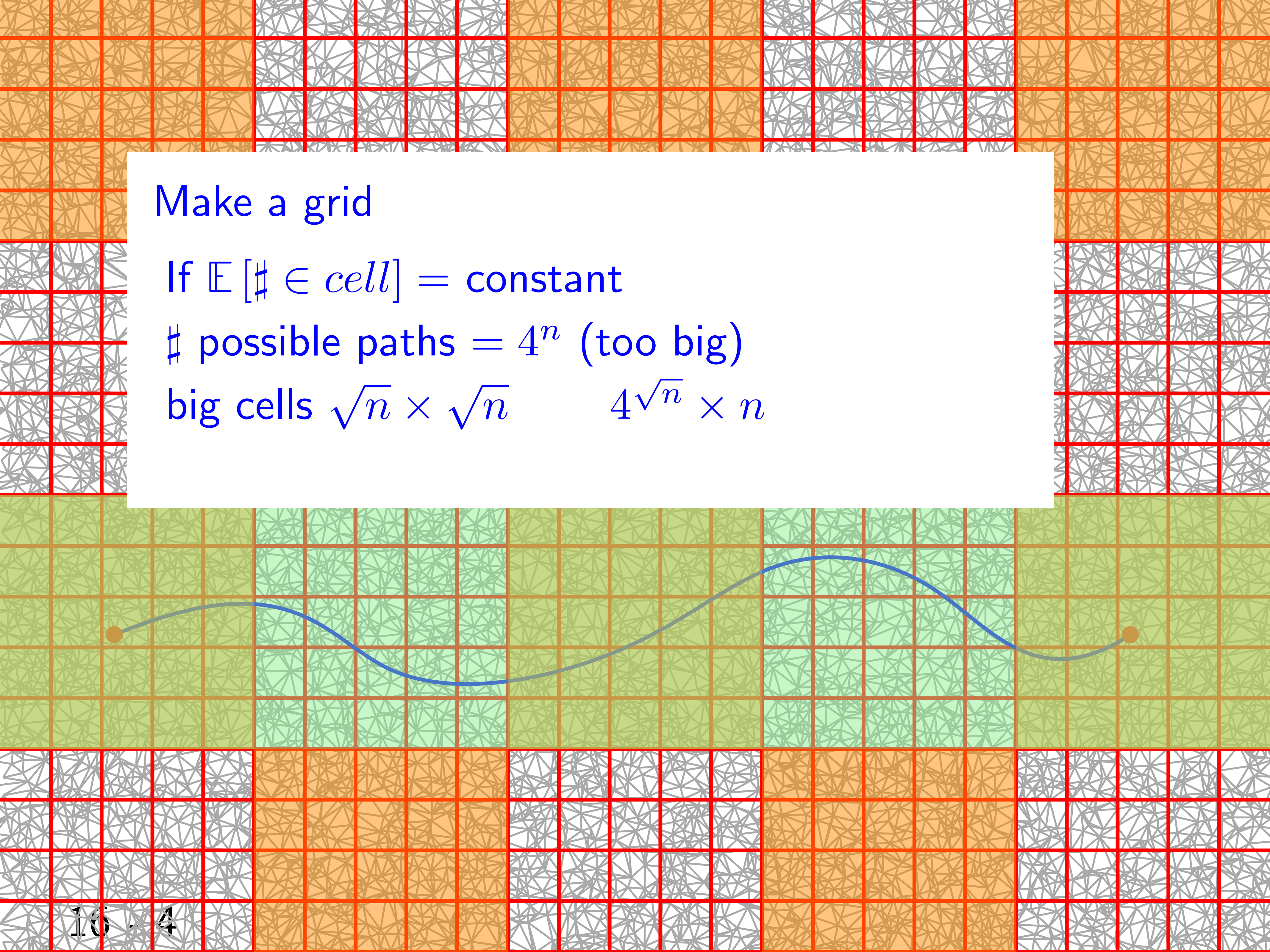


Make a grid

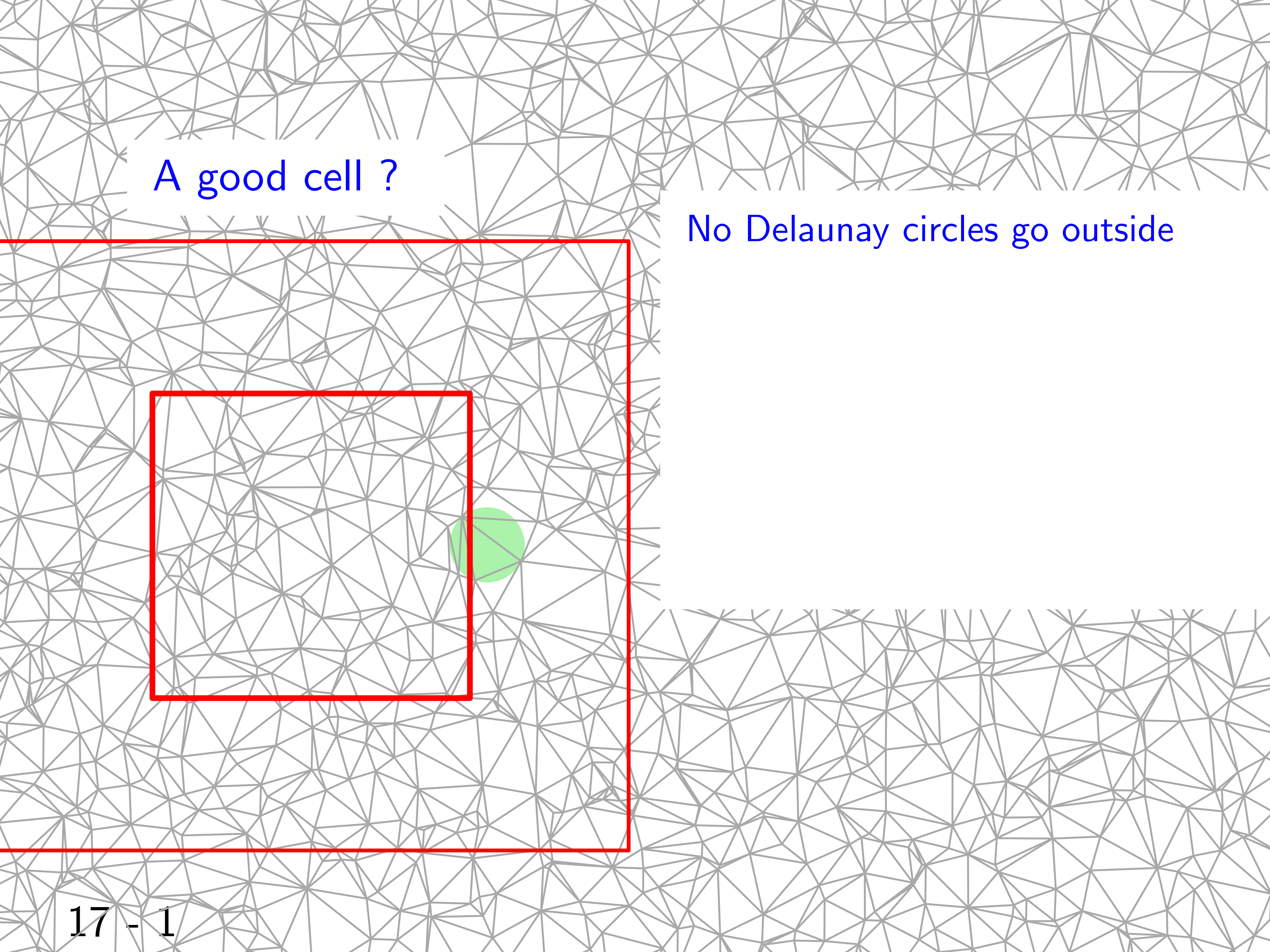
If $\mathbb{E}\left[\sharp \in cell\right] =$ constant

$\sharp$ possible paths $= 4^n$ (too big)

