MPRI course "Algorithms and combinatorics for geometric graphs" 2026
2nd exercise sheet, Éric Colin de Verdière

Exercise 1. Surface classification.

Identify the surfaces with the following schemata, each made of a single relation. (Note: The simplest method is to determine whether the surface is orientable or not, to compute the number of vertices, edges, and faces, and then to apply the classification theorem.)

1. $a\bar{a}b\bar{b}$;
2. $abab$;
3. $ab a\bar{b}$;
4. $a_1 a_2 \dots a_n \bar{a}_1 \bar{a}_2 \dots \bar{a}_n$;
5. $a_1 a_2 \dots a_{n-1} a_n \bar{a}_1 \bar{a}_2 \dots \bar{a}_{n-1} a_n$.

Exercise 2. Genus of a graph.

Let G be a connected graph that has no loops and no multiple edges. The *genus* of G, denoted $\gamma(G)$, is the smallest integer $g \geq 0$ such that G can be embedded in a surface of genus g. (Here, all surfaces are assumed to be connected, compact, orientable, and without boundary.)

1. Assume that G is embedded on a surface of genus $\gamma(G)$. Is the embedding necessarily cellular? Justify.
2. Assume that G is cellularly embedded on a surface of genus g. Does this always imply $g = \gamma(G)$? Justify.
3. Assume that G is cellularly embedded on a surface of genus g, in such a way that each face is a triangle. Does this always imply $g = \gamma(G)$? Justify.