big cells $\sqrt{n} \times \sqrt{n}$ $\qquad 4^{\sqrt{n}} \times n$

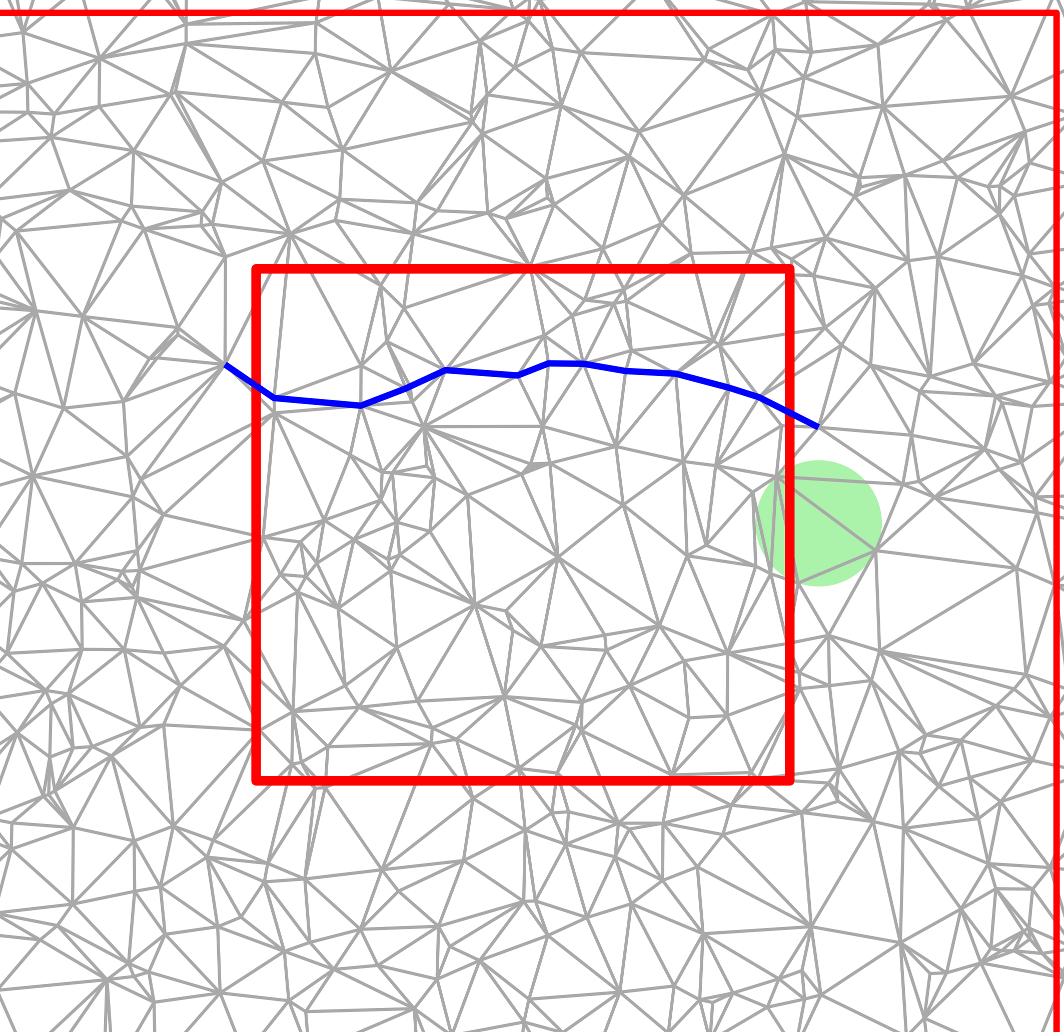


A good cell ?



No Delaunay circles go outside

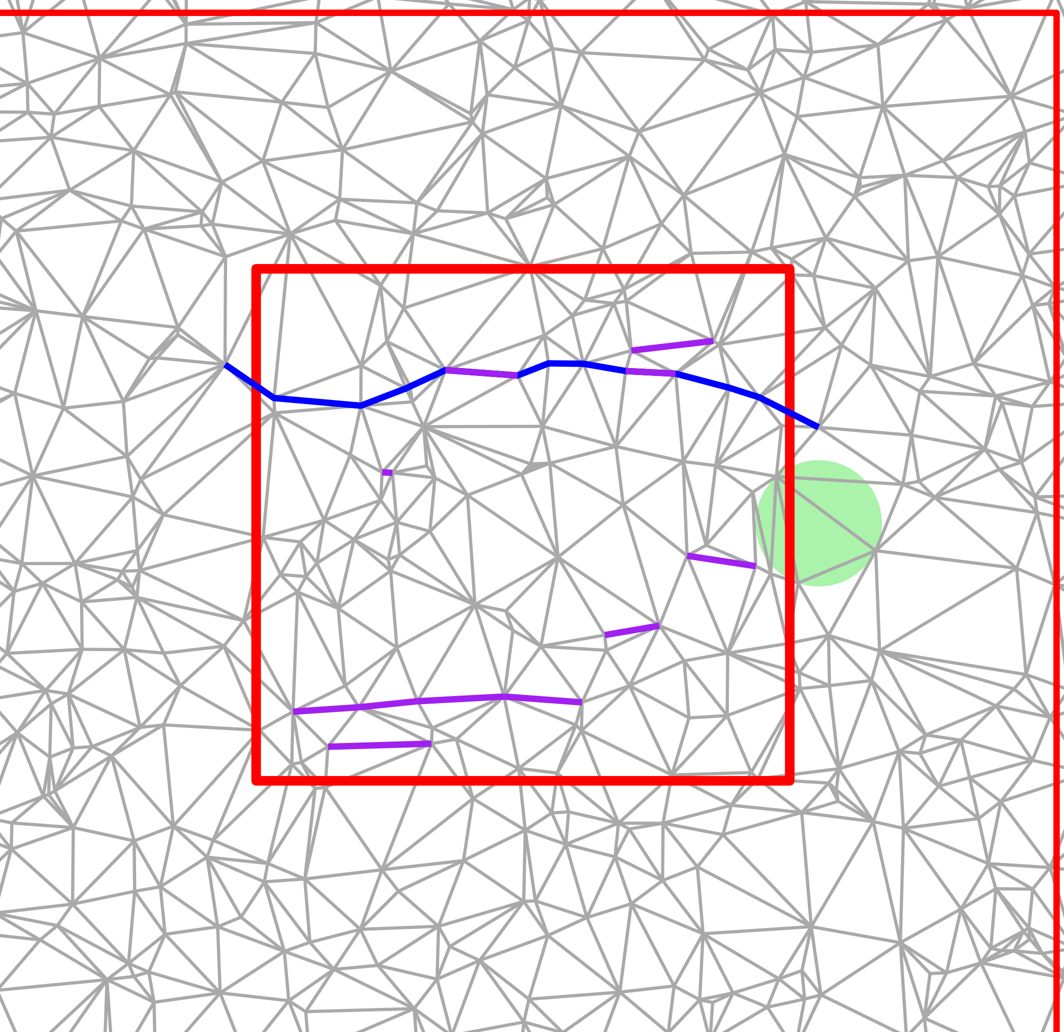
A good cell ?



No Delaunay circles go outside

No short path from left to right

A good cell ?

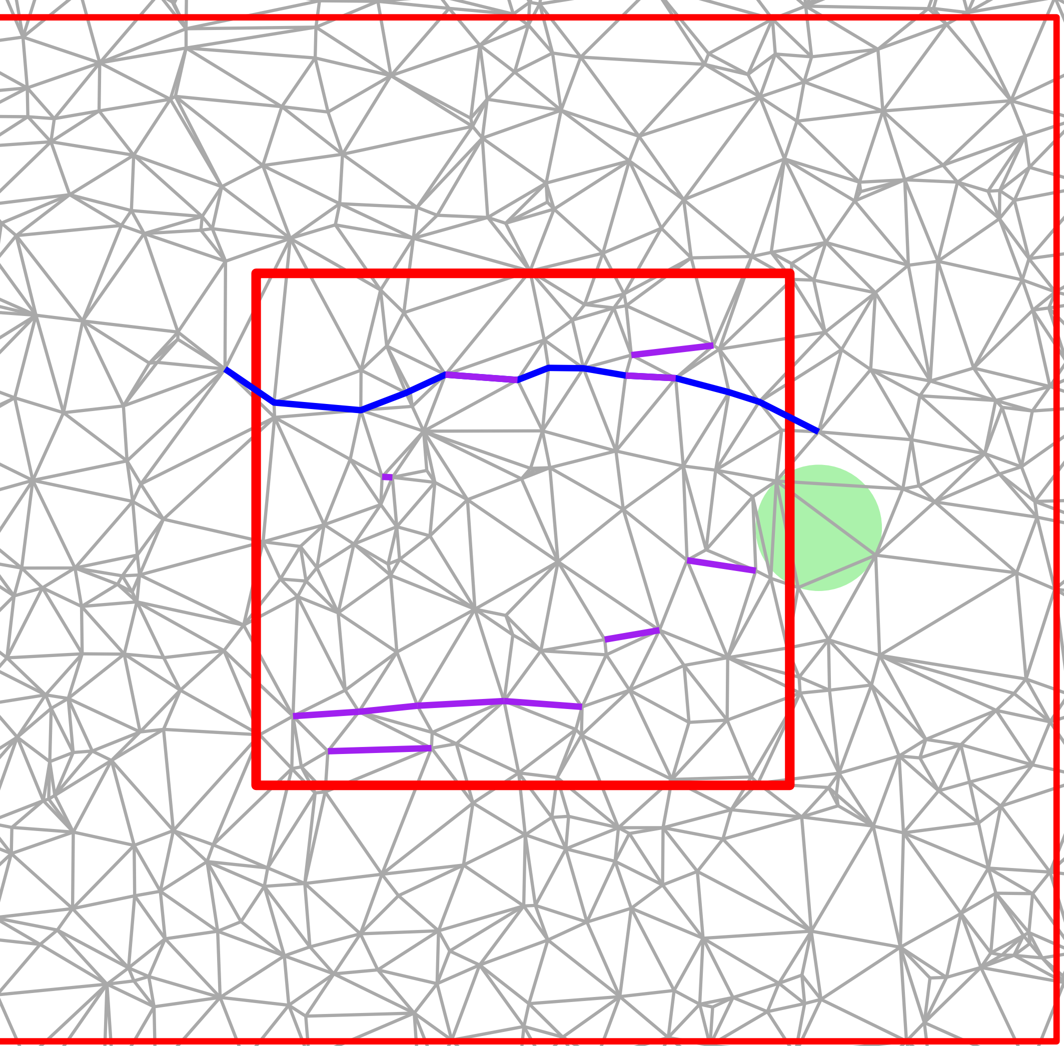


No Delaunay circles go outside

~~No short path from left to right~~

Not enough edges
to make a short path

A good cell ?



No Delaunay circles go outside

~~No short path from left to right~~

Not enough edges
to make a short path

Choose ♯ points in cell, 153

Choose what "short path" means

$1 + 10^{-10}$

$\mathbb{P}\left[length \geq 1 + 2.5 \times 10^{-11}\right] \leq O\left(\frac{1}{\sqrt{n}}\right)$

Walking in Delaunay triangulations

Expected length (experiments)		theory
Euclidean length	1	
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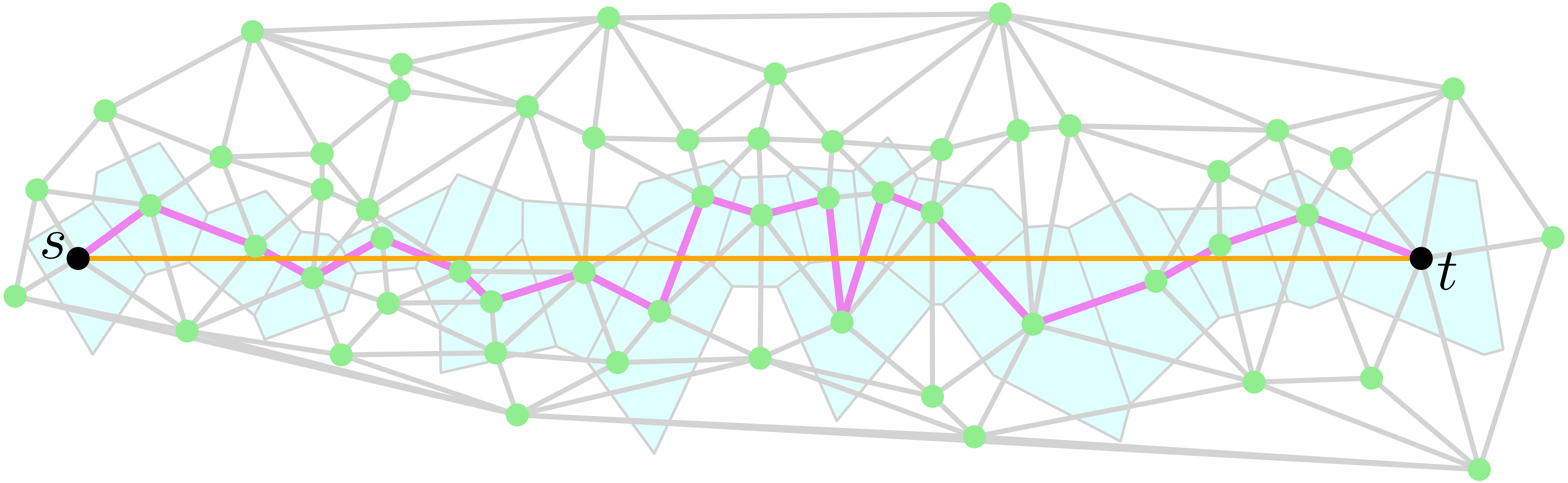
in higher dimension [de Castro & D., 2018]

Walking in Delaunay triangulations

Voronoi path 1.27 $\frac{4}{\pi} \simeq 1.27$ [Baccelli et al., 2000]



Walking in Delaunay triangulations

Voronoi path 1.27 $\frac{4}{\pi} \simeq 1.27$ [Baccelli et al., 2000]

Voronoi path in higher dimension

Walking in Delaunay triangulations

Voronoi path 1.27 $\frac{4}{\pi} \simeq 1.27$ [Baccelli et al., 2000]

$$\mathbb{E}\left[\ell(VP_X)\right] = \frac{1}{2}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{(\mathbb{S}_{d-1})^2} \mathbb{P}\left[B((x,0,\dots,0),r)\cap X = \emptyset\right] \mathbb{1}_{[(x,0,\dots,0)\in[st]]}$$

$$\cdot r\|u_1u_2\|\,|\det(J_\Phi)|\mathrm{d}\alpha_{1,1}\dots\mathrm{d}\alpha_{1,d-1}\mathrm{d}\alpha_{2,1}\dots\mathrm{d}\alpha_{2,d-1}\mathrm{d}r\mathrm{d}x$$

Integral form

Walking in Delaunay triangulations

Voronoi path 1.27 $\frac{4}{\pi} \simeq 1.27$ [Baccelli et al., 2000]

$$\mathbb{E}\left[\ell(VP_X)\right] = \frac{1}{2}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{(\mathbb{S}_{d-1})^2} \mathbb{P}\left[B\left((x,0,\ldots,0),r\right)\cap X=\emptyset\right]\mathbb{1}_{[(x,0,\ldots,0)\in[st]]}$$

$$\cdot r\|u_1 u_2\|\left|\det(J_\Phi)\right| \mathrm{d}\alpha_{1,1}\ldots\mathrm{d}\alpha_{1,d-1}\mathrm{d}\alpha_{2,1}\ldots\mathrm{d}\alpha_{2,d-1}\mathrm{d}r\mathrm{d}x$$

Integral form

Use Taylor expansion to be able to integrate

Walking in Delaunay triangulations

Voronoi path 1.27 $\frac{4}{\pi} \simeq 1.27$ [Baccelli et al., 2000]

$$\frac{\Gamma\left(\frac{d}{2}\right)^4 2^{4d-5} d}{\pi^2 (2d-2)!}\left(1-\frac{d-1}{4d^2-1}\right)\sqrt{2} \leq \mathbb{E}\left[\ell(VP_X)\right] \leq \frac{\Gamma\left(\frac{d}{2}\right)^4 2^{4d-5} d}{\pi^2 (2d-2)!}\left(1+\frac{1}{4d-2}\right)\sqrt{2}$$

Walking in Delaunay triangulations

Voronoi path 1.27 $\frac{4}{\pi} \simeq 1.27$ [Baccelli et al., 2000]

asymptotic behavior between

$$\sqrt{\frac{2d}{\pi}} \quad -\frac{1}{4\sqrt{2d\pi}} + O(d^{\frac{3}{2}})$$

$$+\frac{3}{4\sqrt{2d\pi}} + O(d^{\frac{3}{2}})$$

Walking in Delaunay triangulations

Voronoi path 1.27 $\frac{4}{\pi} \simeq 1.27$ [Baccelli et al., 2000]

asymptotic behavior

$$\sqrt{\frac{2d}{\pi}}$$

Walking in Delaunay triangulations

Voronoi path 1.27 $\frac{4}{\pi} \simeq 1.27$ [Baccelli et al., 2000]

d	k	lower bound	$\simeq$	correct value	upper bound	$\simeq$
3	41	$\frac{788984278470257640690697143}{745000536337515228912680960}\sqrt{2}$	1.49770	1.500	$\frac{452337036471251065807696350}{426448582869060441377603584 0}\sqrt{2}$	1.50007
4	7	$\frac{102494570}{8729721}\frac{\sqrt{2}}{\pi^2}$	1.6823	1.698	$\frac{121774997}{10270260}\frac{\sqrt{2}}{\pi^2}$	1.6990
5	3	$\frac{135}{104}\sqrt{2}$	1.8357	1.875	$\frac{21305}{16016}\sqrt{2}$	1.8812
6	1	$\frac{3014656}{225225}\frac{\sqrt{2}}{\pi^2}$	1.9179	2.04	$\frac{753664}{51975}\frac{\sqrt{2}}{\pi^2}$	2.0778
7	1	$\frac{210}{143}\sqrt{2}$	2.0768	2.2	$\frac{225}{143}\sqrt{2}$	2.2252
8	1	$\frac{2080374784}{134008875}\frac{\sqrt{2}}{\pi^2}$	2.2244	2.3	$\frac{130023424}{7882875}\frac{\sqrt{2}}{\pi^2}$	2.3635

numerical integration

Walking in Delaunay triangulations

Expected length (experiments)		theory
Euclidean length	1	
Shortest path	1.04	$\geq 1 + 10^{-11}$ [Chenavier & D.,2018]
Compass walk	1.07	[Ongoing work]
Shortened V. path	1.16	1.16 numerical integration
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Walking in Delaunay triangulations

Compassl walk

Complicated dependencies

Stretch factor

Walking in Delaunay triangulations

Compass walk

Complicated dependencies

edge lengths

$$\text{Stretch factor } = \frac{\ell_1 + \ell_2 + \ldots \ell_k}{x_1 + x_2 + \ldots x_k}$$

length of horizontal projections

Walking in Delaunay triangulations

Compass walk

Complicated dependencies

$$\text{Stretch factor} = \frac{\ell_1 + \ell_2 + \ldots \ell_k}{x_1 + x_2 + \ldots x_k}$$

If independent: $$\frac{\mathbb{E}\left[\ell\right]}{\mathbb{E}\left[x\right]} = \frac{175\pi}{512} \simeq 1.0738$$

Walking in Delaunay triangulations

Compass walk

Complicated dependencies

Stretch factor $= \dfrac{\ell_1 + \ell_2 + \ldots \ell_k}{x_1 + x_2 + \ldots x_k}$

If independent: $\dfrac{\mathbb{E}[\ell]}{\mathbb{E}[x]} = \dfrac{175\pi}{512} \simeq 1.0738$

Experimental: 1.06777

There is a positive bias

Walking in Delaunay triangulations

Compass walk

Complicated de

Stretch fac

Experimental: $\frac{\mathbb{E}\left[\ell\right]}{\mathbb{E}\left[x\right]} \simeq \frac{1.061}{0.988} \simeq 1.0738$

If independent: $$\frac{\mathbb{E}\left[\ell\right]}{\mathbb{E}\left[x\right]} = \frac{175\pi}{512} \simeq 1.0738$$

Experimental: 1.06777

There is a positive bias

Walking in Delaunay triangulations

Compass walk

Complicated de

Stretch fac

Experimental: $\frac{\mathbb{E}[\ell]}{\mathbb{E}[x]} \simeq \frac{1.061}{0.988} \simeq 1.0738$

at depth 1 $\frac{\mathbb{E}[\ell]}{\mathbb{E}[x]} \simeq \frac{1.116}{1.045} \simeq 1.0681$

at depth 2 $\frac{\mathbb{E}[\ell]}{\mathbb{E}[x]} \simeq \frac{1.117}{1.046} \simeq 1.0678$

If independent: $$\frac{\mathbb{E}[\ell]}{\mathbb{E}[x]} = \frac{175\pi}{512} \simeq 1.0738$$

Experimental: 1.06777

There is a positive bias

Walking in Delaunay triangulations

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Walking in Delaunay triangulations

Expected length (experiments)		theory
Euclidean length	1	[Chenavier & D.,2018]
Shortest path	1.04	$\geq 1 + 10^{-11}$
Compass walk	1.07	
Shortened V. path		
Upper path		
Voronoi path		

Locally defined path

"easy" to analyze

(computation may be difficult)

Walking in Delaunay triangulations

Expected length (experiments) theory

Euclidean length	1	
Shortest path	1.04	$\geq 1 + 10^{-11}$ [Chenavier & D.,2018]
Compass walk		
Shortened V. path		
Upper path		
Voronoi path		

Incrementally defined path

dependency issues

the tree can be analyzed

Walking in Delaunay triangulations

Expected length (experiments) theory

Euclidean length 1 [Chenavier & D.,2018]

Shortest path

Compass walk

Shortened V. path

Upper path

Voronoi path

Incrementally defined path

also dependency issues

no idea about tight bounds

Thank you